IEICE Proceeding Series

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Vol. 1 pp. 304-307
Publication Date: 2014/03/17
Online ISSN: 2188-5079

2012 International Symposium on Nonlinear Theory and its Applications
NOLTA2012, Palma, Majorca, Spain, October 22-26, 2012

Graph analysis on simulate hierarchical complex networks dynamic structure

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Abstract—
The brain is a complex structure that can contain up to several billion neurons connected to each other. One possible way to study its structure is to design neural computation algorithms for simulating a simplified mathematical modelisation with several features of the brain and its neurons inspired by their biological features. Due to the large number of neurons and brain dynamics, the mathematical object representing the simulated brain can be considered as a complex network and a study of its dynamic structure is possible by using graph theory [1] to calculate a number of relevant measures [2] that evolve over time in order to observe emergent properties associated to network dynamics. In this context, we have provided some simulations of bio-inspired complex neural network modeled by a hierarchically organized circuit of evolvable neural networks [3]. This model is based on the observations that the vertebrate brain possess several specific areas organized and connected by a hierarchical topology. Neurons are simulated by leaky integrate and fire spiking neurons interconnected by modifiable synapses according to Spike Time Dependent Plasticity (STDP). An other study [4] suggest that the introduction of a feedback connection between two networks hierarchicaly organized can modify their dynamic structure with unexpected differences between the two networks. We studied two hierarchical topologies based upon the feedforward topology with and without recurrent loops. The purpose of this paper is to present new datas on geometrical properties of the networks by means of graph analysis. Some basic graph measures are represented with time dependency considerations by using the R software package "Igraph" [5]. In this simulation, there is no cell death except apoptosis, we chose to present results based on the analysis of different sets of excitatory neurons selected following their level of activity.

1. Introduction

In this article, we studied two topologies, FFHN and FFHL. A sensory network receive all external stimulations. Then, we considered hierarchicaly neural networks made of 9 processing levels. Each level is composed of two neural networks. A network i from the level L_i will project activity on a network j from the level L_j only if $L_j - L_i$ is equal to 0 or 1. In the topology FFHL, networks of the level 9 can project activity on networks of the level 2. The two topologies are shown in the figure 1.

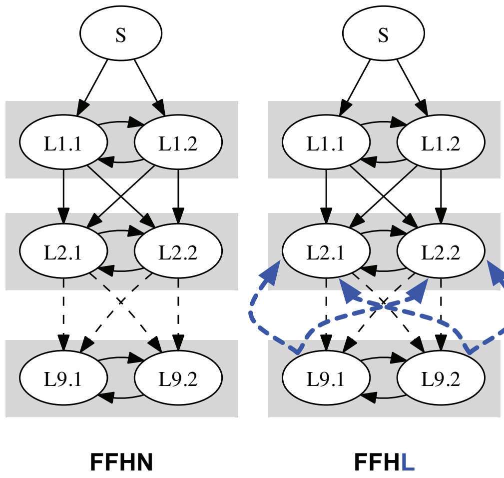

Figure 1: The feedforward topology with (right panel, FFHL) and without (left panel, FFHN) recurrent loops.

2. Simulations

The simulation's parameters were set in order to be bio-inspired. Neurons are either type excitatory or inhibitory and the network is generated so that the proportion of excitatory neurons is close to 80%. The potential of the neuron is increased by excitator neuron spike and decreased by inhibitory neuron spike. If the potential of the neuron exceed the threshold level then the neuron produce a spike to others connected neurons and return to its resting potential. the neuron that has just been fired enters into a refractory period and can't spike during this period. At each time step t, the value of the membrane potential of the i^{th} unit, $V_i(t)$, is calculated [6] such that

$$\begin{aligned} V_i(t+1) &= V_{rest} + B_i(t) \\ &\quad +(1 - S_i(t))\, k_{mem}\, (V_i(t) - V_{rest}) \\ &\quad + \sum_j \omega_{ji}(t) \end{aligned} \tag{1}$$

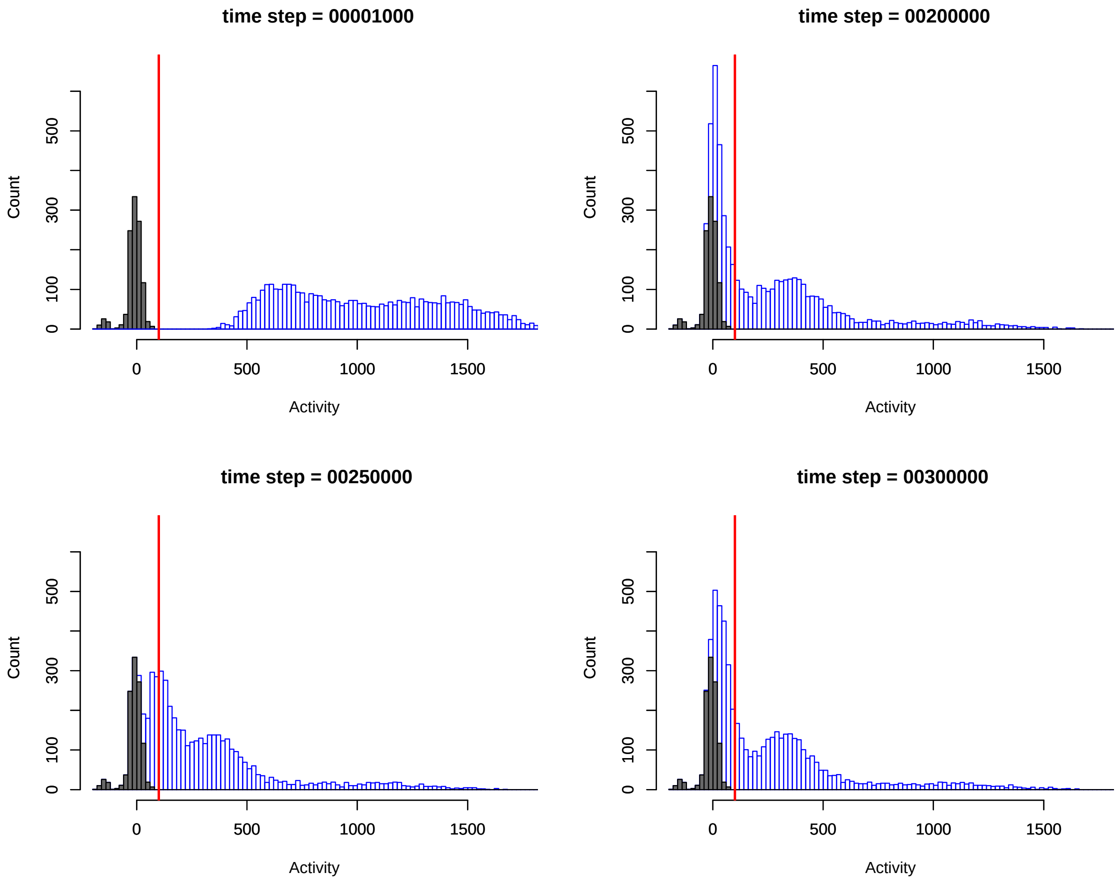

Figure 2: Histogram of neuron's activity at 4 selected moments, #1000 (4521 neurons selected), #200000 (2953 neurons selected), #250000 (3710 neurons selected) and #300000 (2982 neurons selected). The death neurons (killed by apoptosis) are colored in black and the red line is one of the four selection limit used in this study. The activity of a neuron is measured by the potential of the neuron added by the sum of the strengths of edges from other neurons.

where V_{rest} corresponds to the value of the resting potential ($-78mV$ in this simulation), $B_i(t)$ is the background activity arriving to the i^{th} unit, $S_i(t)$ is the state of the i^{th} unit (0 for "not fired" and 1 for "fired"), $k_{mem} = exp\left(\frac{1}{\tau_{mem}}\right)$ is the constant associated to the current of leakage for the units, $w_{ji}(t)$ are the post-synaptic potentials of the j^{th} units projecting to the i^{th} unit. The resting potential of the neurons is $-78mV$ and the threshold level is $-40mV$.

Modeling is done using the simulator described in [6, 7]. Structural files designed specifically for structural analysis are provided at the request of the user. We selected 44 moments during lifetime of the network to explore it's parameters in a first analysis and 92 moments for a second analysis. 1024ts represent 1 second. At the time step #3072, the network receive stimulations for 512ts then there is no stimulation for 1024ts and then this pattern is repeated to the end of the simulation. The simulation has be done with a grid of $75 \times 75 = 5625$ neurons and the spike timing dependent plasticity is implemented as in [8].

3. Graph analysis

The simulated network is represented as a graph. The neurons are viewed as vertices and the neurons projections as edges. The adjacency matrix **A** is a representation of a network G. We have

$$A_{ij} = \begin{cases} 1 & \text{if } e_{ij} \in G \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

where e_{ij} is the edge between the vertex i and j. In this case, the edges are directed and the graph is called asymetric because the adjacency matrix is asymetric.

If a weight is assigned to each edge of the graph, this one is called a weighted graph and elements of the adjacency matrix can be different of the value 1. To measure networks evolution, we calculated, using the R package "igraph", the next variables [9] :

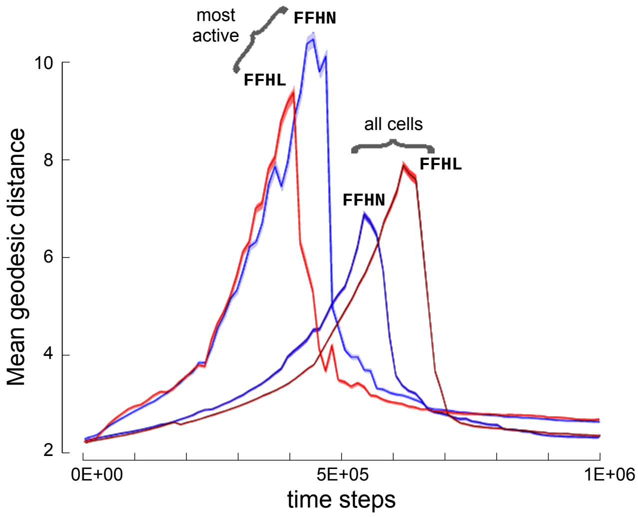

Figure 3: Mean of the geodesic distance for the two topologies and two sets of neurons (*most active* and *all cells*).

- **the average degree :** the degree k_i of a vertex, in a graph G with n vertices, is the number of edges connected to it. We have, in undirected graphs, $k_i = \sum_{j=1}^{n} A_{ij}$ and in directed graphs, $k_i^{in} = \sum_{j=1}^{n} A_{ij}$ and $k_i^{out} = \sum_{j=1}^{n} A_{ji}$.

  Note that we can also see a directed graph as an undirected graph (with a loss of information) to calculate some usefull value so that all measures about the non directed graphs could be relevant to the directed graphs. The average degree is calculated as :

$$\bar{k} = \frac{1}{n}\sum_{i=1}^{n} k_i = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij} \quad (3)$$

- **the mean of the geodesic distance :** the geodesic distance between two vertices in a graph is the number of edges in a shortest path connecting them. There are several algorithms to compute the geodesic distance but the best known is Dijkstra's one [10].

  The mean and the standart deviation of the geodesic distance is calculated with the application of Dijkstra's algorithm on the graph. This algorithm gives, by construction [10], the shortest path for each vertex of the considered graph.

- **the number and the maximum length of strongly connected clusters :** a clustering coefficient is a measure of degree which nodes in a graph tend to cluster together. A directed graph is called strongly connected if there is a path from each vertex in the graph to every other vertex. In particular, this also means a paths in each direction, a path from n_1 to n_2 and a path from n_2 to n_1.

- **the global transitivity :** the global clustering coefficient or the global transitivity of a undirected graph is based on triples. Let be u, v and w three vertices of a undirected gaph. If there is a edge between u and v and between v and w, we can say that u and w has a common neighbor. If the triple is closed, u and w are neigbor thereself and we have a triangle uvw which contain the triples uvw, vwu and wuv. The global clustering coefficient can be defined as the fraction of closed triples, so the division of three times the number of triangles (because they contain three triples) by the number of connected triples :

$$C = \frac{3 * (\text{number of triangles})}{(\text{number of connected triples})} \quad (4)$$

4. Results

The first analysis represent 44 moments from time step #1000 to #350000. Evolution of all measures are plotted as a function of time. We noticed a significant peaks of neural activity at time step #250000. This renewed activity is observed in figure 2. To increase accuracy, we provided other simulations for a second analysis which represent 92 moments from #2560 to #996352. With the additions of different limitations for the selection criteria of the most excitatory neurons, we have found that this peak has no long term impact.

In our simulations, the global transitivity, the number of projections and the average degree did not show emergent behavior and decrease throughout the simulation reaching a stabilization towards the end of the simulation. The measure of maximum length of strongly connected clusters showed that this length tends quickly to 0. Indeed, with decreasing number of projections, the maximum length of strongly connected clusters will also decrease and 0 is reached, for all networks, from around the time step #600000. Indeed, the condition type "strongly connected" is constraining and the complex networks that we have simulated does not keep long strongly connected clusters.

However, the measure of the geodesic distance has shown more interresting behavior in our simulations. As we can see in figure 3, there is a very large peak for all networks. We observe this peak earlier for the most active cells but the largest peak is not alway linked to the same topology (there is an inversion observed in figure 3). In figure 4, we observe that the average degree do not appear to influence the decreasing of the mean of the geodesic distance. We can see an inversion of peak height between the level 2 and 5 for the two topologies.

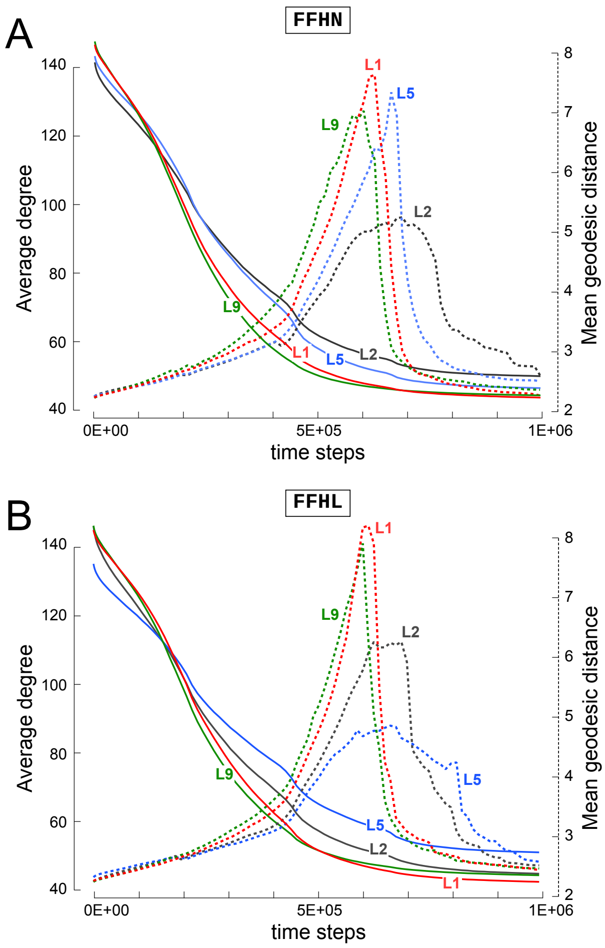

Figure 4: Average degree (solid lines) and Mean of the geodesic distance (dotted lines) as a function of time for level 1, 2, 5 and 9. Each level is represented by a mean of the two networks which composed the level.

5. Conclusion and perspectives

We did this study in order to observe emergent behavior on some basic measures of graph theory. We found an interesting behavior reflected by the mean of the geodesic distance which shows variations that were not expected from other measures. In a first time, the geodesic distance increases with the decreasing of the average degree. Indeed, there are less projections so the mean of the geodesic distance is higher. But, in a second time, the mean of the geodesic distance decreases with the decreasing of the average degree.

One possible explanation for this phenomenon is that the set of neurons that contributes to increase the mean of the geodesic distance has, on average, a low number of connections. They will become completely inactive and they will not influence any more the network, decreasing the mean geodesic distance. This suggests that the network improves its structure and arrives to maturity.

It is clear that this study represent a first step in studies which are targeted at determination of the same measures for other types of bio-inspired simulations. An improve of this study is to explore with greater statistical samples of hierarchically organized circuit simulation and with the development of more sophisticated graph theory measures, particularly in terms of geodesic distance and the detection of clusters according to the weight of the synapses.

References

[1] C. Stam and J. Reijneveld, “Graph theoretical analysis of complex networks in the brain,” *Nonlinear Biomedical Physics*, vol. 1, no. 1, pp. 3+, 2007.

[2] M. Rubinov and O. Sporns, “Complex network measures of brain connectivity: Uses and interpretations,” *NeuroImage*, vol. 52, pp. 1059–1069, Sept. 2010.

[3] O. Chibirova, J. Iglesias, V. Shaposhnyk, and A. Villa, “Dynamics of firing patterns in evolvable hierarchically organized neural networks,” in *Evolvable Systems: From Biology to Hardware* (G. Hornby, L. Sekanina, and P. Haddow, eds.), vol. 5216 of *Lect Notes Comput Sci*, pp. 296–307, Springer Berlin / Heidelberg, 2008.

[4] J. Iglesias, J. García-Ojalvo, and A. E. Villa, “Effect of feedback strength in coupled spiking neural networks,” in *Proceedings of the 18th international conference on Artificial Neural Networks, Part II*, ICANN ’08, (Berlin, Heidelberg), pp. 646–654, Springer-Verlag, 2008.

[5] G. Csardi and T. Nepusz, “The igraph software package for complex network research,” *InterJournal*, vol. Complex Systems, p. 1695, 2006.

[6] J. Iglesias and A. E. P. Villa, “Emergence of preferred firing sequences in large spiking neural networks during simulated neuronal development,” *Int J Neural Syst*, vol. 18, no. 4, pp. 267–277, 2008.

[7] V. Shaposhnyk and A. E. Villa, “Reciprocal projections in hierarchically organized evolvable neural circuits affect eeg-like signals,” *Brain Research*, vol. 1434, no. 0, pp. 266 – 276, 2012.

[8] S. Perrig, J. Iglesias, V. Shaposhnyk, O. Chibirova, P. Dutoit, J. Cabessa, K. Espa-Cervena, L. Pelletier, F. Berger, and A. E. P. Villa, “Functional interactions in hierarchically organized neural networks studied with spatiotemporal firing patterns and phase-coupling frequencies,” *Chin J Physiol*, vol. 53, no. 6, pp. 382–395, 2010.

[9] M. Newman, *Networks : An Introduction*. Oxford University Press, 2010.

[10] E. Dijkstra, *A note on two problems in connexion with graphs*. Numerische Mathematik 1: 269-271, 1959.