THE THEORETICAL TIME COMPLEXITY COST OF ADJACENT CHANNEL INTERFERENCE IN A HEURISTIC ALGORITHM FOR FIXED FREQUENCY ASSIGNMENT

Raúl Chávez Santiago and Vladimir Lyandres
Ben-Gurion University of the Negev
Department of Electrical and Computer Engineering, P.O.B. 653, Beer-Sheva 84105, Israel
E-mail: rachavez@bgumail.bgu.ac.il

1. Introduction

The frequency assignment problem (FAP) in mobile communications systems belongs to the NP-complete combinatorial problems [1], the reason why heuristics are extensively applied to solve it. The FAP has been widely modeled as a binary constraints satisfaction problem (BCSP), and the analogy of this representation with the graph-coloring problem was first averted in [2]. Thus, many heuristics for graph-coloring have been applied to the FAP, being the sequential (greedy) algorithms [3] among the most popular ones. The reasons of this popularity are their programming simplicity, robustness, good running time, and surprisingly good solutions in some cases. Sequential algorithms generally solve the minimum span problem, which consists of finding an assignment that minimizes the difference between the minimum and the maximum used frequencies, denoted as $Sp(\ )$. Several sequential algorithms have been proposed [3]. Based on the results obtained in [4], more sophisticated sequential algorithms as well as their corresponding time complexity analyses were presented in [5]. However, it has been demonstrated in [6] that modeling the FAP as the coloring graph problem with binary constraints might produce solutions that are not necessarily the best ones. The use of nonbinary constraints, such as maximizing the carrier-to-interference ratio (CIR) over all points of the coverage region of a mobile communications system, is an alternative and most effective representation of the FAP. This approach was used in [7] to solve the FAP using a sequential algorithm that takes into account the level of cochannel interference (CCI) to compute a degree of difficulty (DD) that defines an assignment sequence. Unfortunately, this algorithm is not practical for assignments in real systems because of several limitations [8], among which, the fact that it does not take into account the effects of adjacent channel interference (ACI). This paper presents the time complexity analyses of both, the aforementioned algorithm, i.e., considering only CCI, and an improved algorithm that includes ACI [8]. We analyze also the time complexity for practical cases, commonly found in real problems. Towards the end of the paper, some conclusions are elaborated upon.

2. Complexity of the algorithm

2.1 Considering only cochannel interference (CCI)

The algorithm presented in [7] establishes an assignment sequence in descending order according to a DD. Ten different DDs were defined in terms of interference from neighboring cells and the number of frequency requirements per cell, m_i. The most complex DD is given by:

$$\mathrm{DD}_i(10) = \sum_{\substack{j=1 \\ j \neq i}}^{N} x_{ij} m_i \left[\left(\frac{d_{ij}}{R_i} \right)^{-4} + \left(\frac{d_{ji}}{R_j} \right)^{-4} \right], \quad (1)$$

where R_i is the radius of cell i, d_{ij} is the worst-case distance between the interfering cell j and cell i, N is the total number of TXs in the system (we assume that there is only one TX per cell), and x_{ij} is an influence factor whose value is 1 if the cells i and j use the same frequency simultaneously, and 0 otherwise.

The algorithm arranges the TXs in descending order in terms of their DD; this way of defining the assignment sequence is known as *node degree ordering* [4]. The assignment process basically consists of a *requirement-exhaustive* strategy [5], [7]. The first frequency is taken and assigned to the

ISBN: 89-86522-78-0 94460©KEES

first TX in the assignment sequence, i.e., the TX that has the largest DD, after which, all the rest TXs are inspected to determine if the frequency can be reused. Only if a CIR constraint is satisfied, the frequency reuse is allowed. Such constraint is given by:

$$\sum_{j \in b_i} \left(\frac{P_j d_{ij}}{P_i R_i} \right)^{-4} \leq g \quad , \qquad (2)$$

where b_i is the set of all the cells (excluding cell i) that use the same frequency as cell i, P_i is the transmitting power of the TX antenna in cell i, and g is a CIR threshold that depends on the type of service provided [7]. After all the cells have been inspected, the assignment sequence is reordered and then the second channel is considered. This process is repeated iteratively until all the frequency requirements have been satisfied.

Let $M = \sum_{i=1}^{N} m_i$ be the total number of frequency requirements in the system, and consider that *insertion sort* [9], with time complexity $\Theta(N^2)$, is used for node degree ordering. Thus, the frequency assignment algorithm can be written in pseudo-code and analyzed as follows:

	cost	*times*
1. `for` i=1 to N `do`	c_1	N
$m_i^{'} = m_i$;		
`end for`		
2. $f = 1$;	c_2	1
3. `for` i=1 to N `do`	c_3	MN
`if` $m_i^{'} = 0$ `then` $\mathrm{DD}_i = 0$		
`else` apply (1) or (2);		
4. arrange TXs using node degree ordering;	c_4	$M\Theta(N^2)$
5. find i the first TX in the list such that the assignment of frequency f to TX i satisfies (3);	c_5	$M\Theta(N^2)$
6. `if` cell i is found `do`	c_6	MN
`if` $\mathrm{DD}_i \neq 0$ `then`		
$m_i^{'} = m_i^{'} - 1$, $k = m_i - m_i^{'}$,		
$f_{ik} = f$; `goto` step 3		
`else` $Sp(\ ) = \max_{i,k} f_{ik}$;		
`Exit`		
7. `else` $f = f + 1$;	c_7	M
`goto` step 4		

where c_1, c_2, …, c_7 are the cost of their corresponding step, i.e., the number of operations needed to complete it, which might vary for different implementations of the algorithm. The total running time of the algorithm is the sum of running times for each step executed, and it can be reduced to:

$$T_{\mathrm{CCI}}^{\mathrm{total}}(M, N) = O(MN^2 + MN). \qquad (3)$$

This result is in agreement with the analyses in [5]. The time complexity in step 5 was obtained by computing the number of comparisons needed in every cycle, which results in the arithmetic series $1 + 2 + ... + (N-1) = \sum_{i=1}^{N-1} i = N(N-1)/2 = \Theta(N^2)$. If *merge sort* [9], with complexity $\Theta(N \log_2 N)$, is used instead of insertion sort, (4) can be rewritten as:

$$T_{\mathrm{CCI}}^{\mathrm{total}}(M, N) = O(MN^2 + MN \log_2 N + MN). \qquad (4)$$

Fig. 1(a) depicts (3); a similar behavior is observed for (4). In both (3) and (4), M appears as a linear coefficient, thus we can conclude that in this case the faster growth of complexity is due to the number of TXs rather than the number of frequency requirements in the system.

2.2 Considering CCI and adjacent channel interference (ACI)

When both CCI and ACI are considered in the algorithm, some modifications should be introduced. First, the CIR constraint given by (2) must be replaced by:

$$\sum_{j\in b_i} \frac{P_j d_{ij}^{-4}}{P_i R_i^{-4}} + \sum_{g=1}^{n} adj\_factor_g \sum_{k\in b_{i,g}} \left(\frac{P_k d_{ik}^{-4}}{P_i R_i^{-4}} \right) \leq g \ , \tag{5}$$

where $b_{i,g}$ is the set of all the cells (excluding cell i) that use an adjacent channel with an integer spectrum separation g from a channel used in cell i, n is the maximum number of spectrum separation channels to be considered for the computation of ACI, and adj_factor_g is the filter attenuation on an adjacent channel, given by [6]:

$$adj\_factor_g = -a(1+\log_2 g) \quad \forall g > 0 \,, \tag{6}$$

where a is an attenuation constant, typically 18 dB.

Theoretically, all the adjacent channels contribute to the deterioration of the CIR for a given frequency, thus in the worst case, n will be equal to the total number of channels already assigned to a cell at a given iteration of the algorithm. Taking this into account, the number of comparisons required in step 5 of the algorithm result in the arithmetic series $\sum_{i=1}^{M-1} i = M(M-1)/2$, which allows computing the exact running time of step 5 considering both CCI and ACI as $T_{\mathrm{CCI,ACI}}^{\mathrm{step\,5}}(M,N) = c_5 MN(M-1)(N-1)/4$, which compared to the exact running time of step 5 with only CCI, $T_{\mathrm{CCI}}^{\mathrm{step\,5}}(M,N) = c_5 MN(N-1)/2$, leads to the expression of the time complexity cost of ACI in the algorithm, given by:

$$C_{\mathrm{ACI}}(M,N) = T_{\mathrm{CCI,ACI}}^{\mathrm{step\,5}}(M,N) - T_{\mathrm{CCI}}^{\mathrm{step\,5}}(M,N) = c_5 MN(M-3)(N-1)/4 \,. \tag{7}$$

Taking into account that $M(M-1)/2 = \Theta(M^2)$ the growth of the time complexity function of the algorithm considering CCI and ACI with insertion sort is given by:

$$T_{\mathrm{CCI,ACI}}^{\mathrm{total}}(M,N) = O(M^2 N^2 + MN), \tag{8}$$

which is depicted in Fig. 1(b). The time complexity when merge sort is used can be obtained in a straightforward way, whose behavior is similar to that of (8). Comparing Fig. 1(a) and 1(b), we can notice the significant increase of complexity when ACI is taken into account. Moreover, the fast growth of complexity is equally caused by the number of TXs and the number of frequency requirements.

3. Practical considerations

The previous result might deter the incorporation of ACI models into heuristic algorithms for FAP due to the significant increase of complexity that this represents. However, computer simulations in [8] showed that for practical purposes, it is enough to take into account the effect of the first adjacent channels ($g=1$) to obtain reliable assignments that might guarantee interference-free operational environments, due to the very low interference power that further adjacent channels ($g \geq 2$) might contribute, as follows from (7). Nevertheless, for cellular layouts with very high intercell overlapping, second adjacent channels ($g = 2$) might have a slightly noticeable effect. Taking this into account, one might wish to express the time complexity also as a function of the parameter n in (6) that gives the maximum level of ACI considered. Thus, for the algorithm with insertion sort we obtain:

$$T_{\mathrm{CCI,ACI}}^{\mathrm{total}}(M,N,n) = O\left(\frac{1}{2}(2M-n-1)nN^2 + MN \right). \tag{9}$$

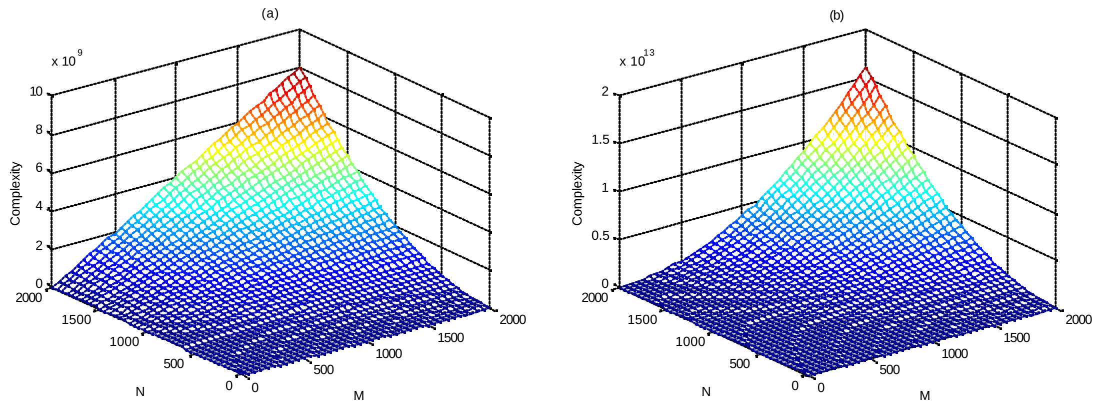

Fig. 1. Time complexity of the sequential algorithm: (a) considering only CCI, and (b) considering CCI and ACI.

Notice that (9) is valid only for $0 < n \le M$, and for $n = 0$ the time complexity must be computed using (3). By using (9) we can estimate the time complexity of the algorithm for practical situations. Numerical calculations for $n = 2$ have shown that, although the complexity of the algorithm increases, its order of magnitude is similar to that of the case when only CCI is taken into account, which means that the problem is perfectly tractable. Furthermore, it must be considered that our analyses were done considering the worst-case assignment, where every frequency requirement is assigned a different channel (no frequency reuse). Moreover, the complexity of the sorting algorithm for node degree ordering (step 4) might vary depending on how much the sequence of assignment is sorted for a given iteration. This fact is closely related to the distribution of frequency requirements in the cells, which indirectly is a function of the spatial traffic distribution in the system's coverage area.

4. Conclusions

We have presented the theoretical upper bounds of time complexity of a heuristic algorithm for fixed frequency assignment. We observed that the order of complexity of such algorithm is determined by the number of base stations in the system, and it is independent of the sort technique used for creating the assignment sequence. Theoretically, the complexity cost of adjacent channel interference in the algorithm is very high, and is determined by the number of channel requirements; but in practical cases, this cost is not big enough to make the problem intractable. Therefore, adjacent channel interference should not be neglected in the implementation of heuristics for frequency assignment.

5. References

[1] W. K. Hale, "Frequency assignment: theory and applications," Proc. IEEE, vol. 68, no. 12, pp. 1497–1514, December 1980.

[2] B. H. Metzger, "Spectrum management technique," presented at 38th ORSA National Meeting, Detroit, MI, Fall 1970.

[3] W. K. Hale, "New spectrum management tools," in Proc. IEEE Int. Symp. Electromagn. Compat., Piscataway, NJ, pp. 47–53, 1981.

[4] J. A. Zoellner and C. L. Beall, "A breakthrough in spectrum conserving frequency assignment technology," IEEE Trans. Electromagn. Compat., vol. EMC-19, no. 3, pp. 313–319, August 1977.

[5] K. L. Yeung and T.–S. P. Yum, "Fixed channel assignment optimization for cellular mobile networks," IEICE Trans. Commun., vol. E83-B, no. 8, pp. 1783–1791, August 2000.

[6] N. W. Dunkin and S. M. Allen, "Frequency assignment problems: representations and solutions," Univ. London, UK, Tech. Rep. CSD–TR–97–14, June 1997.

[7] J.–L. C. Wu and L.–Y. Wey, "Channel assignment for cellular mobile networks with nonuniform cells," IEE Proc. Commun., vol. 145, pp. 451–456, December 1998.

[8] R. Chávez, E. Gigi and V. Lyandres, "An improved heuristic algorithm for frequency assignment in nonhomogeneous cellular mobile networks," presented at IEEE 60th Veh. Technol. Conf., VTC 2004/Fall, Los Angeles, CA, September 26–29, 2004.

[9] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein, Introduction to Algorithms, 2nd ed. MIT Press: Cambridge, MA, 2001, pp. 15–61.