ELECTROMAGNETIC THEORY OF FOUR–PORT (CRUCIFORM) WAVEGUIDE JUNCTION WITH A CILINDRICAL ROD

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The four–port (cruciform) H–plane waveguide junction containing perfectly conducting cylindrical rod in the main arm is considered. The geometry of a problem is presented in Fig.1. *a* and *b* are the widths of wide sides of main and side waveguide arms, respectively, *c* is the radius of cylindrical rod, Δ is its separation distance from the left wall of the arm, ℓ is its separation distance from the juncture and *L* is its distance from the plane $z = L$. An incident wave in the main arm is defined by its electric component:

$$E_y^{inc} = \sin\sigma_1(x+\Delta)\, e^{-ih_1 z}, \qquad (-\Delta \le x \le (a-\Delta) \le 0), \tag{1}$$

where: $h_1 = \sqrt{k_1^2 - \sigma_1^2}$, $k = 2\pi/\lambda$, $\sigma_1 = \pi/a$, $\operatorname{Im} h_1 < 0$. The total field in areas 1, 2, 3, 4 and 7 could be written as:

$$E_{y1} = E_y^{inc} + \sum_{m=1}^{\infty}(A_m^- + B_m e^{-ih_m\ell})e^{ih_m z}\sin\sigma_m(x+\Delta), \quad z \le -c, \quad (-\Delta \le x \le (a-\Delta)), \tag{2}$$

$$E_{y2} = E_y^{inc} + E_y^{sc} + \sum_{m=1}^{\infty} B_m \sin\sigma_m(x+\Delta)e^{ih_m(z-\ell)}, \qquad -c \le z \le c, \quad (-\Delta \le x \le (a-\Delta)), \tag{3}$$

$$E_{y3} = E_y^{inc} + \sum_{m=1}^{\infty}\left[A_m^+ e^{-ih_m z} + B_m e^{ih_m(z-\ell)}\right]\sin\sigma_m(x+\Delta), \quad c \le z \le \ell, \quad (-\Delta \le x \le (a-\Delta)), \tag{4}$$

$$E_{y4} = \int_{-\infty}^{\infty} dt \left\{ D(t)\,\mathrm{sh}\left[\sqrt{t^2-k^2}\,(L-z)\right] + \tilde{D}(t)\,\mathrm{ch}\left[\sqrt{t^2-k^2}\,(L-z)\right]\right\} e^{itx}, \quad \ell \le z \le L, \quad -\infty < x < \infty, \tag{5}$$

$$E_y^{sc} = \sum_{\nu=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} X_m\left[H_m^{(2)}(kr_\nu)e^{im\varphi_\nu} - H_m^{(2)}(k\rho_\nu)e^{im\psi_\nu}\right], \quad r_\nu \ge c, \quad \rho_\nu \ge c, \tag{6}$$

where $r_\nu = \sqrt{z^2 + (x-d\nu)^2}$, $\rho_\nu = \sqrt{z^2 + (x - d\nu - 2\Delta)^2}$, $(d = 2a)$, $\varphi_\nu = \operatorname{arctg}\left(\frac{x-d\nu}{z}\right)$, $\psi_\nu = \operatorname{arctg}\left(\frac{x-d\nu-2\Delta}{z}\right)$ are local cylindrical coordinates of real and mirror cylinders [1,2], H_m^2 is m-th kind Hankel function of second order.

$$E_{y7} = \sum_{m=1}^{\infty} N_m \sin\sigma_m(x+\Delta)e^{-ih_m(z-L)}, \qquad z \ge L, \quad (-\Delta \le x \le (a-\Delta)), \tag{7}$$

where: $h_m = \sqrt{k^2 - \sigma_m^2}$, $\sigma_m = m\pi/a$, $\operatorname{Im} h < 0$. The unknown coefficients A^+, A^-, B_m, $D(t)$, $\tilde{D}(t)$ and N_m could be determined applying the following boundary conditions:

$E_{y2} = 0$, $(r_0 = c,\ 0 \le \varphi \le 2\pi)$, $E_{y1} = E_{y2}$, $(z = -c,\ -\Delta \le x \le a - \Delta)$,

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$$E_{y1}=E_{y2},\ (z=-c,\ -\Delta\le x\le a-\Delta),\quad E_{y2}=E_{y3},\ (z=c,\ -\Delta\le x\le a-\Delta),$$

$$E_{y4}=\begin{cases}E_{y3}, & -\Delta\le x\le a-\Delta\\ 0, & x\le-\Delta,\ \ x\ge a-\Delta\end{cases},\quad z=\ell,\qquad \frac{\partial E_{y4}}{\partial z}=\frac{\partial E_{y3}}{\partial z},\quad -\Delta\le x\le a-\Delta,\quad z=\ell,$$

$$E_{y4}=\begin{cases}E_{y7}, & -\Delta\le x\le a-\Delta\\ 0, & x\le-\Delta,\ x\ge a-\Delta\end{cases},\quad z=L,\qquad \frac{\partial E_{y4}}{\partial z}=\frac{\partial E_{y7}}{\partial z},\quad -\Delta\le x\le a-\Delta,\quad z=L. \tag{8}$$

Using projection method (or moments method) the set of equations (8) leads to the following system of algebraic equations for the unknown coefficients X_m, B_p and N_p:

$$X_n+\sum_{m=-\infty}^{\infty}P_{nm}X_m+\sum_{p=1}^{\infty}r_{np}B_p=a_n, \tag{9}$$

$$-B_\mu+\sum_{m=-\infty}^{\infty}\tilde g_{\mu m}X_m+\sum_{p=1}^{\infty}(\tilde Q_{\mu p}B_p+\tilde Q'_{\mu p}N_p)=b_\mu, \tag{10}$$

$$-N_\mu+\sum_{m=-\infty}^{\infty}\tilde g'_{\mu m}X_m+\sum_{p=1}^{\infty}(\tilde Q'_{\mu p}B_p+\tilde Q_{\mu p}N_p)=c_\mu, \tag{11}$$

$$(n=0,\pm1,\pm2,...;\quad \mu=1,2,3,..)$$

where: $a_n=-\,i^{-n}\sin(\sigma_1\Delta+n\xi_1)\,\dfrac{J_n(kc)}{H_n^{(2)}(kc)}$, $\quad b_\mu=-(\delta_{\mu1}+\tilde Q_{\mu1})e^{-ih_1\ell}$, $\quad c_\mu=-\tilde Q'_{\mu1}e^{-ih_1\ell}$,

$$P_{nm}=\frac{J_n(kc)}{H_n^{(2)}(kc)}\left[-i^{n-m}H_{n+m}^{(2)}(2k\Delta)+Z_{n-m}(kd,0)-(-1)^m Z_{n+m}(kd,2k\Delta)\right],$$

$$r_{np}=\frac{J_n(kc)}{H_n^{(2)}(kc)}e^{-ih_p\ell}\begin{cases}i^n\sin(\sigma_p\Delta-n\xi_p), & p<[D]\\ \dfrac{1}{2i}\left[e^{i\sigma_p\Delta+n\xi'_p}-(-1)^n e^{-(i\sigma_p\Delta+n\xi'_p)}\right], & p>[D]\end{cases}$$

$$\tilde g_{\mu m}=g'_{\mu m}+\sum_{p=1}^{\infty}g'_{pm}\tilde Q_{\mu p},\qquad \tilde g'_{\mu m}=\sum_{p=1}^{\infty}g'_{pm}\tilde Q'_{\mu p},\qquad g'_{\mu m}=\frac{4e^{-ih_\mu\ell}}{h_\mu a}g_{\mu m},$$

$$g_{\mu m}=\begin{cases}i^m\sin(\sigma_\mu\Delta+m\xi_\mu), & p<[D]\\ \dfrac{1}{2i}\left[(-1)^m e^{i\sigma_\mu\Delta-m\xi'_\mu}-e^{-(i\sigma_\mu\Delta-m\xi'_\mu)}\right], & p>[D]\end{cases}$$

$$\xi_p=\mathrm{Arc\,tg}\left(\frac{p}{\sqrt{D^2-p^2}}\right),\quad \xi'_p=\mathrm{Arth}\left(\frac{\sqrt{D^2-p^2}}{p}\right),\quad h_\mu=\sqrt{k^2-\sigma_\mu^2},$$

$$\tilde Q_{\mu p}=-2Q_{\mu p}/(ih_\mu a),\quad \tilde Q'_{\mu p}=2Q'_{\mu p}/(ih_\mu a),\quad h'_\mu=\sqrt{k^2-(\mu\pi/b)^2},\ \zeta_p=\sqrt{\sigma_p^2-k^2}$$

$$Q_{\mu p}==\sigma_p\left\{\frac{\zeta_p}{\sigma_p\,th(\zeta_p b)}\frac{a}{2}\delta_{\mu p}-i\frac{(-1)^n\sigma_n}{b}\sum_{s=1}^{\infty}\frac{(h_s'^2-k_3^2)\left[(-1)^\mu-e^{-ih'_s a}\right]}{h'_s(h_s'^2-\sigma_\mu^2)(h_s'^2-\sigma_p^2)}\left[(-1)^\mu+(-1)^p\right]\right\},$$

$$Q'_{\mu p}=\sigma_p\left\{\frac{\zeta_p}{\sigma_p\,th(\zeta_p b)}\frac{a}{2}\delta_{\mu p}-i\frac{\sigma_n}{b}\sum_{s=1}^{\infty}\frac{(h_s'^2-k_3^2)\left[(-1)^\mu-e^{-ih'_s a}\right]}{h'_s(h_s'^2-\sigma_\mu^2)(h_s'^2-\sigma_p^2)}\left[(-1)^\mu+(-1)^p\right]\right\},$$

The relation between the modal coefficients $A_p^{\pm}$ and multipole coefficient X_m is found:

$$A_p^{\pm} = \frac{4}{ah_p} \sum_{m=-\infty}^{\infty} \begin{cases} i^{\pm m} \sin(\sigma_p \Delta \pm \mathrm{arctg}\xi_p) X_m, & p<[D] \\ \frac{1}{2i}\left[(-1)^m e^{i\sigma_p\Delta \mp m\xi'_p} - e^{-i\sigma_p\Delta \pm m\xi'_p}\right] X_m, & p<[D] \end{cases} .$$

The power balance equation in cruciform waveguide junction is expressed as:

$$\sum_{m=1}^{M} (\hat{P}_{1m} + \hat{P}_{7m}) + \sum_{n=1}^{N} (\hat{P}_{5n} + \hat{P}_{6n}) = 1 \ ,$$

where $\hat{P}_{1m} = \frac{h_m}{h_1} | A_m |^2$ and $\hat{P}_{7m} = \frac{h_m}{h_1} | N_m |^2$ are normalized powers in the main and transmitted arms; $\hat{P}_{5n}$ and $\hat{P}_{6n}$ are the same powers in side arms, while M and N are the numbers of propagating modes in the main and side waveguide arms, respectively.

In the case of dielectric rod, the field inside the rod is expressed as $E_y^{ins} = \sum_{m=-\infty}^{\infty} Y_m J_m(kr) e^{im\varphi}$, $0 \le r \le c$, $0 \le \varphi \le 2\pi$, where Y_m are the new unknowns to be found. The relation between the Y_m and X_m is:

$$Y_m = \left\{ \frac{2i}{\pi kc} \left[J'_n(kc) J_m(k'c) - \frac{J_n(kc) J'_n(k'c)}{W} \right] \right\} X_m \ ,$$

where $k' = k\sqrt{\varepsilon / \varepsilon_0}$, ε is the permittivity of the rod, $\varepsilon_0 = (1/36\pi) \cdot 10^{-9}\, F/m$, $W = \sqrt{\varepsilon_0 / \varepsilon}$.

The solution of boundary problem leads to the same algebraic system as (9), (10), (11), but with the following replacements:

$$\frac{J_n(kc)}{H_n^{(2)}(kc)} \to \frac{J_n(kc) J'_n(k'c) - W J'_n(kc) J_n(k'c)}{H_n^{(2)}(kc) J'_n(k'c) - W H_n^{(2)\prime}(kc) J_n(k'c)} \ .$$

Fig. 2 illustrates the dependence of transmitted and reflected powers on the parameter $2a/\lambda$ at $\varepsilon/\varepsilon_0 = 3.8$, $2c/a = 0.425$, $\ell/a = 0.36$, $b/a = 1$ and $\Delta/a = 0.5$. From this figure it follows that in the frequency range $1.30 \le (2a/\lambda) \le 1.85$ action regime of four–port waveguide junction is optimal, as far as the value of reflected power is very low. Fig. 3 has been constructed for the following parameters: $d/a = 0.20$, $\ell/a = 0.31$, $2a/\ell = 1.796$.

References

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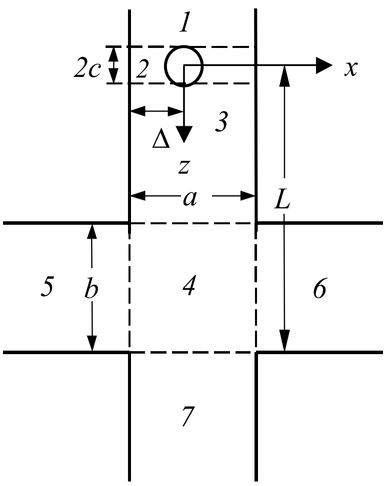

Fig. 1. Cruciform waveguide junction with a cylindrical rod in the main arm

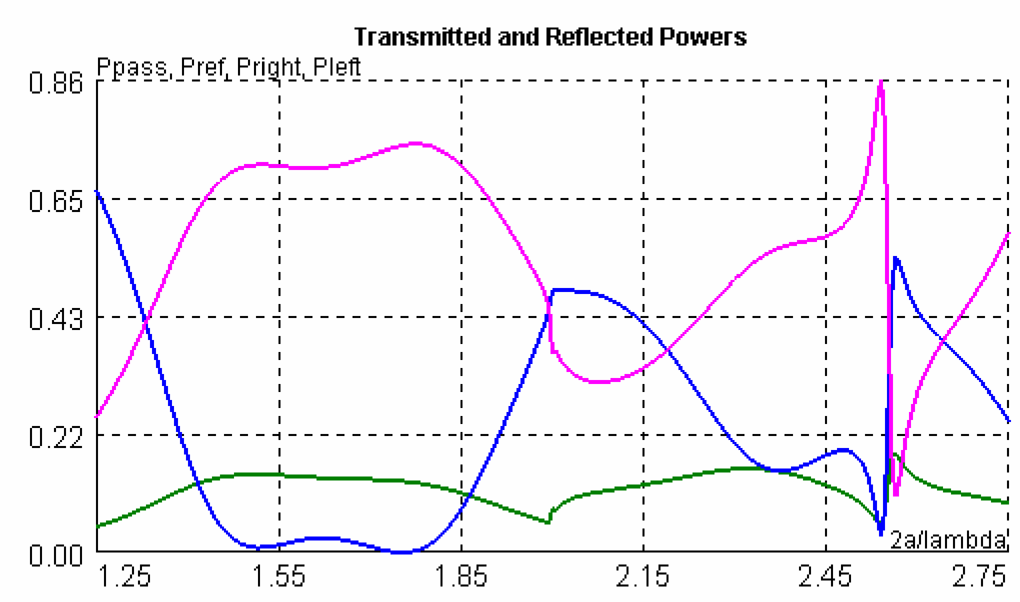

Fig. 2. Scattering and energetic characteristics of 4-port waveguide junction with dielectric rod for $b/a=1$ at optimal rod parameters $\varepsilon'=3.8$, $2r/a=0.425$, $\ell/a=0.36$.

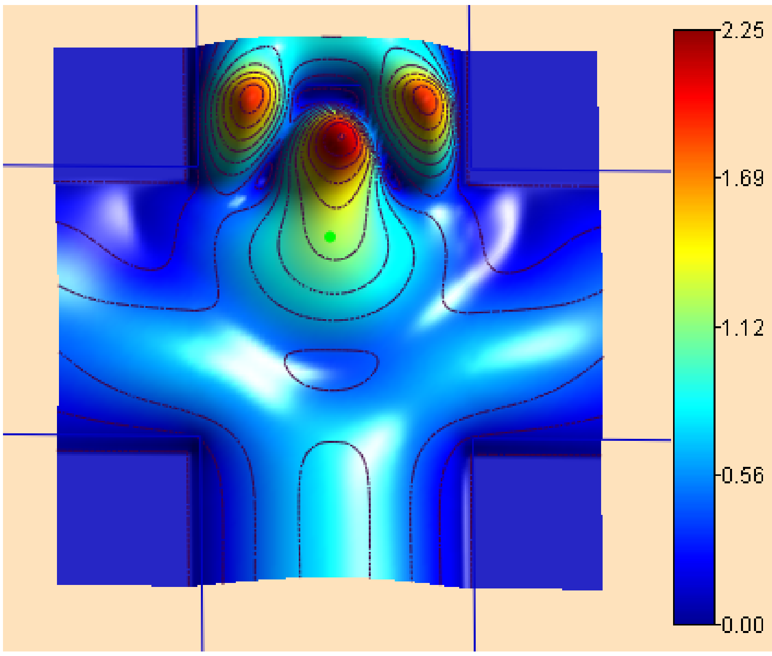

Fig. 3. Structure of electric field in the 4-port waveguide junctions with a strip and dielectric layer at optimal parameters of discontinuities $b/a=1$, $\varepsilon_1'=\varepsilon_3'=1$, $\varepsilon_2'=3.8$.