EVOLUATION OF THE ANGULAR SPECTRUM OF SCATTERED ELECTROMAGNETIC WAVES BY THE ABSORPTIVE INHOMOGENEOUS SLAB

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1. Introduction

Absorption of the energy of electromagnetic waves scattered in the chaotic media has a substantial effect on statistical characteristics [1–3]. In case of asymmetric formulation of the problem, scattering may significantly distort the angular power spectrum (APS) of the scattered radiation [4,5]. The location of a point source and a receiver on opposite sides of a randomly inhomogeneous absorptive slab is under the great interest from the practical point of view. In such formulation, the APS of the received scattered radiation significantly depends on both the medium properties and the mutual arrangement of the source and the receiver with respect to the slab. Similar problem was solved earlier in the small-angle approximation using the method of complex geometrical optics [4,5]. However, in these papers, the layer of an absorptive chaotic medium was assumed to be relative thin, in which, in spite of multiple scattering on smooth inhomogeneities, the width of the APS and the shift of its maximum with respect to the direction to the source remained to be small. At the same time, the solution of this problem without such assumption has great practical significance. As the analytical methods of solution are not yet available, information about the properties of the scattered radiation could be obtained either through the experiment or through the numerical calculations. In this paper, the problem is numerically solved using the method of statistical modeling, which provides nearly detailed information about the statistical characteristics of the scattered radiation.

2. Formulation of the problem and method of solution

The point source is assumed to be located in a homogeneous nonabsorbing medium with the permittivity $\varepsilon = \varepsilon_0$ at a distance L_1 above slab of a randomly inhomogeneous absorptive medium having a thickness L_2. The permittivity of the slab is $\varepsilon = \varepsilon_0 + i\varepsilon'' + \varepsilon_1(\mathbf{r})$, where $\varepsilon_1(\mathbf{r})$ describes the fluctuations of the real part of the permittivity inside the slab and ε'' is the imaginary part of the permittivity, describing wave absorption in the layer. The source has a conical direction pattern. The receiver is located in the homogeneous nonabsorbing medium $\varepsilon = \varepsilon_0$ in the plane XZ at a distance L_3 below the slab. For sake of convenience ε_0 is assumed to be unity (lower atmosphere). The line–of–sight connecting the source and the receiver makes an angle θ with Z-axis. This angle and the apex angle of the light cone are considered to be fixed, while the source and the receiver are moving with respect to the layer. The characteristic size of inhomogeneities in the slab is assumed to be much larger than the radiation wavelength and $\varepsilon_1 << \varepsilon_0$. The APS was determined as a Fourier transform of the correlation function of the scattered field and has the Gaussian form in case of strong phase fluctuations. The equations for the statistical moments of the angular spectrum, namely, for the shift of the center of gravity and for the variance have been obtained in [1–5]. Dependences of the moments on the mutual location of the source and the receiver relative to the layer have been studied by numerical calculations for the Gaussian model of the spectrum of permittivity fluctuations. This paper continues and generalizes investigations started earlier in [1–5]. For solution of the problem, statistical simulation (the Monte Carlo method) has been applied. We have employed the so-called weighted modification of the Monte Carlo method. As the radiation is propagating along the model path (ray

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tube), absorption is taken into account as a parameter that contributes to the effect of this propagation path on the angular power distribution of the scattered radiation.

3. Numerical analysis of the angular spectrum

Investigation of numerical simulation of the APS is based on the power-law spatial spectrum of permittivity fluctuations:

$$\Phi(k)=C\begin{cases}\left(\frac{\sqrt{2}}{90}k_0\right) & k\in\left[0,\frac{\sqrt{2}}{90}k_0\right]\\ k^{-2.3} & k\in\left[\frac{\sqrt{2}}{90}k_0,\sqrt{2}k_0\right]\\ 0 & k>\sqrt{2}k_0\end{cases}\quad, \tag{1}$$

where the coefficient C could be determined from the following normalization condition:

$$\int_{-\infty}^{+\infty}\int\int\Phi(\mathbf{k})d^3\mathbf{k}=4\pi\int_0^{+\infty}k^2\Phi(k)dk=<\varepsilon_1^2>\ , \tag{2}$$

and for the chosen model of the spectrum of fluctuations is equal to $C=0.045189<\varepsilon_1^2>k_0^{-0.7}$.

It is known [6] that the single scattered phase function $\sigma(\alpha,\varphi)$ is connected with the spatial spectrum of fluctuations $\Phi(k)$ as: $\sigma(\alpha,\varphi)=\pi k_0^4\Phi(k)/2$, where α and φ are the zenithal and azimuth angles between the directions of propagation of the initial and single scattered waves, k is the module of the difference between the wave vectors of the single scattered waves and the initial wave, k_0 is the module of the wavevector of electromagnetic radiation in the vacuum. Since the scattered medium is assumed to be statistically isotropic, the scattering phase function is independent of the azimuth angle φ: $\sigma(\alpha,\varphi)=\sigma(\alpha)$, consequently, $\sigma(\alpha)=\pi k_0^4\Phi[(k_0/2)\sin(\alpha/2)]/2$. This scattering phase function describes rather well as the processes of single scattering of light in seawater, as well as infrared radiation from the water droplets in clouds and in living tissue. In the algorithm of statistical modeling this dependence is the probability density of the ray turn by the zenithal angle α in a particular act of scattering. The azimuth angle of the ray turn in the plane normal to the wave vector in the ray tube is uniformly distributed in the range from 0 to 2π before the scattering event. Fig. 1 illustrates the dependence of single scattering phase function (solid curve) and cross section of the APS (dashed curve) of scattered radiation $I(s_x,s_y)\big|_{s_y=k_y/k=0}$ versus projection of the unit vector of the wave normal $s_x=k_x/k$ on the X–axis without absorption at $\sigma_s L_2=40$, $\sigma_s L_1=210$ and $\sigma_s L_3=80$.

In all numerical experiments, θ is assumed to be equal to 36.89^0, and the absorption is determined by the so-called photon survival probability $\Lambda=\sigma_s/(\sigma_s+\sigma_a)$ [7], where σ_s and σ_a are the extinction coefficients due to scattering and the absorption, respectively. The parameter σ_s is calculated by integration of the scattering phase function over all possible scattering directions: $\sigma_s=\int\sigma(\alpha,\varphi)d\Omega=(1/2)\pi k_0^4\int\Phi(k)d\Omega$, where $d\Omega$ is a small solid angle, in which the scattering occurs. The cross section is normalized to its maximum value. In the radiative transfer theory, the parameter reciprocal to the extinction coefficient corresponds to the mean length of the rectilinear path of the radiation between the two acts of scattering on the medium inhomogeneities. Therefore, the parameter $\sigma_a=k_0\varepsilon''$ has the meaning of the reciprocal to the path length at which the amplitude of radiation decreases e (e = 2.71…) times due to absorption in the medium. We carried out a series of calculations for the layer thickness $\sigma_s L_2=40$ and for the absorption corresponding to the survival probability $\Lambda=0.5$. Numerical simulations have illustrated that the APS has nearly the Gaussian form. The variance $D[s_x]$ is calculated as a second-order central statistical moment of s_x. The center of gravity $M[s_x]$ is calculated as the first-order statistical moment of the resulting APS:

$$D[s_x] = \frac{\int_{-1}^{1}\int_{-1}^{1} (s_x - M[s_x])^2 \, I(s_x, s_y) ds_x ds_y}{\int_{-1}^{1}\int_{-1}^{1} I(s_x, s_y) ds_x ds_y}, \qquad M[s_x] = \frac{\int_{-1}^{1}\int_{-1}^{1} s_x I(s_x, s_y) ds_x ds_y}{\int_{-1}^{1}\int_{-1}^{1} I(s_x, s_y) ds_x ds_y}. \qquad (3)$$

Numerical simulations have shown that the center of gravity is displaced to the opposite direction with respect to the normal in the initial range of L_1 at $L_3 \neq 0$. Furthermore, when $L_1, L_3 >> L_2$, the center of gravity firstly is shifting from the normal and then tends toward it. The displacement could not be observed when $L_1 = L_3$. Fig. 2 illustrates the dependence of the center of gravity $M[s_x]$ of the angular distribution of $I(s_x, s_y)$ versus L_3 at: $\sigma_s L_2 = 40$ and $\Lambda = 0,5$; $\sigma_s L_1 = 40$ (1 curve) and $L_1 = 0$ (2 curve). From figure it follows that the dependence of the statistical moment $M[s_x]$ on the distance L_3 from the receiver to the slab boundary has non–monotonic behaviour. Fig. 3 presents the dependence of variance $D[s_x]$ of the angular distribution $I(s_x, s_y)$ versus L_1 at: $\sigma_s L_2 = 40$, $\sigma_s L_3 = 80$ and photon survival probability $\Lambda = 0,5$ (1 curve) $\Lambda = 1$ (2 curve). The line parallel to the X-axis corresponds to the limiting case $L_1 \to \infty$. The dependence of the variance versus L_1 in the absence of absorption is also plotted by the curve 2 ($\Lambda = 1$). Fig. 4 illustrates the dependence of variance $D[s_x]$ of the angular distribution $I(s_x, s_y)$ as a function of the L_3 – distance between the receiver and the slab at $\sigma_s L_2 = 40$ and $\Lambda = 0,5$: $\sigma_s L_1 = 40$ (1 curve) and $\sigma_s L_1 = 0$ (2 curve).

4. Conclusion

From the numerical results some important conclusions could be drawn. For a relative thin layer, in which the conditions of the small-angle approximation are fulfilled, but multiple scattering occurs, the APS of scattered radiation has nearly Gaussian form, in spite of a non-Gaussian character of the spectrum of dielectric permittivity fluctuations. Numerical experiment, carried out for a relative thick slab, has shown that the spectrum of the received radiation could be strongly distorted by the location of the source and the receiver relative to the slab even without absorption. For the thick absorptive layer statistical moments depend on the mutual location of the source and the receiver with respect to the absorptive slab. Analyses of the APS shows that it has strongly non-Gaussian form and several peaks corresponding to certain directions of the wave propagation could be observed. The propagation of the radiation along these directions (maximums of the APS) may be virtually impossible depending on the position of the source or the receiver with respect to the layer. This result has been obtained for the first time.

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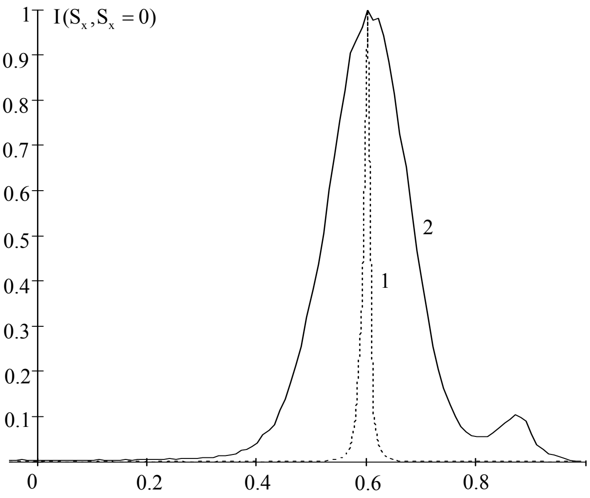

Figure 1

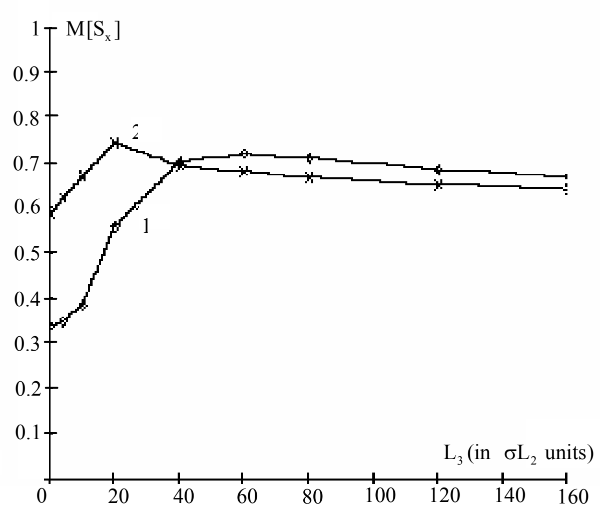

Figure 2

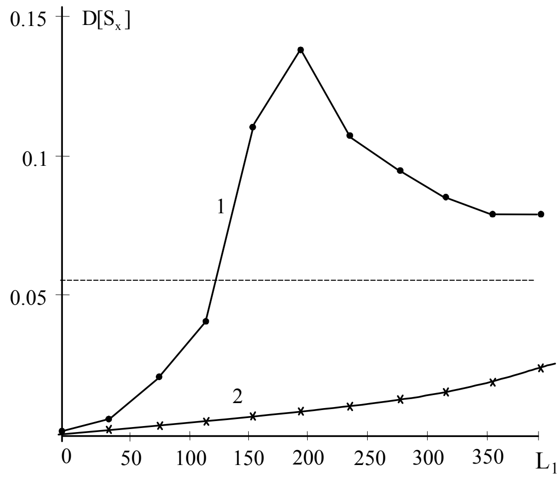

Figure 3

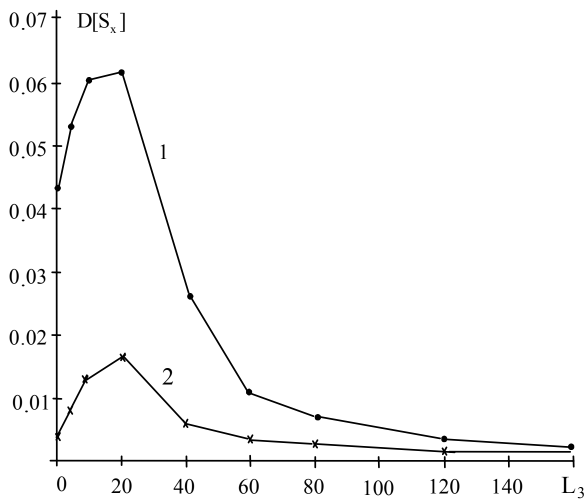

Figure 4