GEOMETRICAL OPTICS FIELDS CALCULATION BY THE MODIFIED EDGE REPRESENTATION LINE INTEGRALS

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1. Introduction

The purpose of this paper is to discuss, from the numerical point of view, on the equivalence between the line integration of the Modified Edge Representation (MER) currents on an infinitesimal contour around the Stationary Phase Point (SPP) and the Geometrical Optics fields. The MER currents are singular at the SPP, where the SPP is a point bellowing to the scatterer, represented as the reflection point (at the Illuminated Region) or the intersection point between the source and observer (at the Shadow Region).

Following Young-Maggi-Rubinowicz-Miyamoto-Wolf and [1]-[3], the Physical Optics (PO) surface integral, for the case of Herztian dipole incidence, is decomposed numerically into the MER Line Integrals. Basically, one line integral is along the periphery of the illuminated region (E_{MER}^{Per} hereafter) and the other one is around the infinitesimal contour, centered on the SPP (E_{MER}^{SPP} hereafter) [4].

According to the observer position, the PO scattered field is composed by the diffracted and the Geometrical Optic field (GO). The PO surface to MER line integrals reduction, when the scattered field is composed only by the diffracted field, was mathematically demonstrated [1]. On the other hand, when the scattered field is composed not only by the diffracted field but also for the GO contributions, the meaning of the MER line integration is not clear and the equivalence between the PO surface-to-MER line-integration has not been confirmed for all observation points. In 2004 [4] an extension of the MER theory has been presented to cover last case, witch is the most general. Numerically, it was showed the extraction or representation of GO fields by MER line integrals. This development was presented, at its first stage, for one observation position and considering the symmetry of the problem respect to the reference system. Using the MER line integration along an infinitesimal contour of the SPP (E_{MER}^{SPP} hereafter), the GO reflected field for the reflection region and the incident wave, with the minus sign, for the shadow region were obtained. In this article, the study of the E_{MER}^{SPP}, for any observation position is presented for first time.

It is numerically demonstrated the good agreement between the E_{MER}^{SPP} and the GO field for all observation positions, also it can be concluded that E_{MER}^{Per} recovers unequivocally the diffracted fields for all regions, performing the extension for [1] and [3]. For the reflection region is shown the equivalence of E_{MER}^{SPP} to the GO and its dependence with the curvature of the surface. For the case of the shadow region, the equivalence is also shown and moreover, it is possible to conclude the independence on the curvature of the surface. The general and important results derived are:

1. The PO surface integral is decomposed into the GO and the diffraction terms via the MER line integrals.
2. MER line integration around the SPP gives the contribution of the GO fields, representing the reflecting wave or the minus incident wave depending upon the observation position.

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3. The MER line integral along the illuminated region extracts only the diffraction.
4. The classical ideas proposed by Young in 1802 are recalled and for the practical use.
5. The calculation time of the scattered field is reduced.
6. For cases when the GO fields are needed to be calculated, the estimation of the radii of curvature of the reflected wave front is avoided.

2. PO Surface Integral and Diffracted Field Definitions

The scattered field E^S is uniformly defined everywhere as total field minus incident field. It may be expressed as the integral of the currents on the surface of the scatterer as:

$$E^S = j\frac{k\eta}{4\pi}\oiint_S \hat{r}_o \times (\hat{r}_o \times J)\frac{e^{-jkr_o}}{r_o}dS \tag{1}$$

where r_o is the distance from the integration point to the observer and $\hat{r}_o$ is the unitary vector toward the observer. The time factor $e^{j\omega t}$ has been suppressed. When the PO approximation is used, the surface electric current J is approximated by $J_{PO} = 2\hat{n}\times H^i$ on the lit surface for the case of a conductive scatterer, $\hat{n}$ is the unit vector normal to the surface and H^i is the unperturbed incident magnetic field. At the non illuminated region, it is assumed $J_{PO} = 0$. The source and the scatterer geometrically define three observation regions in the space as is shown in Fig.1. Regions I and III are the reflection and the shadow regions, respectively, for which the SPP falls inside the illuminated region. Region II is the space for which SPP does not exist.

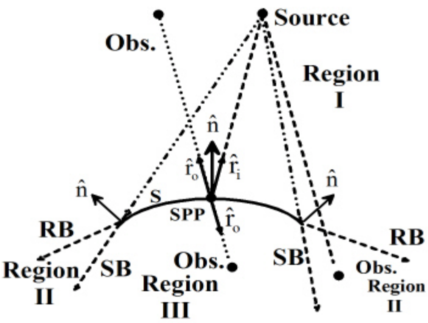

Fig. 1 GO regions

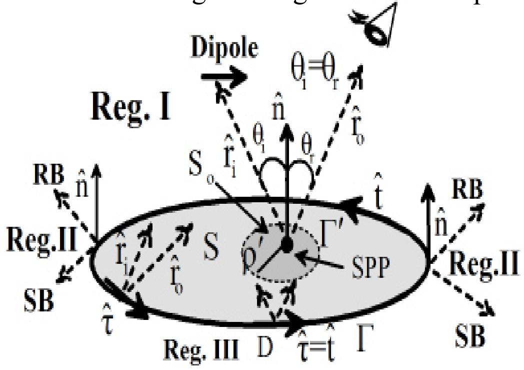

Fig. 2 Vector $\hat{\tau}$ definition

3. Modified Edge Representation Line Integrals

In MER, a unitary vector ($\hat{\tau}$) is defined to satisfy the diffraction law for given directions of incidence and observation:

$$\begin{aligned}(\hat{r}_i + \hat{r}_o)\cdot\hat{\tau} &= 0\\ \hat{n}\cdot\hat{\tau} &= 0\end{aligned} \tag{2}$$

The vector $\hat{\tau}$ depends only on the local values (locality principle), its definition is related with the source-observer positions as well as the normal vector ($\hat{n}$) to the surface at the point of interest. In Fig. 2 the vector $\hat{\tau}$ definition is illustrated; only at the diffraction points, if any, the vector $\hat{\tau}$ coincides with $\hat{t}$ (D in Fig.2).

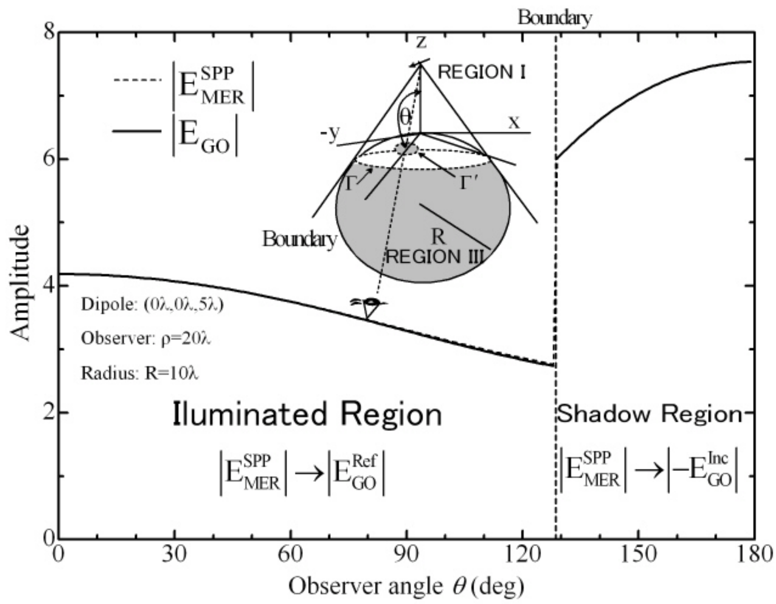

Fig. 3. Convergence of E_{MER}^{SPP} to E_{GO}. Entire region

The significance of the MER theory is the reduction of the PO surface integral into line integrals. Based on the Stokes' theorem and the high frequency approximation, for the case when the SPP does not exist on the illuminated region (Region II), the PO diffracted fields have been approximated by [1] as:

$$E_{MER}^{Diff} = jk\eta(A+B) \tag{3}$$

where the notation E_{MER}^{Diff} indicates the line integration of MER currents along the periphery of the illuminated region:

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{k}{4\pi} \oint_{\Gamma} \begin{bmatrix} \hat{r}_o \times \hat{r}_o \times J_{MER}^{I} \\ \hat{r}_o \times M_{MER}^{I} \end{bmatrix} \frac{e^{-jk(r_i + r_o)}}{r_o} dl \tag{4}$$

and the MER currents are defined as:

$$J_{MER}^{I} = \frac{\{\hat{r}_o \times (\hat{r}_o \times J_o)\} \cdot \hat{\tau}}{j(1-(\hat{r}_o \cdot \hat{\tau})^2)(\hat{r}_i + \hat{r}_o)\cdot(\hat{n} \times \hat{\tau})} \hat{t} \qquad M_{MER}^{I} = \frac{(\hat{r}_o \times \text{-}J_o)\cdot \hat{\tau}}{j(1-(\hat{r}_o \cdot \hat{\tau})^2)(\hat{r}_i + \hat{r}_o)\cdot(\hat{n} \times \hat{\tau})} \hat{t} \tag{5}$$

Where $J_o = 2\hat{n} \times H^i$ including only the radiation term. The derivation of the MER currents at the illuminated boundaries of the scatterer, the vector $\hat{\tau}$ instead of $\hat{t}$ has been utilized. MER reduces the PO surface integral (1) into the line integrals (4) of the equivalent currents (5) with remarkable accuracy [1].

4. Discussion on the PO Surface Integral Reduction in Terms of the MER Line Integrals

Important property has been observed on the MER currents and it has been discussed from 2004 [4]. As for the regions I and III where SPPs exist on the scatterer surface, the MER currents are singular there, due to $(\hat{r}_i + \hat{r}_o)\cdot(\hat{n} \times \hat{\tau}) \to 0$ and the mathematical treatment for region II, based upon Stokes theorem in [1] is no valid, it is needed other procedure to applied the MER .

The surface integral in (1) is written as two surface integrals, one for S and the other for So, Fig.2. The surface integral S has not singularities, based upon [1], this integration is rewritten as two MER line integrals. The first Line integral (E_{MER}^{Per}) is along the periphery Γ of the illuminated region and the other is on Γ' around the SPP (E_{MER}^{SPP}). In this sense the original PO surface integration has been decomposed by two MER line integrals (E_{MER}^{Per}, E_{MER}^{SPP}) and one PO surface integral on the surface So. When the limit $\rho' \to 0$ is considered, the surface integral So vanishes. Now the integral in (1) is expressed by MER Line integrals only.

$$E_{PO} = E_{MER}^{Per} + E_{MER}^{SPP} \tag{6}$$

When the limit is taken, only if the integral E_{MER}^{SPP} does not vanishes [4], the equivalence may be written as: $E_{MER}^{Per} \leftrightarrow E_{PO}^{Diff}$ and $E_{MER}^{SPP} \leftrightarrow E_{GO}$. Fig.3 shows, the results for the case of a sphere of radius $R=10\lambda$. The incidence is a spherical wave; the source is not far from the scatterer and the surface. For the entire region is observed good accuracy between the E_{MER}^{SPP} and the E_{GO}. For the Region I it is recovered the GO reflected field and for the case of Region III, the shadow is created

5. Conclusions

In this article, the MER line integration around the SPP is discussed numerically for the all observation region. It is concluded the good agreement $E_{MER}^{SPP} \leftrightarrow E_{GO}$. The development is possible due to the particular property of the MER currents, the singularity at the SPP. The singularity permits to recover the GO fields by the line integration. The present work recalls the classical ideas proposed by Young in 1802 and it may arise as a new methodology for the GO field computation from general surfaces. Also this results permit to conclude the accuracy between the PO surface to MER line integral reduction and they can be written with (6) as:

$$E_{MER}^{Per} = E_{PO}^{Diff} \quad \text{for all Regions} \tag{7}$$

and

$$E_{MER}^{SPP} = \begin{cases} E^{Ref} & \text{for Region I} \\ 0 & \text{for Region II} \\ -E^{Inc} & \text{for Region III} \end{cases} \tag{8}$$

6. References

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