COMPLEX PERMITTIVITY OF THIN PCB SUBSTRATE MEASURED BY OPEN-ENDED COAXIAL PROBE

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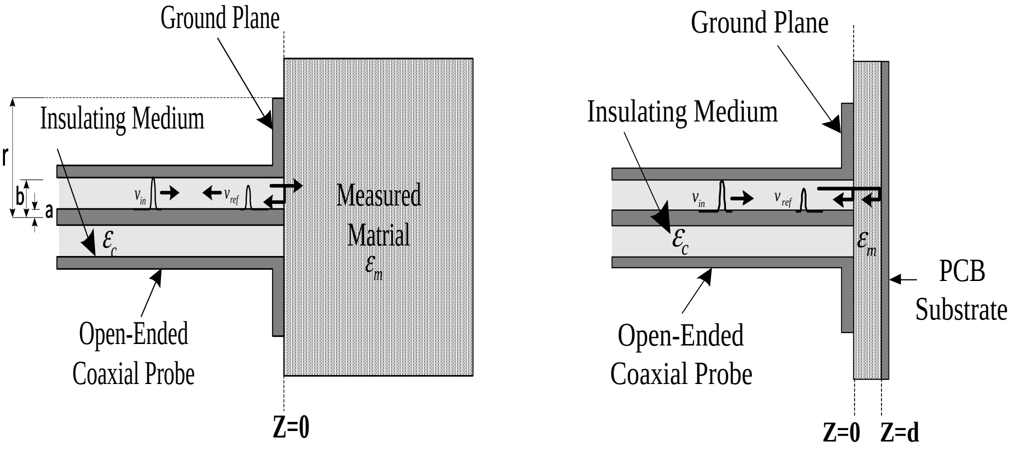
1. Introduction

Open-ended coaxial probe has been used to convert complex permittivity of material in a wide range of frequency. Virtual transmission line model[1] is an effective conversion model, which has been used in our laboratory. But it can convert complex permittivity of a material, which has larger thickness than 3 cm[2]. Therefore a new conversion model is needed to convert complex permittivity of thin substrate.

Applying the image theory to the equivalent source model of our measurement situation, we formulate a simple integral equation to reflection coefficient. Then the corresponding complex permittivity is obtained by solving the integral equation numerically. For a given complex permittivity of thin PCB substrate, the reflection coefficient is calculated by employing the dispersive FDTD method. Inserting the calculated reflection coefficient into the above conversion procedure provides the complex permittivity very close to the given value. In contrast, the reflection coefficients measured by four different open-end coaxial probes render some deviation between those converted complex permittivities. This measurement error is analyzed here.

2. Formulation

Complex permittivity of a PCB substrate may be calculated from the reflection coefficient, which is measured by the contacted open-ended coaxial probe. The radius of inner conductor of our open-ended coaxial probe ‘a’, the radius of outer conductor ‘b’, and the radius of flange ‘r’ are 0.635mm, 2.04mm, and 12mm, respectively. Virtual transmission line model used in our laboratory needs the reflection coefficient measured by open-ended coaxial probe, as shown in Fig.1(a). It implies the thickness of material to be infinite. We find that the thickness larger than 3cm may be considered infinite. Therefore thin substrate like PCB substrate needs a new conversion model because of the multiple reflections between open-ended coaxial probe and PCB ground plane in Fig.1(b).



(a) Thick material (b) Thin material

Fig. 1. Comparison of measuring situation according to thickness of material

An equivalent source model of our measurement situation is constructed using the image theory, as shown in Fig.2[3],[4].

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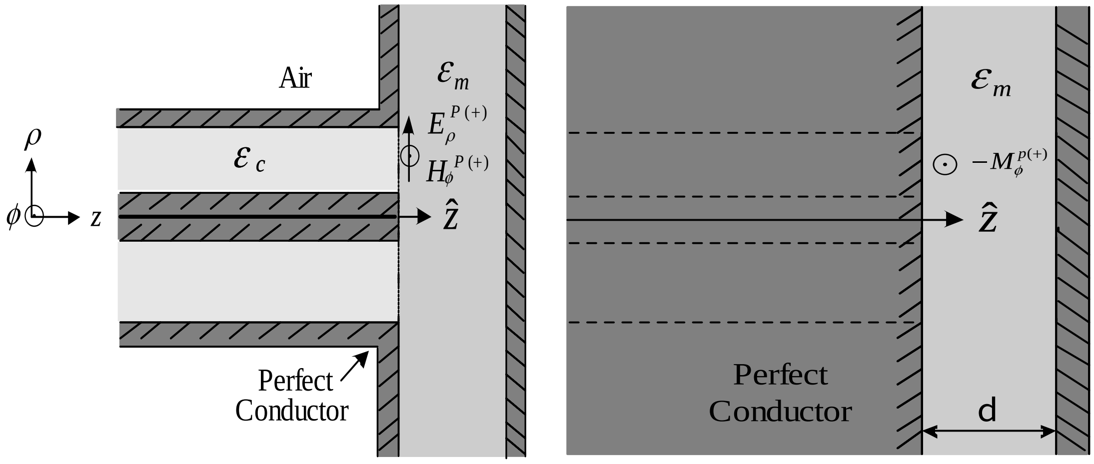
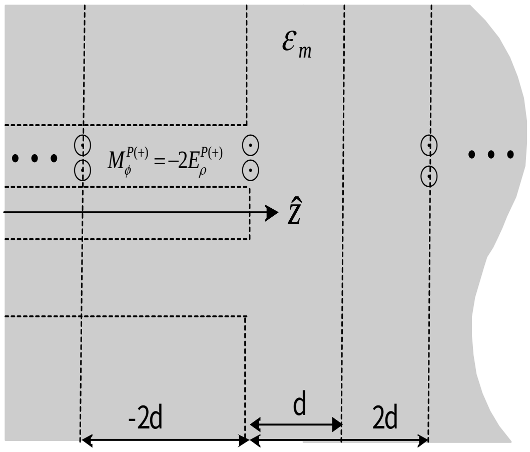
The original problem in Fig.2(a) is simplified into an equivalent magnetic current $M_{\phi}^{P(+)}$ in Fig.2(b). Applying the image theory to Fig.2(b), one may obtain infinite number of $M_{\phi}^{P(+)}$ in Fig.2(c).



(a) Original

(b) Equivalent current on the probe opening



(c) Image currents due to PCB ground plane

Fig. 2. Images of equivalent magnetic current on the probe opening in case that the thickness of PCB substrate is finite.

Then the integral equation to reflection coefficient and complex permittivity is formulated as

$$\frac{1}{\pi\rho} - w\varepsilon_0\varepsilon_c \sum_{n=0}^{N} \frac{f_n(\rho)}{N^2{}_n\beta_n} \int_a^b E_\rho{}^{p(+)}(\rho')f_n(\rho')\rho' d\rho' = \frac{jw\varepsilon_0\varepsilon_m}{\pi} \int_a^b \int_0^\pi E_\rho^{p(+)}(\rho') \frac{e^{-jk_m r}}{r} \rho' \cos\phi' d\phi' d\rho' \tag{1}$$

$$+ \sum_{\substack{l=-\infty \\ (l \neq 0)}}^{\infty} \int_a^b \int_0^\pi E_\rho^{P(+)}(\rho') \frac{e^{-jk_m r}}{r} \rho' \cos\phi' d\phi' d\rho']$$

where,

$$f_n(\rho) = [J_1(\gamma_n\rho)Y_0(\gamma_n a) - Y_1(\gamma_n\rho)J_0(\gamma_n a) \tag{2a}$$

$$N_n^2 = \frac{2}{\pi^2\gamma_n^2}[\frac{J_0^2(\gamma_n a)}{J_0^2(\gamma_n b)} - 1] \tag{2b}$$

$$\beta_n = -j\sqrt{\gamma_n^2 - k_k^2} \tag{2c}$$

In (1), $E_\rho^{p(+)}$ is the electric field introduced between open-ended coaxial probe and PCB substrate. And $f_n(\rho)$, N_n^2 , and β_n represent eigenfunction of TM_{0n} mode, normalization coefficient, and attenuation coefficient of TM_{0n} mode, respectively. In our conversion procedure, the given value is input admittance or reflection coefficient as

$$y_{in} = \frac{1-\Gamma_m}{1+\Gamma_m} = \frac{2}{\int_a^b E_\rho^{p(+)}(\rho')d\rho'} - \frac{2\pi}{[\frac{\mu_0}{\varepsilon_0\varepsilon_c}\ln(\frac{b}{a})]} \tag{3}$$

where $E_\rho^{p(+)}$ is the solution to the above integral equation (1).

3. Results and Discussion

In order to verify the accuracy of our conversion procedure, the reflection coefficient of PCB substrate with a given complex permittivity is calculated numerically by employing the dispersive FDTD method. For example, PCB substrate is with relative permittivity ε_m = 3.4 and conductivity σ_m = 0.25. And five different PCB substrates are taken by the thickness 'd'= 0.3mm, 0.6mm, 0.9mm, 1.2mm, and 2.4mm. The integral equation in (1) is calculated using moment method and Muller's iterative method[5]. Calculated complex permittivities at frequency of 3.56GHz are represented in Table 1.

Table 1. Converted complex permittivities and percentage errors

Thickness [mm]	Number of higher mode	Complex permittivity		Error[%]	
		ε_m	σ_m	ε_m	σ_m
0.3	2	3.4644	0.2483	1.8948	0.68
0.6	2	3.4566	0.2477	1.665	0.792
0.9	3	3.3960	0.2404	0.1166	3.8056
1.2	3	3.4343	0.2442	1.011	2.3116
2.4	4	3.3531	0.2468	1.379	1.28

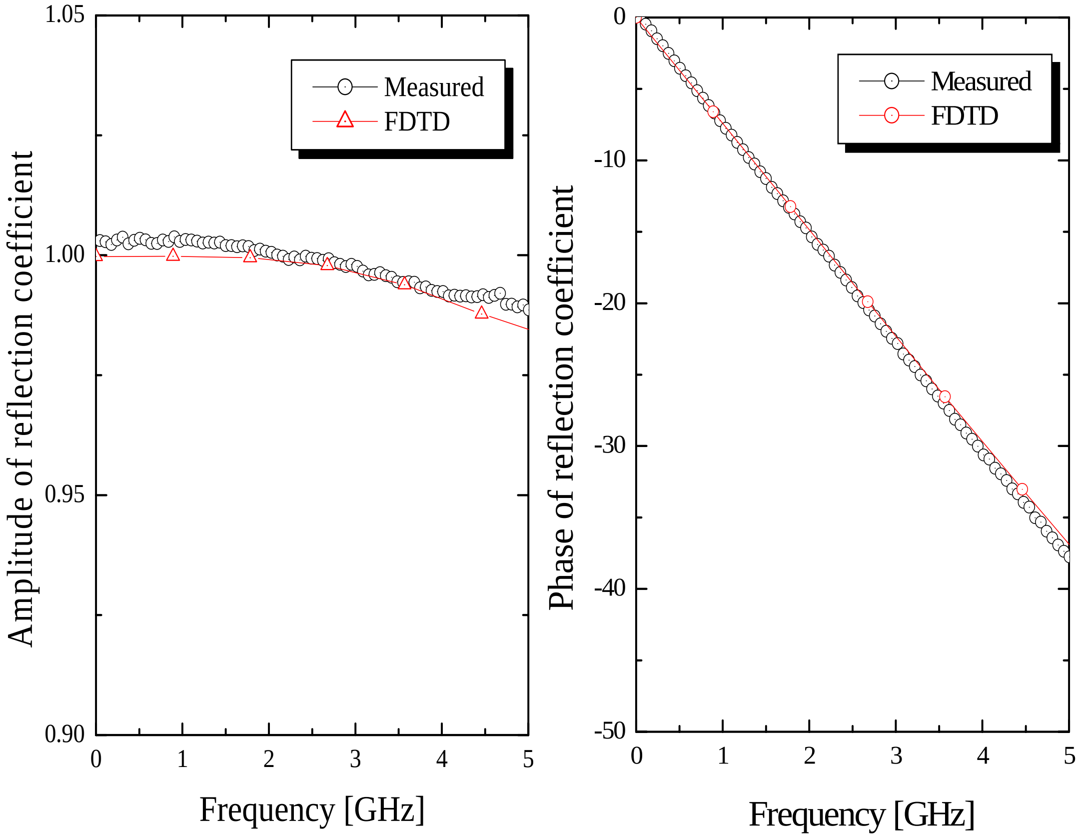
In Table 1, it may be found that the number of higher mode is different according to the thickness of PCB substrate. And the number of higher mode increases as the thickness increases[5]. The reason is that the multiple reflections between open-ended coaxial probe and PCB ground plane cancel out higher mode significantly as its thickness decreases.

Next the complex permittivity of PCB substrate is obtained from the measured reflection coefficients. For our actual measurement, PCB substrate is chosen by RO4003 with relative permittivity ε_m =3.38, conductivity σ_m =0, and thickness 'd'=0.3048mm. Reflection coefficient of RO4003 is measured by employing four different open-ended coaxial probes with the same specification. And those relative permittivities are converted at frequency of 3.55GHz. The converted results are represented in Table 2. The converted relative permittivities from four open-end coaxial probe are different, and reveal some deviation from the given value.

Table 2. Converted relative permittivities according to four different probes

	Probe-1	Probe-2	Probe-3	Probe-4
Reflection coefficient [Magnitude]	0.99435	0.98898	0.98798	0.98993
Reflection coefficient [Phase]	-26.99	-30.341	-27.197	-30.208
Converted ε_m	2.9916	3.4307	3.0199	3.4128

In Table 2, one may find that the probe-1 provides the worst conversion result. Hence we assume the complex permittivity of RO4003 to be ε_m =2.9916 and σ_m =0. Applying the assumed value to the dispersive FDTD simulation, we recalculate the corresponding reflection coefficient. Fig.3 shows that the recalculated reflection coefficient approaches to the original reflection coefficient measured by the probe-1 very closely. It assures the accuracy of our conversion procedure.



(a) Amplitude

(b) Phase

Fig. 3. Comparison between measured and recalculated reflection coefficients in case of probe-1

As shown in Table 2, the manufacturing error posed in our open-ended coaxial probes should be reduced to measure the reflection coefficient of PCB substrate accurately.

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