TE scattering from finite rectangular grooves in a conducting plane using overlapping T-block analysis

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Abstract

TE plane-wave scattering from finite rectangular grooves in a conducting plane is systematically analyzed with the overlapping T-block method. Multiple rectangular grooves are divided into several overlapping T-blocks to obtain the fast CPU time, CAD applicability, and wide versatility. The scattered fields are obtained in simple closed forms including a fast-convergent integral.

1 Introduction

TE plane-wave scattering from finite rectangular grooves in a conducting plane is a fundamental problem and has been extensively studied [1-5]. In the present work, we introduce a novel approach based on the overlapping T-block method for the scattering from finite rectangular grooves in a conducting plane. The dispersion analysis [6] of overlapping T-blocks are extended to the scattering analysis of finite rectangular grooves. The main advantage of the overlapping T-block method is that scattering relations of finite rectangular grooves are obtained as simple closed forms without the need of the integral equation technique [1,2]. The overlapping T-block method allows us to obtain a simple yet numerically efficient series solution including a fast-convergent integral.

2 Field Analysis of a Single Groove

Consider a rectangular groove with the TE plane-wave incidence shown in Fig. 1. The time-factor $e^{-i\omega t}$ is suppressed throughout. The incident and reflected E_z fields are shown as, respectively,

$$E_z^i(x,y) = \exp[ik_2(\sin\theta_i x - \cos\theta_i y)] \quad (1)$$

$$E_z^r(x,y) = -\exp[ik_2(\sin\theta_i x + \cos\theta_i y)] \quad (2)$$

where $k_2 = \omega\sqrt{\mu_2\epsilon_2} = 2\pi/\lambda_2$ and θ_i is an incident angle of the TE plane-wave. In regions (I) $(-d < y < 0)$ and (II) $(y > 0)$, the E_z components are

$$E_z^I(x,y) = \sum_{m=1}^{\infty} p_m \sin a_m(x+a)\sin\xi_m(y+d)\Big[u(x+a) - u(x-a)\Big] \quad (3)$$

$$E_z^{II}(x,y) = \sum_{m=1}^{\infty} p_m \sin(\xi_m d)\Big[E_m(x,y) + R_m^E(x,y)\Big] \quad (4)$$

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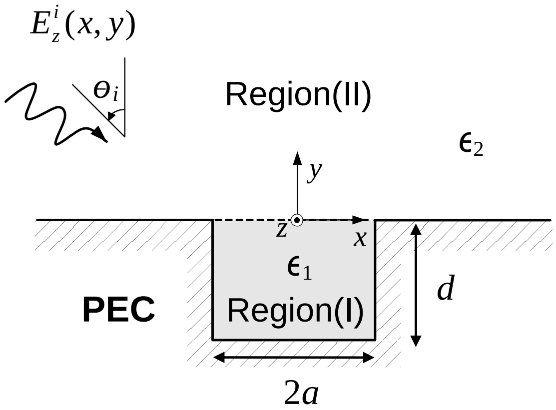

Figure 1: Geometry of rectangular groove.

where $a_m = m\pi/(2a)$, $\xi_m = \sqrt{k_1^2 - a_m^2}$, $k_1 = \omega\sqrt{\mu_1\epsilon_1} = 2\pi/\lambda_1$, and $u(\cdot)$ is a unit step function. Based on the radiation condition and the $E_z^I(x,y)$ component, we represent $E_m(x,y)$ in (4) as

$$E_m(x,y) = e^{i\eta_m y}\sin a_m(x+a)\Big[u(x+a) - u(x-a)\Big] \tag{5}$$

where $\eta_m = \sqrt{k_2^2 - a_m^2}$. By utilizing the Green's function relation in [6] and deforming the integral path, we obtain the fast-convergent integral as

$$\begin{aligned} R_m^E(x,y) &= \frac{k_2 a_m}{\pi}\int_0^\infty \frac{(1+2vi)\eta\sin(\eta y)}{\zeta(\zeta^2 - a_m^2)}\Big[(-1)^m e^{i\zeta|x-a|} - e^{i\zeta|x+a|}\Big]\,dv \\ &= \frac{k_2 i}{2}\int_{-a}^{a} \frac{yH_1^{(1)}(k_2\sqrt{(x-x')^2+y^2})}{\sqrt{(x-x')^2+y^2}}\sin a_m(x'+a)\,dx' - E_m(x,y) \end{aligned} \tag{6}$$

where $\eta = k_2 v(v-i)$ and $\zeta = \sqrt{k_2^2 - \eta^2}$. The total electric field is, therefore, given as

$$T_E(x,y) = E_z^I(x,y) + E_z^{II}(x,y). \tag{7}$$

When $\rho = \sqrt{x^2+y^2} \to \infty$, (4) becomes

$$E_z^{II}(\rho,\theta) \sim \frac{e^{i(k_2\rho - \pi/4)}}{\sqrt{2\pi k_2\rho}} k_2\cos\theta \sum_{m=1}^{\infty} p_m \sin(\xi_m d) F_m(-k_2\sin\theta; a) \tag{8}$$

where $\theta = \tan^{-1}(x/y)$.

3 Field Analysis of Multiple Grooves

It is possible to apply the overlapping T-block approach to the geometry of multiple rectangular grooves shown in Fig. 2(a). We first divide the multiple grooves in Fig. 2(a) into several overlap-

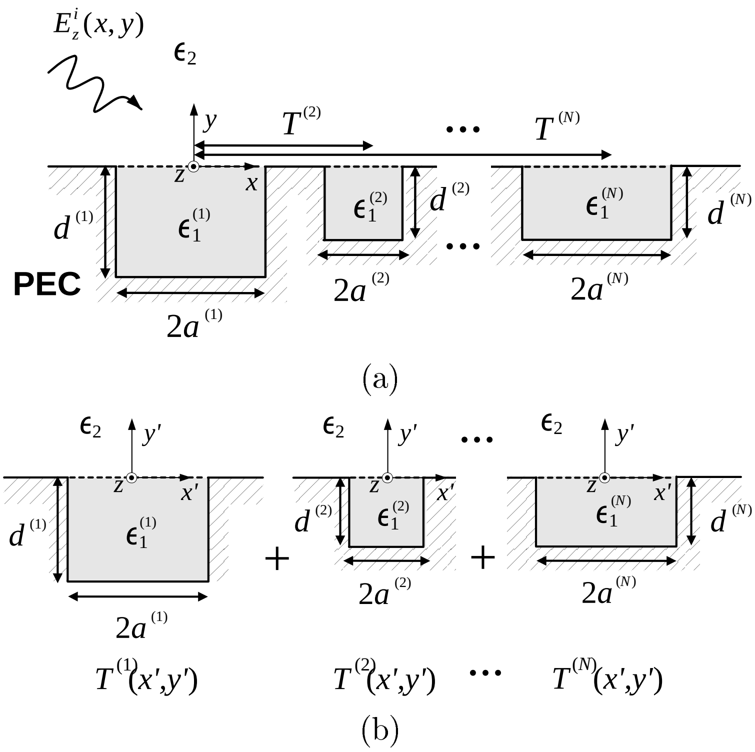

Figure 2: Geometry of (a) multiple rectangular grooves and (b) overlapping T-blocks.

ping T-blocks as shown in Fig. 2(b), thus facilitating the CAD applicability. The superposition procedures are utilized in [6]. The E_z fields of Fig. 2(a) are represented as

$$\begin{aligned} E_z(x,y) &= T_E^{(1)}(x,y) + T_E^{(2)}(x - T^{(2)}, y) + \cdots + T_E^{(N)}(x - T^{(N)}, y) \\ &= \sum_{n=1}^{N} T_E^{(n)}(x - T^{(n)}, y) \end{aligned} \tag{9}$$

where $T^{(1)} = 0$ and N is the number of grooves.

4 Numerical Computations

To understand the scattering characteristics of finite rectangular grooves, we define a backscattered echowidth as

$$\sigma = \lim_{\rho\to\infty} 2\pi\rho \left| \frac{E_z^{II}(\rho,\theta)}{E_z^i(\rho,\theta_i)} \right|^2 \tag{10}$$

Table 1 represents the behaviors of a normalized backscattered echowidth versus an incident angle, θ_i. It is seen that our higher-mode solutions ($m = 5,\ 7$) converge to the more higher modes ($m = 9,\ 11$). A dominant-mode solution is quite accurate only near to the normal incidence. Beyond $\theta_i = 30°$, higher-mode solutions ($m = 5,\ 7$) should be used to compute the backscattered echowidth. Fig. 3 illustrates the $E_z(x,y)$ field continuity to verify our approach.

Table 1: Behaviors of normalized backscattered echowidth, σ/λ_0 [dB] versus incident angle, θ_i for $a = 1.1\lambda_0$, $d = 1.6\lambda_0$, and $\lambda_1 = \lambda_2 = \lambda_0$.

Angle	$m = 1$	$m = 3$	$m = 5$	$m = 7$	$m = 9$	$m = 9$
0°	14.64	15.37	15.32	15.31	15.31	15.31
30°	-12.11	7.73	8.07	8.08	8.09	8.10
60°	-43.43	-8.54	9.36	9.28	9.26	9.24

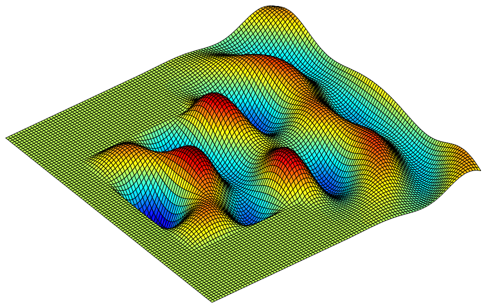

Figure 3: $E_z(x, y)$ field distributions for $\theta_i = 0°$, $a = 1.1\lambda_0$, $d = 1.6\lambda_0$, and $\lambda_1 = \lambda_2 = \lambda_0$

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