Reflection and transmission of nonaligned coaxial lines: TM-mode analysis

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1 Introduction

Open-ended coaxial lines have been extensively used to estimate the permittivity and permeability of dielectric materials since its estimation method is relative accurate, simple, and nondestructive. Techniques to measure the permittivity and permeability with coaxial lines require the insertion of a material sample between coaxial lines. Reflection and transmission behavior of a dielectric material inserted between two coaxial lines can be used to estimate the permittivity of a dielectric material. Scattering from a dielectric slab material between aligned two-port flanged coaxial lines has been analyzed [1] where the distance between the centers of coaxial lines is zero. The purpose of the present paper is to investigate scattering from a dielectric slab between nonaligned two-port flanged coaxial lines where the distance between centers of coaxial lines is nonzero. The technique of mode matching and Hankel transform [1, 2] is used in this paper to evaluate scattering from a dielectric slab between non-aligned two-port flanged coaxial lines. Only TM-modes will be considered with the assumption that the fields inside the coaxial line have no azimuthal ϕ-variation. The assumption of no azimuthal ϕ-variation substantially simplifies the problem formulation. Some computation is performed to check the validity of our formulation as well as its applicability to the problem of permittivity estimation.

2 Field Analysis

The problem geometry is shown in Fig. 1 where two coordinate systems, ρ and ρ', are used for each coaxial line. The inner and outer radii of the coaxial line are a and b, respectively. The permittivity and permeability of the dielectric material between a and b are ϵ and μ, respectively. Regions (I) and (III) denote the coaxial line interior ($a \le \rho \le b$, $z \le 0$) and ($a \le \rho' \le b$, $z \ge d$), respectively. Region (II) denotes the dielectric slab ($0 \le z \le d$). The wavenumbers for region (I) and (III) are $k = \omega\sqrt{\mu\epsilon}$, and the wave number for region (II) is $k_1 = \omega\sqrt{\mu_1\epsilon_1}$. Assume a TEM wave is incident from below a coaxial line in region (I). It is reasonable to assume that the scattered field in a coaxial line can be represented in terms of TEM and TM modes if the radius of the coaxial line is small compared with the wavelength and the distance r between the center of two coaxial lines is large compared with the diameter $2b$ ($r > 2b$). The field in region (I) consists of incident and reflected waves. The reflected wave consists of a TEM wave and higher order $\mathrm{TM}_{0\mathrm{n}}$ waves. Then the field in region (I) is

$$H_\phi^I(\rho, z) = \frac{e^{ikz}}{\rho} + c_0\frac{e^{-ikz}}{\rho} + \frac{1}{\mu}\sum_{n=1}^{\infty} c_n k_z R_1(\xi_n\rho)e^{-ik_z z} \tag{1}$$

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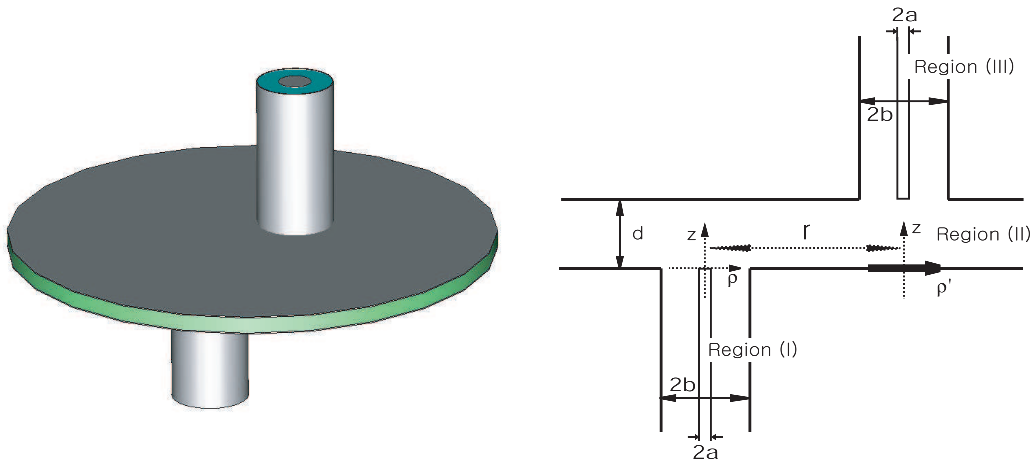

FIG. 1: Structure and geometry

where $\eta = \sqrt{\frac{\mu}{\epsilon}}$, $k_z = \sqrt{k^2 - \xi_n^2}$, $R_1(\xi_n \rho) = J_1(\xi_n \rho) N_0(\xi_n b) - N_1(\xi_n \rho) J_0(\xi_n b)$, and c_n is the unknown modal coefficient. Similarly, the field in region (III) can be expressed as

$$H_\phi^{III}(\rho', z) = l_0 \frac{e^{ik(z-d)}}{\rho'} + \frac{1}{\mu} \sum_{n=1}^{\infty} l_n R_1(\xi_n \rho') e^{ik_z(z-d)} \tag{2}$$

where l_n is the modal coefficient. The field in region (II) is a superposition of scattered fields from regions (I) and (III). The field in region (II) can be represented as

$$\begin{aligned} H_\phi^{II} = & \frac{1}{\mu_1} \int_0^\infty \left[\tilde{H}^+(\zeta) e^{i\kappa z} + \tilde{H}^-(\zeta) e^{-i\kappa z} \right] J_1(\zeta\rho) \zeta \, d\zeta \\ & + \frac{1}{\mu_1} \int_0^\infty \left[\tilde{H'}^+(\zeta) e^{i\kappa(z-d)} + \tilde{H'}^-(\zeta) e^{-i\kappa(z-d)} \right] J_1(\zeta\rho') \zeta \, d\zeta \end{aligned} \tag{3}$$

where $\kappa = \sqrt{k_1^2 - \zeta^2}$.

To determine the unknown coefficients c_n, l_n, $\tilde{H}^+(\zeta)$, $\tilde{H}^-(\zeta)$, $\tilde{H'}^+(\zeta)$, and $\tilde{H'}^-(\zeta)$, six boundary conditions are necessary. The tangential electric field continuities at $\rho = 0$, $z = d$ and $\rho' = 0$, $z = 0$ give

$$\left[\tilde{H}^+(\zeta) e^{i\kappa d} - \tilde{H}^-(\zeta) e^{-i\kappa d} \right] = 0 \tag{4}$$

$$\left[\tilde{H'}^+(\zeta) e^{-i\kappa d} - \tilde{H'}^-(\zeta) e^{i\kappa d} \right] = 0. \tag{5}$$

Applying Hankel transform $\int_0^\infty (\cdot) J_1(\zeta\rho) \rho \, d\rho$ to the continuity of tangential electric field at $z = 0$ yields

$$\left[\tilde{H}^+(\zeta) - \tilde{H}^-(\zeta) \right] = \frac{\mu_1 \epsilon_1}{\kappa \epsilon} \left[k(1 - c_0) I_0 - \frac{1}{\mu} \sum_{n=1}^{\infty} c_n k_z I_n \right] \tag{6}$$

where

$$I_0 = \frac{J_0(\zeta a) - J_0(\zeta b)}{\zeta} \tag{7}$$

$$I_n = 2\zeta \frac{-J_0(\zeta b) J_0(\xi_n a) + J_0(\zeta a) J_0(\xi_n b)}{\pi \xi_n (\zeta^2 - \xi_n^2) J_0(\xi_n a)} . \tag{8}$$

Similarly, the continuity of the tangential electric field at $z = d$ gives

$$\left[\tilde{H'}^{+}(\zeta) - \tilde{H'}^{-}(\zeta)\right] = \frac{\mu_1\epsilon_1}{\kappa\epsilon}\left[kl_0I_0 + \frac{1}{\mu}\sum_{n=1}^{\infty} l_nk_zI_n\right] . \tag{9}$$

The continuity of tangential magnetic field at $z = 0$ for $a < \rho < b$ requires

$$H_\phi^I(\rho, 0) = H_\phi^{II}\Big|_{z=0}. \tag{10}$$

Substituting $\tilde{H}^{+}(\zeta)$, $\tilde{H}^{-}(\zeta)$, $\tilde{H'}^{+}(\zeta)$, and $\tilde{H'}^{-}(\zeta)$ into (10) and manipulating the expressions gives a set of simultaneous equations for the discrete modal coefficients c_n, and l_0, l_1, l_2, $\cdots$, l_n.. Similarly, the continuity of magnetic field at $z = d$ for $a < \rho' < b$ gives another set of simultaneous equations for the discrete modal coefficients. The explicit expressions for the simultaneous equations for the discrete modal coefficients are available in [3].

3 Numerical Computations

To examine the validity of our formulation, numerical simulations have been performed. Figures 2(a) and 2(b) depict the comparison between our results and the simulated results of CST-MWS when $r > 2b$ and $r < 2b$, respectively. Figure 2(a) shows that our solutions agree well with the simulated result of CST-MWS when the cross sections of two open-ended coaxial lines do not overlap ($r > 2b$). Figure 2(b) shows that the discrepancies exist between our results and the simulated result of CST-MWS when the cross sections of two open-ended coaxial lines overlap ($r < 2b$). The discrepancies for $r < 2b$ is attributed to the fact that our theory approximately assumes no field variation in the azimuth ϕ direction within the coaxial line. When $r < 2b$, the fields within the coaxial line tends to become nonuniform in the azimuthal direction and higher order modes other than $\mathrm{TM_{0n}}$ modes are necessary for accurate field representation.

4 Conclusion

An approximate theory to estimate the reflection and transmission of nonaligned flanged coaxial lines is numerically investigated. Our theoretical model assumes no field variation in the ϕ-direction utilizing only TEM and $\mathrm{TM_{0n}}$ modes. The analysis technique is based on the Hankel transform and mode matching. Our solutions agree well with the simulated result of CST-MWS when the cross sections of two open-ended coaxial lines do not overlap ($r > 2b$). When the cross sections of two open-ended coaxial lines overlaps ($r < 2b$), an exact full-wave formulations using TE and TM modes is required for accurate calculation.

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References

[1] J. B. Jarvis and M. D. Janezic "Analysis of a two-port flanged coaxial holder for shielding effectiveness and dielectric measurements of thin films and thin materials", *IEEE Trans. Electromagn. Compat.,* vol.38, pp. 67-70, Feb. 1996.

[2] H. J. Eom, *Electromagnetic Wave Theory for Boundary-Value Problems,* Springer, Berlin, 2004.

[3] J. K. Paek, "Analysis of Reflection and Transmission of Thin Dielectric Slab between Two-port Flanged Coaxial Line", Master dissertation, Department of Eletrical Engineering and Computer Science, Korea Advanced Institute of Science and Technology, Daejeon, 2005.

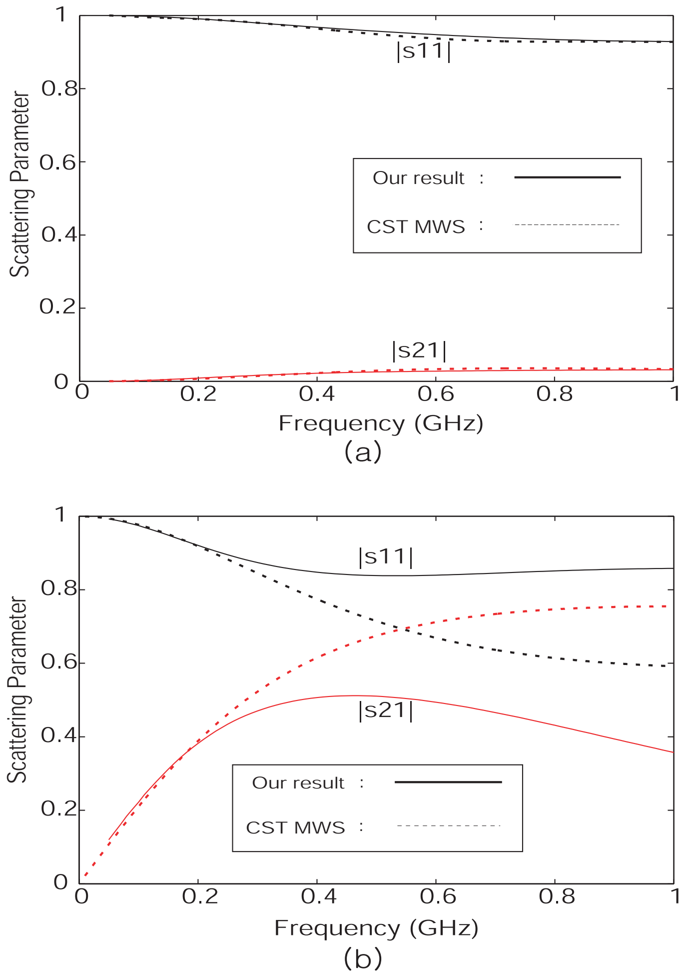

FIG. 2: The components of $|S_{11}|$ and $|S_{21}|$ as functions of frequency with b=38.404mm, b/a=2.3, d=1.8mm, ϵ=1, ϵ_1=2.04, μ, μ_1=1. The dotted lines denote simulated value of CST-MWS and the solid lines denote theoretical predictions. (a) : $r = 100mm$, (b) : $r = 20mm$