ELECTROMAGNETIC WAVE PROPAGATION THROUGH DOUBLY DISPERSIVE SUBWAVELENGTH METAMATERIAL HOLE

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1. Introduction

Unusual electromagnetic wave transmission though subwavelength aperture has been one of the important present research subjects of current electromagnetics and optics [1]. Usually, these subwavelength apertures in the novel metals and the incident electromagnetic waves are in the region of visible and infrared wavelengths. In these wavelengths, the materials can be viewed as plasma media. Excitation and de-excitations of plasma waves at both ends of the subwavelength apertures can be an explanation of a principle of the extraordinary transmission through subwavelenth apertures [2]. These plasma waves can also propagate through the subwavelength hole which is surrounded by the metamaterial. The study on the metamaterials is another hot topic in various microwave / optical device applications as well as in fundamental electromagnetics. In our present work, we investigate the electromagnetic wave propagation along the subwavelength hole which is surrounded by doubly dispersive metamaterials. We consider the electromagnetic dispersion of the waveguiding systems and the propagating power along the subwavelength hole. Some of newly obtained physical effects are addressed and discussed.

2. Characteristics Equation of Doubly Dispersive Metamaterial Hole

Fig. 1 shows the schematic view of the doubly dispersive circular metamateria hole with its diameter $2a$. The inner region of the hole is free space and the surrounding region is composed of the doubly dispersive metamaterial. The dielectric and magnetic constants of the metamaterial can be given by $\varepsilon_r = 1 - \omega_p^2 / \omega^2$ and $\mu_r = 1 - F\omega^2 / \left(\omega^2 - \omega_0^2\right)$, respectively [3], where the parameters are assumed as $\omega_p / 2\pi = 10$ GHz, $\omega_0 / 2\pi = 4$ GHz, and $F = 0.56$. Fig. 2 shows the plots of the material expressions. The characteristic equation of the metamterial hole waveguide can be derived as follows from the standard steps of deriving procedure of conventional dielectric rod waveguides. Axial field component of the inner region of the hole is governed by $I_m(\cdot)$ instead of $J_m(\cdot)$.

$$\left[\frac{\varepsilon_{r1}}{k_1}\frac{I_m'(k_1a)}{I_m(k_1a)} - \frac{\varepsilon_{r2}}{k_2}\frac{K_m'(k_2a)}{K_m(k_2a)}\right]\left[\frac{\mu_{r1}}{k_1}\frac{I_m'(k_1a)}{I_m(k_1a)} - \frac{\mu_{r2}}{k_2}\frac{K_m'(k_2a)}{K_m(k_2a)}\right] = \left[\frac{m\beta}{k_0a}\left(\frac{1}{k_1^2} - \frac{1}{k_2^2}\right)\right]^2 \tag{1}$$

Subscripts 1 and 2 represent the free space $(r < a)$ and metamaterial $(r > a)$ regions, respectively. $k_i = \sqrt{\beta^2 - k_0\mu_{ri}\varepsilon_{ri}}$ is the propagation constant in the radial direction. k_0 is the free space wave number.

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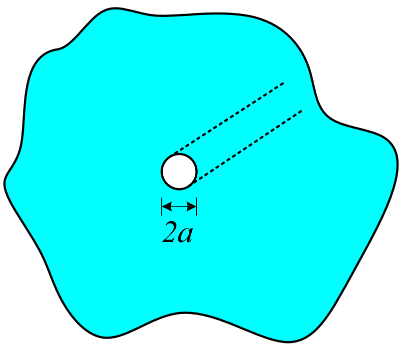

Fig. 1. Metamaterial circular hole with $r = a$.

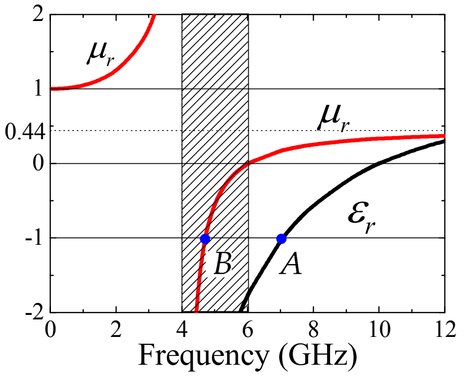

Fig. 2. Material constants of the surrounding metamaterials.

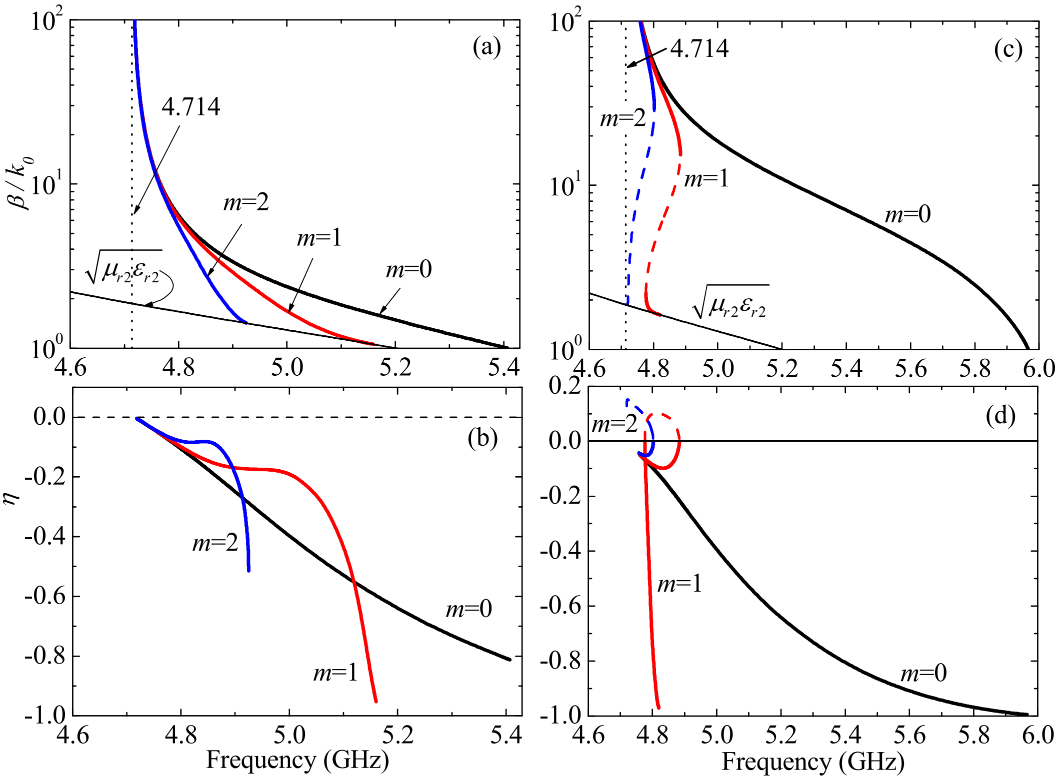

Fig. 3. Dispersion characteristics and normalized power flux for the TE-like modes. (a) Dispersion curves of the metamaterial hole with $a = 10.0$ mm , (b) normalized energy flux of the metamaeterial hole with $a = 10.0$ mm , (c) dispersion curves of the metamaterial hole with $a = 1.0$ mm , and (d) normalized power flux of the metamaeterial hole with $a = 1.0$ mm .

β is the propagation constants in the axial direction. m is the azimuthal eigen value. The discrete eigenvalue solution of (1) forms the guided modes of the metamaterial hole and their normalized energy flux can be defined as $\eta = \left(P_1 + P_2\right) / \left(\left|P_1\right| + \left|P_2\right|\right)$ [3], where $P_1 = \int_0^a \left(\vec{E}_1 \times \vec{H}_1^*\right) rdr$ and $P_2 = \int_a^\infty \left(\vec{E}_2 \times \vec{H}_2^*\right) rdr$, respectively.

3. Numerical Results

Fig. 3 shows the dispersion characteristics and their corresponding normalized energy flux for the TE-like modes. Fig. 3 (a) is the dispersion characteristics in the case of $a = 10.0$ mm . Dispersion curves are existed above the

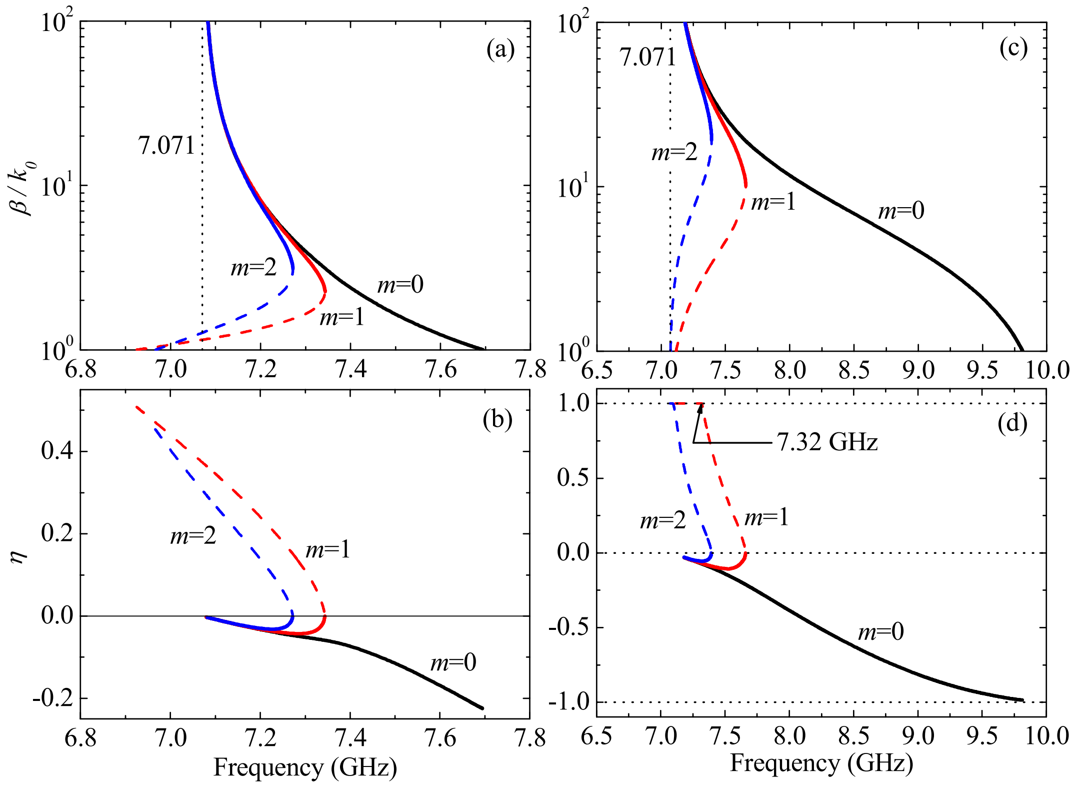

Fig. 4. Dispersion characteristics and normalized power flux for the TM-like modes. (a) Dispersion curves of the metamaterial hole with $a = 10.0$ mm, (b) normalized power flux of the metamaeterial hole with $a = 10.0$ mm, (c) dispersion curves of the metamaterial hole with $a = 1.0$ mm, and (d) normalized power flux of the metamaeterial hole with $a = 1.0$ mm.

frequency of 4.714 GHz, which is corresponding to the point "*B*" in Fig. 2. All the guided mode solutions were existed in the double negative ($\varepsilon_{r2} < 0$ and $\mu_{r2} < 0$) region of Fig. 2. In the case of the metamaterial column waveguides, the guided mode solutions are all existed below this critical frequency [4]. The negative slopes of the dispersion curves indicate the backward waves [5]. The backward waves can also be identified by the normalized energy flux, *i.e.*, $\eta < 0$ for this case as shown in Fig. 3 (b). The cutoff of TE$_{01}$ ($m = 0$) mode is on the $\beta / k_0 = 1.0$, the cutoff of hybrid modes such as EH$_{11}$ ($m = 1$) and EH$_{21}$ ($m = 2$) modes are on the $\beta / k_0 = \sqrt{\mu_{r2}\varepsilon_{r2}}$. We also found the monomodal property for each azimuthal eigenvalue, whereas there exist higher order modes in the case of the metamaterial columns [4]. If we reduce the radius of the metamaterial hole to $a = 1.0$ mm, forward waves are generated in the hybrid modes as shown in the Fig. 3 (c) and (d). Fig. 4 shows the dispersion characteristics and their corresponding normalized energy flux for the TM-like modes. The guided mode solutions are obtained in the ε-negative region ($\varepsilon_{r2} < 0$ and $\mu_{r2} > 0$) of Fig. 2. Fig. 4 (a) shows the dispersion curves of the metamaterial hole with $a = 10.0$ mm. TM$_{01}$ mode has only backward wave portion and the hybrid modes can have both the forward and backward wave portion as shown in Fig. 4 (a) and (b). In the case of $a = 1.0$ mm as shown in Fig. 4 (c), the cutoff frequencies shift toward higher frequencies. Overall values of the β / k_0 are increased compared with those of $a = 10.0$ mm case. In the hybrid mode case, below certain critical frequency, normalized energy flux becomes unity. For the HE$_{11}$ mode, the fractional power flows are shown in Fig. 5 for detailed analysis. Below the frequency of 7.32 GHz, power flows in inner and outer regions are all positive, because the positive values of the Poynting vectors near the surface is dominant as shown in Fig. 5 "A".

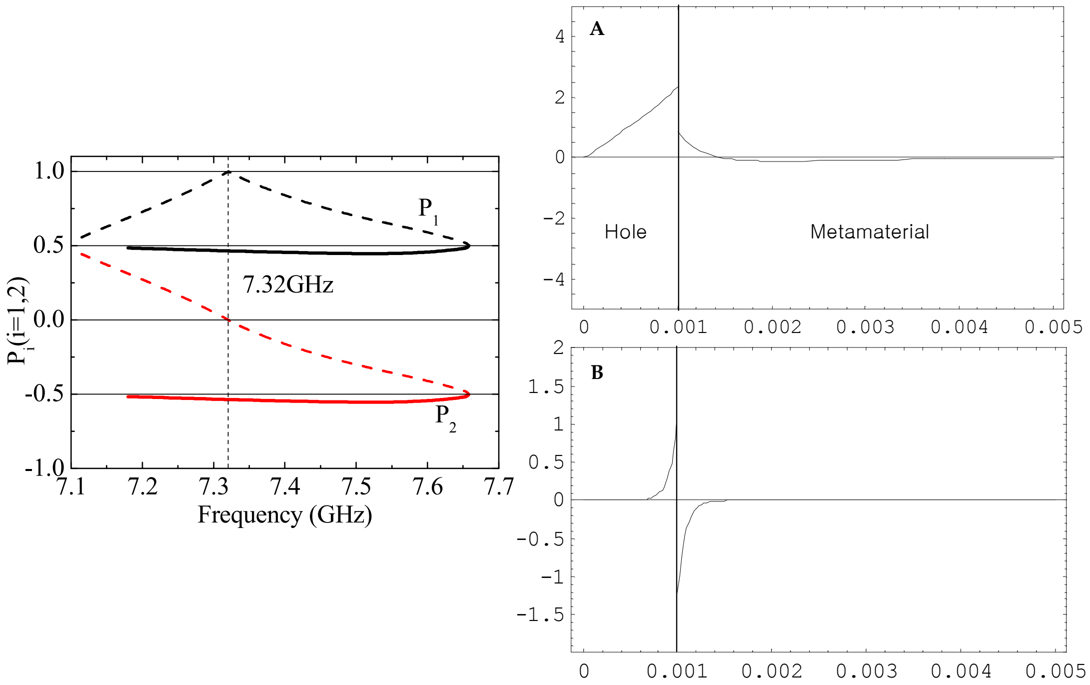

Fig. 5. Fractional power of HE$_{11}$ mode for $a = 1.0$ mm case (left) and its spatial Poynting vector distributions at 7.32 GHz (right). (A: backward, B: forward) Amplitude is in arbitrary units.

4. Conclusions

In this work, we investigated the electromagnetic dispersion characteristics and power distributions on the subwavelength hole which is surrounded by the doubly dispersive metmaterials. Two kinds of hole radii are considered. Both TE-like and TM-like surface guided modes are existed only in principal mode and their higher order modes are not existed. In any case, TM$_{01}$ and TE$_{01}$ modes support only backward waves and higher order modes can support forward and backward wave modes in certain situations. We have also found that all the fractional power flows are positive below certain frequency due to the negative power cancellation in metamaterial regions.

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