A NEW SCATTERING ENHANCEMENT IN A RANDOM MEDIUM FOR H-WAVE INCIDENCE

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1 Introduction

When a body is embedded in a random medium, the radar cross-section (RCS) of the body may be remarkably different from that in free space. This special phenomenon is called backscattering enhancement, and has been one of the important subjects for radar engineering, remote sensing, astronomy and bioengineering. Backscattering enhancement has been investigated from an academic point of view[1–3] and thereby been said to be a fundamental phenomenon in a random medium and to be produced by statistical coupling of incident and scattered waves. If the body is regarded as a single point and the backscattering enhancement occurs prominently, RCS of the body has generally been taken to be nearly twice as large as that in free space.

On the other hand, when the average scattered intensity is enhanced in the backward direction, it is possible to predict that the intensity decreases in the neighborhood of the backward direction from the law of energy conservation and the statistical independency of scattered waves at points separated widely from each other[1].

To make clear numerically the prediction as well as the RCS characteristics for a practical body scattering, we analyzed a bistatic RCS of a conducting body in a random medium. Our approach[4, 5] is based on general results of both independent studies on the surface current on a conducting body in free space and on the wave propagation and scattering in a random medium. A non-random operator, called current generator, is introduced to get the surface current from any incident wave. The operator depends only on the body surface and can be constructed by Yasuura's method. On the other hand, the wave propagation in a random medium is expressed by use of Green's function in the medium. Here, a representative form of the Green's function is not required but the moments are done for the analysis of average quantities concerning observed waves. We apply a two-scale asymptotic procedure[3, 6] to get the fourth moment of Green's functions. As a result, we have obtained numerically results agreed with the law of energy conservation, and shown some interesting behaviors of bistatic RCS caused by statistical coupling between incident and scattered waves[7, 8, 10].

Above numerical results are limited to E-wave incidence. Here, to investigate effects of the polarization of incident wave on the bistatic RCS, we assume H-wave incidence under the same situation as used for E-wave incidence and calculate numerically the RCS.

2 Formulation

Consider a two-dimensional problem of electromagnetic wave scattering from a perfectly conducting circular cylinder embedded in a continuous random medium, as shown in Fig.1. Here L is the thickness of the random medium surrounding the cylinder and is assumed to be larger enough than the size of the

ISBN: 89-86522-78-0 94460©KEES

cylinder cross-section. The random medium is assumed to be described by the dielectric constant ε, the magnetic permeability μ and the electric conductivity σ, which are expressed as

$$\varepsilon = \varepsilon_0[1 + \delta\varepsilon(\boldsymbol{r})] \ , \ \ \mu = \mu_0 \ , \ \ \sigma = 0 \ , \tag{1}$$

where ε_0, μ_0 are constant and $\delta\varepsilon(\boldsymbol{r})$ is a random function with

$$\langle \delta\varepsilon(\boldsymbol{r}) \rangle = 0 \ , \tag{2}$$

$$\langle \delta\varepsilon(\boldsymbol{r}_1) \cdot \delta\varepsilon(\boldsymbol{r}_2) \rangle = B(\boldsymbol{r}_1 - \boldsymbol{r}_2) \ . \tag{3}$$

Here the angular brackets denote the ensemble average and $B(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ is the correlation function of the random function. For numerous cases, it can be approximated as

$$B(\boldsymbol{r}_1 - \boldsymbol{r}_2) = B_0 \exp\left[-\frac{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^2}{l^2} \right] \ , \tag{4}$$

$$B_0 \ll 1 \ , \ \ kl \gg 1 \ , \tag{5}$$

where B_0, l are the intensity and scale-size of the random medium fluctuation, respectively, and $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber in free space. Under the condition (5), depolarization of electromagnetic waves due to the medium fluctuation can be neglected; and the scalar approximation is valid. In addition, the forward multiple scattering approximation is valid, and hence the backscattering by the random medium becomes negligible. In the present analysis, consequently we do not need to consider the re-incidence of backscattered waves by the random medium on the cylinder[4, 5].

Suppose that the current source with the time factor $\exp(-j\omega t)$ suppressed throughout this paper is a line source, located at $\boldsymbol{r}_\mathrm{T}$, far from and parallel to the cylinder. Then the incident wave is expressed by Green's function in a medium containing the random medium and free space $G(\boldsymbol{r}, \boldsymbol{r}_\mathrm{T})$ whose dimension coefficient is understood. Using the current generator Y that transforms any incident wave into the surface current on the cylinder, we can give the average intensity of scattered waves u_s for H-wave incidence as follows [4, 5]:

$$\begin{aligned} \langle |u_\mathrm{s}|^2 \rangle \ = \ & \int_S \mathrm{d}\boldsymbol{r}_1 \int_S \mathrm{d}\boldsymbol{r}_2 \int_S \mathrm{d}\boldsymbol{r}'_1 \int_S \mathrm{d}\boldsymbol{r}'_2 \Big\{ Y(\boldsymbol{r}_1; \boldsymbol{r}'_1) Y(\boldsymbol{r}_2; \boldsymbol{r}'_2) \\ & \frac{\partial}{\partial n_1} \frac{\partial}{\partial n_2} \langle G(\boldsymbol{r}; \boldsymbol{r}_1) G(\boldsymbol{r}'_1; \boldsymbol{r}_\mathrm{T}) G^*(\boldsymbol{r}; \boldsymbol{r}_2) G^*(\boldsymbol{r}'_2; \boldsymbol{r}_\mathrm{T}) \rangle \Big\} , \end{aligned} \tag{6}$$

where S denotes the cylinder surface, $\partial/\partial n_i$ $(i = 1, 2)$ does the outward normal derivative at $\boldsymbol{r}_i$ on S, and the asterisk the complex conjugate. The Y can be calculated by Yasuura's method[4, 5] and expressed in an infinite series for a circular cylinder[9]:

$$Y(\boldsymbol{r}; \boldsymbol{r}') \ = \ \frac{\mathrm{j}}{k\pi^2 a^2} \sum_{n=-\infty}^{\infty} \frac{\exp\{\mathrm{j}n(\theta - \theta')\}}{J_n(ka)\{\partial H_n^{(1)}(ka)/\partial(ka)\}} \tag{7}$$

where J_n is the Bessel function of order n and $J_n(ka) \neq 0$; that is, the internal resonance frequencies are excepted. The $H_n^{(1)}$ is the Hankel function of first kind.

The fourth moment of Green's functions in (6) can be written as

$$\begin{aligned} & \langle G(\boldsymbol{r}; \boldsymbol{r}_1) G(\boldsymbol{r}'_1; \boldsymbol{r}_\mathrm{T}) G^*(\boldsymbol{r}; \boldsymbol{r}_2) G^*(\boldsymbol{r}'_2; \boldsymbol{r}_\mathrm{T}) \rangle = \\ & \qquad G_0(\boldsymbol{r}; \boldsymbol{r}_1) G_0^*(\boldsymbol{r}; \boldsymbol{r}_2) G_0(\boldsymbol{r}'_1; \boldsymbol{r}_{1\mathrm{T}}) G_0^*(\boldsymbol{r}'_2; \boldsymbol{r}_{2\mathrm{T}}) \cdot m_\mathrm{s} \ , \end{aligned} \tag{8}$$

where G_0 is Green's function in free space[2]. The m_s includes multiple scattering effects of the random medium and can be obtained by two-scale method [3, 6–8]; as a result, it is

$$m_\mathrm{s} \ = \ \frac{k}{2\pi z} \iint_{-\infty}^{\infty} \mathrm{d}\eta \mathrm{d}\rho \exp\left\{ -\frac{jk}{z} \eta[\rho - (x - x_\mathrm{T})] \right\} P(\rho, \eta) \ , \tag{9}$$

where

$$
\begin{aligned}
P(\rho,\eta) &= \exp\Big\{-\frac{\sqrt{\pi}k^2lz}{8}\int_0^{L/z}\mathrm{d}t\Big(\ D[a(\sin\theta_1'-\sin\theta_2')t+\eta t] \\
&\quad +D[a(\sin\theta_1-\sin\theta_2)t+\eta t] \\
&\quad -D[a(\sin\theta_1'-\sin\theta_1)t-\rho(1-t)+\eta t] \\
&\quad -D[a(\sin\theta_2'-\sin\theta_2)t-\rho(1-t)-\eta t] \\
&\quad +D[a(\sin\theta_1'-\sin\theta_2)t-\rho(1-t)] \\
&\quad +D[a(\sin\theta_2'-\sin\theta_1)t-\rho(1-t)]\Big)\Big\}\ , \qquad (10)
\end{aligned}
$$

$$
D(x) = 2B_0\left[1-\exp\left(-\frac{x^2}{l^2}\right)\right]\ . \qquad (11)
$$

3 Numerical results

We calculated the bistatic RCS of a conducting circular cylinder ($ka = 1$) with different parameters of the random medium by using (6), and illustrated the numerical results to study the effects.

In Fig.2, the bistatic RCS σ is shown as a function of β for three cases of kl: $kl = 20\pi$, 2000π and 5000π, where the fluctuation intensity and thickness of the random medium are fixed at $B_0 = 5\times10^{-7}$ and $kL = 8\times10^4\pi$, respectively. For all the three cases, σ tends to that in free space σ_0 if β is large enough, and the integral value of σ with respect to β is equal to that of σ_0. This fact means that the results agree with the law of energy conservation.

In the case of $kl = 2000\pi$, the oscillation of σ becomes a wonted shape (refer to [7, 8, 10]): there are a backscattering enhancement peak where σ becomes about 2.6 times as large as σ_0 and a depression outside the peak where σ is less than σ_0. In the cases of $kl = 20\pi$ and $kl = 5000\pi$, σ at $\beta = 0$ are almost equal to σ_0, which means there is almost no backscattering enhancement. We know that the effect of multiple scattering becomes weak as the scale-size increases, because the number of multiple scattering in a random medium becomes smaller with an increase in scale size, when the random medium thickness is fixed. It is easy to understand that the effect of backscattering enhancement in the case of $kl = 5000\pi$ is less than that in the case of $kl = 2000\pi$, but the problem of how to explain the case of $kl = 20\pi$ still remains to be solved.

We note that the RCS in the case of $kl = 20\pi$ displays a complicated behavior with increasing β. It is depressed in the neighborhood of backward direction, and enhanced just outside the depression. The enhancement extends to a wider region although the peak is not so high. This result hints that the scattering enhancement phenomenon may occur not just in the backward direction but in the other directions. To estimate the effect of multiple scattering from the contribution to bistatic RCS, we calculate the variance of σ from

$$
\frac{1}{0.06}\int_0^{0.06}\left(\frac{\sigma-\sigma_0}{\sigma_0}\right)^2\mathrm{d}\beta \qquad (12)
$$

for the three cases. The results become 6.9×10^{-2}, 4.1×10^{-2} and 6.0×10^{-3} for cases of $kl = 20\pi$, $kl = 2000\pi$ and $kl = 5000\pi$, respectively. The fact shows that the effect of multiple scattering on RCS reduces with increasing kl.

As well known, both σ and σ_0 depend on the polarization of incident wave: however, σ/σ_0 for H-wave incidence is almost the same as obtained for E-wave incidence because the spatial coherence length of incident wave at the cylinder is much larger than the size of the cylinder.

4 Conclusion

We discussed the scattering characteristic of a conducting circular cylinder embedded in a random medium by changing the scale-size of the medium. The numerical results of bistatic radar cross-section

(RCS) show that sometimes the scattering enhancement phenomenon may not occur in the backward direction but in the other directions, where a scattering depression region may exist in the neighborhood of backward direction and scattering enhancement may be observed outside the depression region. The region of the enhancement may be much wider than that of the well known backscattering enhancement, although the enhancement peak is not so high. The complicated oscillation of bistatic RCS is considered to be caused by statistical interference of incident and scattered waves. For all numerical results, the intergral value of the bistatic RCS with respect to β is almost equal to that in free space, which fact shows that the results agree with the law of energy conservation.

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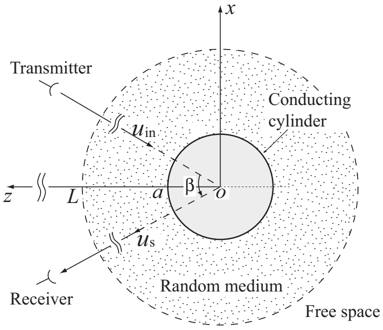

Fig.1 Geometry of the scattering problem from a conducting cylinder surrounded by a random medium.

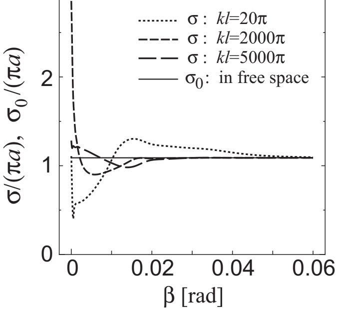

Fig.2 Bistatic RCS (σ) of the cylinder in random media with different scale-sizes and that in free space (σ_0) .