ANALYSIS OF ELECTROMAGNETIC WAVE PROPAGATION INTO REINFORCED CONCRETE WALLS USING THE PARABOLIC EQUATION METHOD

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Abstract: In this paper, a new method of studying wave penetration into and propagation through reinforced concrete walls using the parabolic equation method is presented. The influence of several parameters of wall such as thickness, inter-bar spacings and steel diameters on the plane wave transmission and reflection coefficients is studied. The good agreement between the simulation results and available published ones validates the proposed method.

1. Introduction

With the rapid growth of wireless communication systems, the need to accurately model of urban and indoor propagation channels has increased. The ray tracing method is widely used as a site-specific method for effective and optimum design of wireless networks. In this method, wall structures are modeled as dielectric slabs with Fresnel transmission and reflection coefficients, which is a good model for homogeneous walls. In many modern buildings, reinforced concrete walls are one of the commonly used architectural elements which are made up of steel bars embedded in concrete slabs, where the ray tracing method is not adequate to accurately model radio wave propagation through them.

Transmission and reflection characteristics of nonhomogeneous walls have been studied by many numerical and analytical methods [1]-[3]. In [1] a modal approach was used for modeling wave propagation through concrete block walls. The finite-element method (FEM) and the finite-difference time-domain (FDTD) method have been used for analyzing this problem in [2] and [3], respectively. Both of them are complicated and time consuming.

In this paper we use the parabolic equation method (PEM) which is very efficient for studying such problems. To our best knowledge, PEM has not been used for modeling radiowave propagation through nonhomogeneous periodic walls. The aim of the method presented here is to characterize reflection and transmission properties of reinforced concrete walls under plane wave incidence. We analyze the effect of some parameters namely wall thickness, rebar spacings and their diameters on the transmission coefficients of these kinds of reinforced concrete walls.

2. Problem description and formulation

In a source free medium, the y-independent wave equation for a field component Ψ can be written as

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial z^2} - \gamma^2 \Psi = 0 \quad (1)$$

where $\gamma = \alpha + j\beta$ is the complex propagation constant and a time dependence of $e^{-j\omega t}$ is assumed. We wish to extract the paraxial approximation of the wave equation (1) in the lossy medium. Accordingly, we define the reduced function $u(x,z)$ associated with the paraxial direction x by

$$u(x,z) = e^{j\beta x} \Psi(x,z) \quad (2)$$

Therefore the wave equation in terms of u is

$$\frac{\partial^2 u}{\partial x^2} - 2j\beta \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial z^2} - \left(\gamma^2 + \beta^2\right) u = 0 \quad (3)$$

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By factorization with respect to $\frac{\partial}{\partial x}$, the pseudo-differential equation corresponding to the wave propagating in the $+x$ direction is

$$\frac{\partial u}{\partial x} - j\beta(1-Q)u = 0 \tag{4}$$

where we define the pseudo-differential operator Q by

$$Q = \frac{\gamma}{j\beta}\sqrt{1-\frac{1}{2\gamma^2}\frac{\partial^2}{\partial z^2}} \tag{5}$$

Equation (4) can be solved by marching techniques, given the initial field and the boundary conditions at the top and bottom of the domain. This equation has the formal solution

$$u(x+\Delta x, z) = e^{j\beta(1-Q)}u(x,z) \tag{6}$$

Application of the first-order Taylor expansions of the square-root term in (6) and then Padé (1,0) approximation of the exponential functions [4], results in the following expression

$$\left(1-\frac{1}{2\gamma}\frac{\partial^2}{\partial z^2}\right)u(x+\Delta x, z) = e^{-\alpha\Delta x}u(x,z) \tag{7}$$

By discretizing (7) with respect to z, we obtain a finite-difference equation for u in the form

$$-bu_{n+1}^{m+1} + (1+2b)u_n^{m+1} - bu_{n-1}^{m+1} = e^{-\alpha\Delta x}u_n^m \tag{8}$$

where *m* and *n* are position indices in the x and z directions respectively and $b = \frac{\Delta x}{2\gamma\Delta z^2}$.

This equation can be written as a matrix equation of the general form

$$\mathbf{A}\mathbf{u}^{m+1} = e^{-\alpha\Delta x}\mathbf{u}^m \tag{9}$$

which allows the field at a given range to be found from the field at the previous range.
The general reinforced concrete wall structure to be analyzed is shown in Fig. 1. The problem can be solved by applying PEM to a single cell and periodic boundary conditions along boundaries of the cell. In the case of normally incident plane wave, symmetric boundary conditions must be applied along these boundaries by changing the first and last rows of the matrix **A**. This means that if we have N mesh points along the z axis, then

$$\mathbf{A}_{1,2} = -2b, \qquad \mathbf{A}_{N,N-1} = -2b \tag{10}$$

Since we assume perfectly conducting rebar, u must be zero at the mesh point on the rebar surface and inside it. To satisfy these conditions the matrix values representing the field *u* at these mesh points must be set to zero. For applying PEM, the wall can be approximated as a three-layer periodic structure, where the bar with diameter D is approximated by a square bar with side length equal to the diameter [5]. A plane wave which is normally incident onto the wall in Fig. 1, produces a transmitted and a reflected field on its left side. We save the reflected wave and set the transmitted field as an initial field for PEM, and compute the field by the marching method along the x direction until the wave line crosses the rebar left side. At this range, other transmitted and reflected waves are generated where we set the transmitted field as an initial field for the next marching step and save the reflected field. We continue the marching method until the right side of the wall, where another transmitted and reflected fields are generated. At this step, the transmitted field is saved and the reflected field is set as the initial field. Then the paraxial direction is changed to $-x$ and the algorithm is continued in the reverse direction in a similar manner up to the left surface of the bar, where the reflected field from the previous stage is added to the field of the present stage. The algorithm is continued until the maximum amplitude of the matrix values **u** becomes less than a threshold. The total transmitted and reflected fields can be computed by adding all of the transmitted and reflected fields on the right and left sides of the wall, respectively.

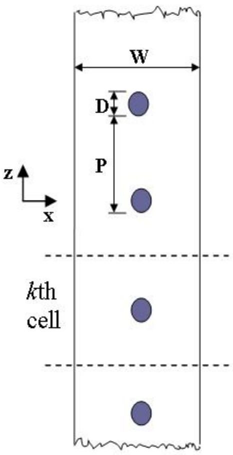

Fig. 1. General reinforced concrete wall onfiguration.

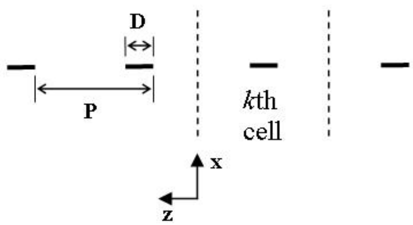

Fig. 2. Array of perfect electric conductor strips.

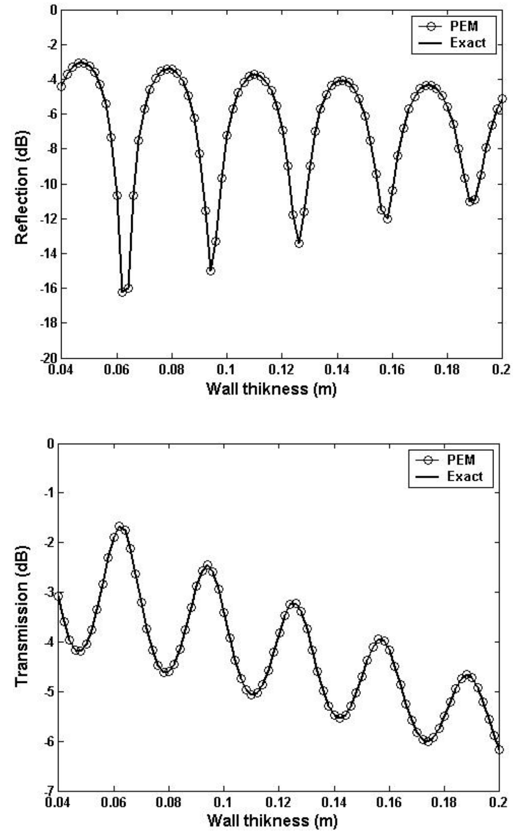

Fig. 3. Reflection and transmission coefficient for a wall with $\varepsilon_r = 7 - j0.3$ at 1800 MHz.

3. Numerical results

To validate the proposed PEM, we apply it to a homogenous wall with complex dielectric constant of $\varepsilon_r = 7 - j.3$ at operating frequency of 1800 MHz. As can be seen in Fig. 3 the PEM results for reflection and transmission coefficient coincide with the exact values. The conclusion here is that the proposed method with the symmetric boundary conditions described above can be used to accurately calculate transmission and reflection coefficients for a homogenous wall.

Next, we apply the proposed method to a periodic structure. We consider the problem of electromagnetic scattering from an infinite 2 dimensional array of parallel coplanar perfect electric conductor strips shown in Fig. 2. We assume $\gamma = ik_0$, where k_0 is the wave number in free-space. The PEM results are obtained for D=1 mm at 1 GHz for different inter-strip spacings. They are compared in Fig. 4 with the moment method results are given in [6]. Good agreement between the two sets of results indicates that the proposed algorithm is an efficient method for studying plane wave incidence onto periodic structures.

To investigate the effect of a grid of bars embedded in a homogeneous concrete medium, the PEM method is applied to the reinforced concrete wall of Fig. 1 with $\varepsilon_r = 7 - j0.3$ and steel diameter of 2 mm at 1800 MHz. Figs. 5 and 6 show the transmission coefficient for rebar spacings equal to 0.1 and 0.2 m, respectively. In Fig. 7 the transmission coefficients are compared for two different steel diameters of 2 and 4 mm. It can be seen that the effect of steel diameter variation is not negligible at this frequency.

4. Conclusion

To study the plane wave incidence onto reinforced concrete walls, a method based on the parabolic equation has been employed. Due to the periodicity of the structure, the studied domain is reduced to an elementary cell by the use of periodic boundary conditions. It was shown that the effect of the rebar on

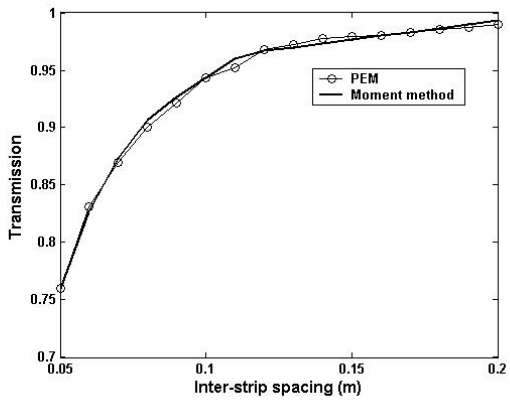

Fig. 4. Transmission coefficient of perfect electric conductor strip array at 1 GHz for D=2 mm .

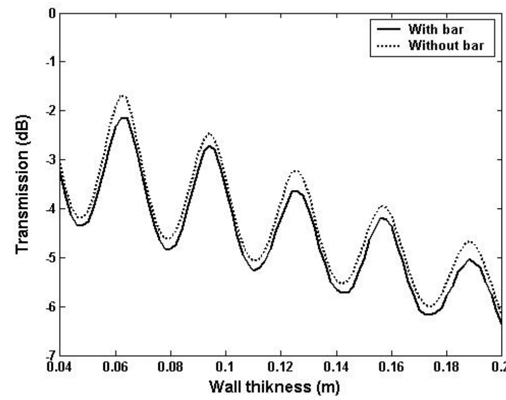

Fig. 5. Transmission coefficient of the wall at 1800 MHz for P=.2 m and D=2 mm .

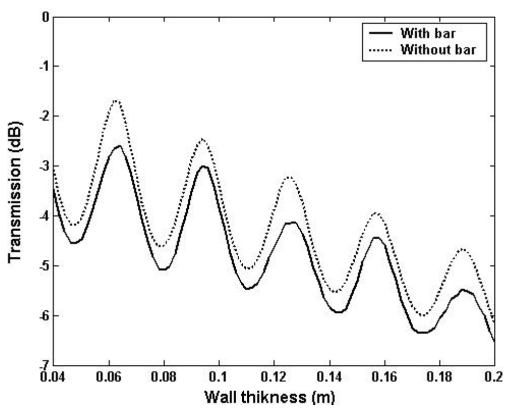

Fig. 5. Transmission coefficient of the wall at 1800 MHz for P=.1 m and D=2 mm .

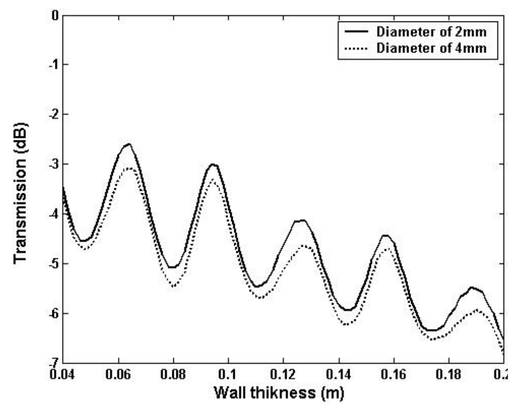

Fig. 7. Transmission coefficient of the wall at 1800 MHz for P=.1 m, D=2 mm and D=4 mm.

propagation of radiowaves can not be neglected in studying such structures.

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