THE DOA ESTIMATION OF INCOHERENTLY DISTRIBUTED SOURCE USING THE PROPAGATOR METHOD

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1. Introduction

In most applications of array processing, source parameter estimation methods are based on point source modeling, where it is assumed that the energy arriving at a sensor array originates from multiple point sources. In terms of direction finding, this means that the source energy is assumed to be concentrated at discrete angles which are referred to as DOAs. Based on this assumption, several high-resolution direction finding methods have been proposed to estimate the source DOAs. However, in numerous applications such as sonar, radar, and wireless communications, signal scattering phenomena may cause angular spreading of the source energy. Hence, in such cases, the distributed source model is more appropriate than the point source one[1].

Several techniques have been proposed for distributed source parameter estimation, such as the maximum likelihood technique[2], the DSPE[3], the DISPARE[4], the covariance matching estimation technique[5] and the subspace fitting concept[6] etc. However, some of these methods lead to a multidimensional nonlinear optimization problem and others need singular value decomposition, all of which require high computational complexity.

In this paper, a deterministic approach using the first order Taylor series expansion of the nominal steering vector of the distributed source is proposed. Then the array manifold consists of a linear combination of the nominal steering vector and its gradient. With the assumption of small angular spread, we introduce the propagator method(PM) as a possible alternative to MUSIC method for distributed source nominal DOA estimation . We find that at high and medium SNR, the PM performs quite like MUSIC. At low SNR, the PM outperforms MUSIC. In addition, the PM has reduced computational complexity with a least squares process and without any eigenvalue decomposition of the covariance matrix of the received signals.

2. Distributed source model

Consider the case of a spatially distributed source impinging on an ULA. The signal is the superposition of all contributions due to local scatterers and can thus be written as

$$\mathbf{x}(t) = \sum_{l=1}^{L} \gamma_l(t)\mathbf{a}\left(\theta_0 + \tilde{\theta}_l(t)\right)s(t) + \mathbf{n}(t) \tag{1}$$

where $\gamma_l(t)$ is the random complex gain of the point source, the steering vector can be modeled as $\mathbf{a}\left(\theta_0 + \tilde{\theta}_l(t)\right)$, θ_0 is the nominal DOA of the distributed source, $\tilde{\theta}_l(t)$ is a zero-mean random angular deviation from the nominal DOA, $\mathbf{n}(t)$ is the additive white Gaussian noise.

3. The first order Taylor series expansion of the nominal steering vector

With the assumption of small angular spread, a Taylor expansion of the steering vector gives

$$\begin{aligned}\mathbf{a}(\theta) &= \sum_{l=1}^{L} \gamma_l(t)\mathbf{a}\left(\theta_0 + \tilde{\theta}_l(t)\right) \\ &\approx \sum_{l=1}^{L} \gamma_l(t)\left(\mathbf{a}(\theta_0) + \tilde{\theta}_l(t)\mathbf{d}(\theta_0)\right)\end{aligned} \tag{2}$$

where the complex gain is included and $\mathbf{d}(\theta_0)$ is the gradient of the steering vector,

ISBN: 89-86522-78-0 94460©KEES

$$\mathbf{d}(\theta)=\frac{d\mathbf{a}(\theta)}{d\theta} \tag{3}$$

It is reasonable to assume that $\sum\gamma_l$ is non-zero and thus the steering vector can be written as

$$\mathbf{a}(\theta)\approx\mathbf{a}(\theta_0)+\rho_i\mathbf{d}(\theta_0) \tag{4}$$

where $\rho_i=\sum\gamma_l\tilde{\theta}_l(t)\big/\sum\gamma_l$. For small angular spread, ρ_i is regarded as small.

For an ULA with *m* omnidirectional elements separated by $\vartriangle$ wavelengths, the steering vector and its derivative are

$$\mathbf{a}(\theta)=\left[1,e^{-j2\pi\vartriangle\sin\theta},\cdots,e^{-j2\pi\vartriangle(m-1)\sin\theta}\right]^T \tag{5}$$

$$\mathbf{d}(\theta)=[0,-j2\pi\vartriangle\cos\theta e^{-j2\pi\vartriangle\sin\theta},\cdots,-j2\pi\vartriangle(m-1)\cos\theta e^{-j2\pi\vartriangle\sin\theta}]^T \tag{6}$$

where the superscript *T* denotes the transpose of a matrix. Substituting (5) and (6) into (4) , The *n*th row of (4) may now be approximated as

$$\begin{aligned}&\left(a(\theta_0)+\rho_i\mathbf{d}(\theta)\right)_n\\&=e^{-j2\pi(n-1)\vartriangle\sin\theta_0}(1-\rho_i j2\pi\vartriangle(n-1)\cos\theta_0)\\&\approx e^{-j2\pi(n-1)\vartriangle\sin\theta_0}e^{-j\rho_i 2\pi\vartriangle(n-1)\cos\theta_0}\end{aligned} \tag{7}$$

So the steering vector can be written as

$$\mathbf{a}(\theta)\approx[1,e^{-jw_i},\cdots,e^{-j(m-1)w_i}]^T \tag{8}$$

where $w_i=2\pi\vartriangle\left(\sin\theta_0+\rho_i\cos\theta_0\right)$. As the angular spread goes to zero, ρ_i also goes to zero. Then (8) reduces to the conventional plane wave steering vector.

4. The PM for the distributed source

For incoherently distributed source, the noise subspace is generally degenerate(i.e., equal to the zero vector) and the whole observation space is occupied by the signal components. In such case, the direct application of conventional high resolution array processing methods will produce erroneous results.

In this paper, the PM is applied to distributed source parameter estimation for the first time. The PM is a subspace-based method which does not require the eigendecomposition of the covariance matrix of the received signals. The use of a partition in the PM to subdivide the whole array into two subarrays is a technique that has similarities with ESPRIT[9]. However, the PM presents several new features

1) The two subarrays may be quite different and the only constraint is that the number of sensors for one of them should be at least equal to the number of distinct incident wavefronts.
2) The sensors may be arranged in any order.

The definition of the PM is based on the partition of the steering vectors(8) according to

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}_1\\\mathbf{A}_2\end{bmatrix} \tag{9}$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are matrices of dimension $N\times N$ and $(M-N)\times N$, respectively. M is the number of the sensors, N is the number of distinct incident wavefronts.

The linear dependence holds between the first N rows of the steering vector and the others,

$$\mathbf{P}^H\mathbf{A}_1=\mathbf{A}_2 \tag{10}$$

or

$$\left[\mathbf{P}^H \quad , \quad -\mathbf{I}_{M-N}\right]\mathbf{A}=\mathbf{Q}^H\mathbf{A}=\mathbf{0} \tag{11}$$

where $\mathbf{P}$ is the propagator operator, $\mathbf{I}_{M-N}$ is the identity matrix of dimension $(M-N)$, $[\bullet]^H$ denotes the conjugate transpose of a matrix. The propagator operator depends only on propagation parameters, such as those characterizing wavefronts geometry, antenna shape, and sensors complex gains, not on the complex amplitudes of the sources. So we introduce the following partition of the data matrix. It can be as follows,

1) The received data matrix is

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}(1) & \mathbf{x}(2) & \cdots & \mathbf{x}(K) \end{bmatrix} \tag{12}$$

2) The partition of the data matrix is

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} \tag{13}$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ are matrices of dimension $N \times N$ and $(M-N)\times N$ respectively.

3) The estimation of the propagator operator can be obtained by minimizing the cost function

$$\min \left\| \mathbf{X}_2 - \widehat{\mathbf{P}}^H \mathbf{X}_1 \right\|^2 \tag{14}$$

$$\widehat{\mathbf{P}} = \left(\mathbf{X}_1 \mathbf{X}_1^H \right)^{-1} \mathbf{X}_1 \mathbf{X}_2^H \tag{15}$$

According to (11), the estimation of the nominal DOAs of the distributed sources can be obtained by minimizing the follow expression

$$d(\theta) = \left\| \hat{\mathbf{Q}}^H \mathbf{A}(\theta) \right\| \tag{16}$$

5. Simulation Results

In this section, we provide numerical results to compare the performances of our proposed PM with those of MUSIC estimator. We consider an ULA with 8 elements separated by a half wavelength.

In the first example, it is the case of a single distributed source. This source is assumed to be Gaussian azimuthal power distribution with the nominal DOA 5° and angular spread 1.5°. The number of snapshots is 500. For angular spread is small, we are interested in nominal DOA. Fig.1 shows the one-dimensional spectrum for the nominal DOA estimation. Fig.2 shows the root-mean-squared error(RMSE) of the estimation of the nominal DOA versus the SNR. From Fig.2, it is clear that the performance of the PM outperforms MUSIC when the SNR is low. For moderate and high SNR, the PM performs like MUSIC. So the major advantages of the PM are better performance at low SNR and lower computational cost.

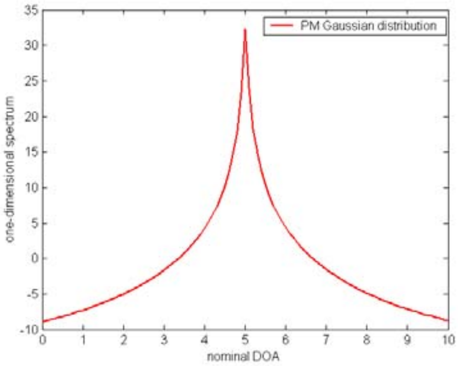

Fig.1. the nominal DOA estimation of a single distributed source

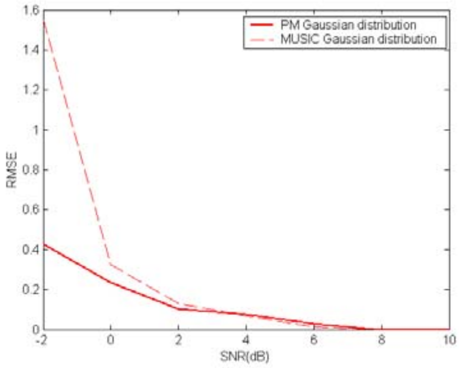

Fig.2. RMSE of the nominal DOA estimate of a single distributed source versus the SNR

In the second example, we assume two distributed sources. One of them is Gaussian azimuthal power distribution with nominal DOA 3° and angular spread 1.5°. The other source is uniform azimuthal power distribution with nominal DOA 8° and angular spread 1.5°. Fig.3 shows the one-dimensional spectrum for the nominal DOAs estimation. Fig.4 shows the RMSE of the estimation of the nominal DOAs versus the SNR. From Fig.4, when the SNR is low, it is clear that MUSIC can't give the ideal estimation. However, the RMSE of the estimation of the DOAs by the PM is smaller. In particular, the performance of the PM is better in uniform azimuthal power distribution. When the SNR is getting high, the performance of the PM is close to MUSIC. We can conclude the application of the PM is not restricted to the single source case. The sources may have different forms of distribution.

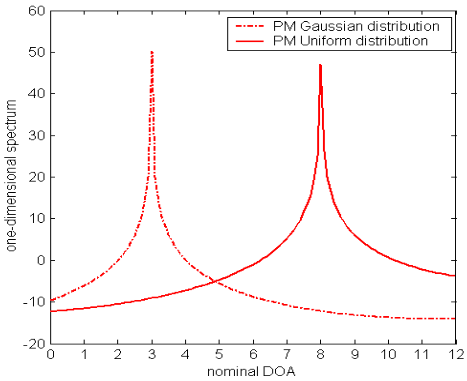

Fig.3. the nominal DOA estimation of two distributed sources

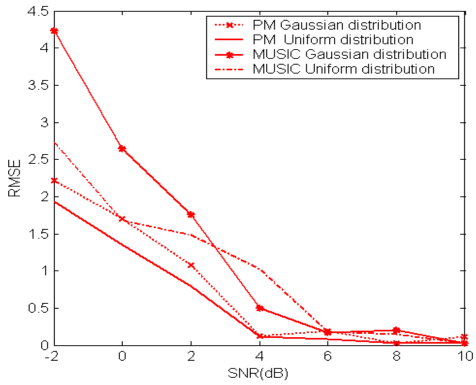

Fig.4. RMSE of the nominal DOA estimate of two distributed sources versus the SNR

6. Conclusions

In this paper, we considered the estimation of the nominal DOA of incoherently distributed source with a view to provide the validity of the algorithm at low SNR and lower computational complexity. Toward this end, the first order Taylor series expansion of the nominal steering vector of the distributed source is applied and the PM is introduced to estimate the nominal DOA. The algorithm presents several new features

1) It does not require the eigendecomposition of the covariance matrix of the received signals.
2) It has a substantially better performance at low SNR.
3) It's applicable to the multisource scenarios with different azimuthal power distribution.

Simulation figures clearly demonstrate that the PM is shown to be valid at low SNR and consistently enjoy a good performance at medium and high SNR as compared with MUSIC.

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