Matrix Pencil Method Using Fourth-Order Cumulant

Woojin Jang, Weiwei Zhou, Yisu Wang, Jinhwan Koh
Department of Electronic Engineering, Engineering Research Institute,
GyeongSang National University
Jinju, Gyeongsangnam-do, 660-701, Korea
jwjflys@hanmail.net
jikoh@gsnu.ac.kr

1. Introduction

In recent years, higher order cumulant-based methods, such as fourth-order cumulant, have received increasing interest due to their advantage over second-order statistics. The Matrix Pencil method(MPM) can be used when the received signals of arrays are approximated by sums of complex exponentials[1][2]. In 1991, Yingbo Hua first applied higher order statistics, third-order moment, to the Matrix Pencil method[3], but the performance of these method was not enough to satisfy. When order is two or high, high-order cumulants of a Gaussian processes become all zero[4]. Utilizing this property, one can enhance the performance of MPM. The fourth-order cumulant for harmonic retrieval were presented in [5]-[7].

In this paper, we applied the fourth-order cumulant to the Matrix Pencil method. By substituting the fourth-order cumulant of the original array input data the Gaussian additive noise to the signals can be suppressed. The average of conventional MPM has been compared to the proposed method. Simulation result shows that the fourth-order cumulant method generates better performance in finding DOA than the conventional MPM.

2. Proposed Matrix Pencil method using the fourth-order cumulant

In the Matrix Pencil method, the sampled signal x(k) is to be modeled by a sum of complex exponentials, i.e.,

$$x(k)=\sum_{i=1}^{M} R_i e^{\omega_i k T_s}=\sum_{i=1}^{M} R_i z_i^{\,k} \quad ,(1)$$

where R_i = Residues or complex amplitudes, ω_i = Angular frequencies, $e^{\omega_i T_s}=z_i$ for i = 1,2,...,M. and we assumed the damping factor is not important.

The objective is to find the best estimates of M, R_i and z_i from $x(kT_s)$ [6][7]. We assume that the received signal of the respective array is used, which is expressed by (1) and is time series, thus it can be described as the following expression

$$x_n(t)=\sum_{i=1}^{M} R_i e^{j\omega_i (t+n\tau)} \ , (2)$$

where n=0,1,2,⋯, N-1, N denotes the number of arrays, and τ is time delay. If the

ISBN: 89-86522-78-0 94460©KEES

respective input signals have zero mean, the fourth-order cumulant can be estimated from the followings:

$$\begin{aligned} C_4 &= Cum\left\langle x_n(t), x_{n+1}(t), x_{n+2}(t), x_{n+3}(t) \right\rangle \\ &= E\{x_n(t), x_{n+1}(t), x_{n+2}(t), x_{n+3}(t)\} - E\{x_n(t), x_{n+1}(t)\}E\{x_{n+2}(t), x_{n+3}(t)\} \qquad (3) \\ &\quad - E\{x_n(t), x_{n+2}(t)\}E\{x_{n+1}(t), x_{n+3}(t)\} - E\{x_n(t), x_{n+3}(t)\}E\{x_{n+1}(t), x_{n+2}(t)\}, \end{aligned}$$

where "Cum" is the abbreviation of cumulant, this expression implies that the fourth-order cumulant requires knowledge of all moments up to order 4.

To attain the sum of the complex exponentials related to only time τ, we must exchange $Cum\left\langle x_n(t), x_{n+1}(t), x_{n+2}(t), x_{n+3}(t) \right\rangle$ for $Cum\left\langle x_n^*(t), x_n^*(t), x_n(t), x_{n+1}(t) \right\rangle$, then the expression (3) can be rewritten by

$$\begin{aligned} C_4 &= Cum\left\langle x_n^*(t), x_n^*(t), x_n(t), x_{n+1}(t) \right\rangle \\ &= E\{x_n^*(t), x_n^*(t), x_n(t), x_{n+1}(t)\} - E\{x_n^*(t), x_n^*(t)\}E\{x_n(t), x_{n+1}(t)\} \qquad (4) \\ &\quad - E\{x_n^*(t), x_n(t)\}E\{x_n^*(t), x_{n+1}(t)\} - E\{x_n^*(t), x_{n+1}(t)\}E\{x_n^*(t), x_n(t)\}. \end{aligned}$$

The right term remains to only the following expression.

$$Cum\left\langle x_n^*(t), x_{n+1}^*(t), x_{n+1}(t), x_{n+1}(t) \right\rangle = -\sum_{i=1}^{M} R_i^4 e^{j\omega_i \tau} \qquad (5)$$

It turns out that the fourth-order cumulant has the sum of the complex exponentials, which is related to τ, such as (1).

Accordingly, we can write the fourth-order cumulant matrix C in the following.

$$C = Cum\left\langle x_0^*(t), x_0^*(t), x_0(t) \begin{bmatrix} x_0(t) & x_1(t) & \cdots & x_L(t) \\ x_1(t) & x_2(t) & \cdots & x_{L+1}(t) \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-L-1}(t) & x_{N-L}(t) & \cdots & x_{N-1}(t) \end{bmatrix} \right\rangle \qquad (6)$$

$$= \begin{bmatrix} C_4(0) & C_4(1) & \cdots & C_4(L) \\ C_4(1) & C_4(2) & \cdots & C_4(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ C_4(N-L-1) & C_4(N-L) & \cdots & C_4(N-1) \end{bmatrix}$$

Thus, we can apply this matrix to initial matrix of the Matrix Pencil method[6][7].

3. Numerical Simulations

For the first example, consider a signal of unit amplitude arriving from $\beta_1 = \pi/5.$, i.e., $u = e^{(j\beta_1 t_{i,j})} + N_{i,j}; i = 0,1,2,...,99,\ \text{j=0,1,2,...,10}$, where $N_{i,j}$ denote the noise with Gaussian distributions. Fig. 1 shows that as the SNR increase the error in the estimation of the fourth-order cumulant decreases much more than that of the conventional MPM. The pencil parameter, L, was equal to 5 with N=10 and the size of the pencil matrix was 5 by 5.

For the second example, consider a signal of unit amplitudes arriving from $\beta_1 = \pi/3$ and $\beta_2 = \pi/3 + \beta_d$, i.e., $u = e^{(j\beta_1 t_{i,j})} + e^{(j\beta_2 t_{i,j})} + N_{i,j}; i = 0,1,2,...,99,\ \text{j=0,1,2,...,60}$. The frequency error in the estimation was defined as $\| \beta - \beta_{est} \|_2 / \| \beta_2 - \beta_1 \|_2$, where β_{est} is estimated frequency of β_1 and β_2 and $\beta = [\beta_1 \beta_2]^T$. Fig. 2 shows the Monte Carlo simulation of the angle, β_d, versus the error in the estimation. In Fig. 2, the pencil parameter, L, was equal to 10 and 30, and the size of pencil matrix was 10 by

10 and 30 by 30.

For the third example, consider a signal of unit amplitude arriving from $\beta_1 = \pi/2$ and a jammer arriving at $\beta_2 = \pi/2 + \beta_d$ with amplitude J_{mag}, i.e., $u = e^{(j\beta_1 t_{i,j})} + J_{mag} e^{(j\beta_2 t_{i,j})} + N_{i,j};\ i = 0,1,2,\ldots,99,\ \text{j}=0,1,2,\ldots,50$. Fig. 3 is to find jammer strength that is going to produce an output error of 1% in the estimation of the signal strength. Observe that the fourth-order cumulant gives higher resolution than the MPM.

4. Conclusion

In this paper, the performance of the proposed Matrix Pencil method using the fourth-order cumulant and the conventional Matrix Pencil method for DOA estimation was compared. Simulation results show that the proposed method has better performance than the conventional MPM in terms of SNR, as well as resolution. We can observe that the fourth-order cumulant can successfully suppress the Gaussian noise.

References

[1] Sarkar, T. K. and Pereira, O. “Using the Matrix Pencil Method to Estimate the Parameters of a Sum of Complex Exponentials”, IEEE Antennas and Propagation Magazine, Vol. 37, No. 1, February 1995

[2] Jinhwan KOH, Dongmin LIM, and Tapan K. SARKAR, “DOA Estimation Using Matrix Pencil Method”, IEICE TRANS. , VOL. E85-A, NO. 1, January 2002

[3] Yingbo Hua, “Parameter Estimation of Exponentially Damped Sinusoids Using Higher Order Statistics and Matrix Pencil, IEEE TRANSACTIONS ON SIGNAL PROCESSING. VOL. 39. NO. 7, JULY 1991

[4] Chrysostomos L. Nikias, A. P. Petropulu, “Higher-Order Spectra Analysis, Prentice Hall, 1993

[5] Z. Shi, and F. W. Fairman, “Harmonics Retrieval via State Space and Fourth-Order Cumulants”, IEEE Trans. Signal Processing, Vol. ASSP-42, No. 5, pp.1109-1119, May 1994.

[6] X. Zhang, and Y. Li, “Harmonics Retrieval in Mixed Gaussian and Non-Gaussian ARMA Noises”, IEEE Trans. Signal Processing, Vol. ASSP-42, No. 12, pp.3539-3543, December 1994.

[7] J. A. R. Fonollosa, “Sample Cumulants of Stationary Processes : Asymptotic Results”, IEEE Trans. Signal Processing, Vol. ASSP-43, No. 4, pp.967-977, April 1995.

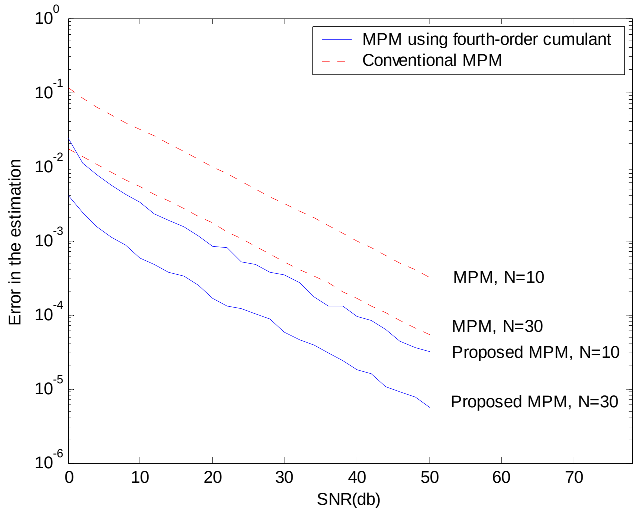

Fig. 1 SNR and error in the estimations, different number of arrays

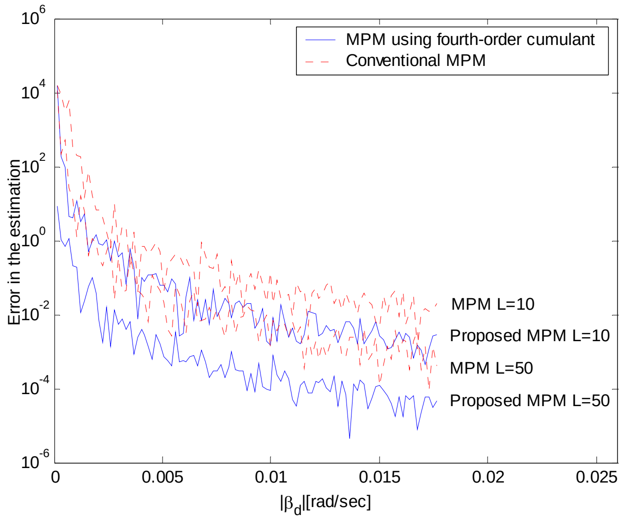

Fig. 2 Resolution and errors in the estimations, different number of arrays

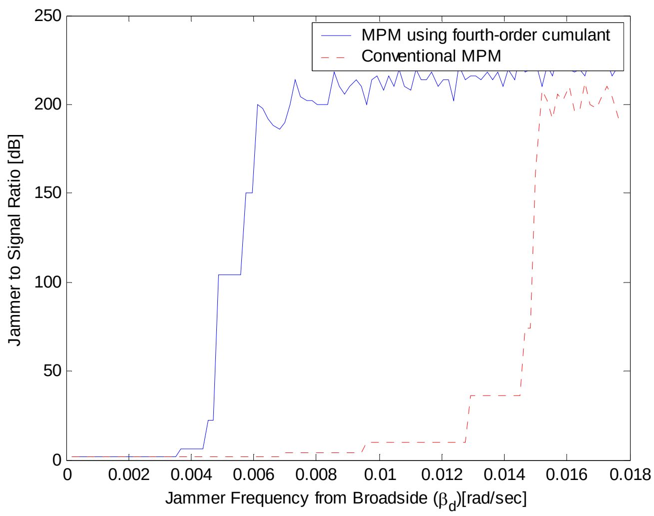

Fig. 3 40dB output SNR added noise criterion