An Estimation of Channel Characteristics for SVD-MIMO Antenna Array

Chul-Keun Park and Kyeong-Sik Min
Department of Radio Sciences and Engineering,
College of Sciences and Engineering, Korea Maritime University,
#1 Dongsam-Dong, Yeongdo-Ku, Busan, 606-791, Korea
E-mail: ck5013@bada.hhu.ac.kr, Tel: +82-51-410-4425

Abstract-This paper presents an analysis of the channel capacity for 2×2 multiple input multiple output (MIMO) antenna systems. In this research, channel capacity is simultaneously analyzed by arbitrary transmitting (Tx) and receiving (Rx) antenna correlation matrix with correlated fading channel matrix. The channel capacity for 2×2 MIMO system is calculated. Considered parameters include receiving signal to noise ratio (SNR), and Doppler spread. In MIMO antenna system, the relation between Tx/Rx correlation matrix and channel state is presented by singular value decomposition (SVD) with polar decomposition. From the simulation results, we confirmed that the maximum channel capacity is achieved by control of theta of channel matrix with polar decomposition.

I. INTRODUCTION

In wireless communications, multiple-input multiple-output (MIMO) techniques have recently emerged as a new paradigm. It has been shown that MIMO is a promising approach that can lead to very large bandwidth efficiencies [1]. The spectral efficiency in the same frequency channel is achieved by using multiple transmitting (Tx) and receiving (Rx) antennas as well as appropriate receiving schemes with spectral channel state. The spectral efficiency depends on various parameters such as the channel state [2]-[4], average received power of the desired signal, thermal condition and implementation related with noise [5]. Moreover, the multidimensional statistical behavior of the MIMO fading channel [6]-[8] is important for evaluation of the system performance. In this work, the fading channel is considered that small-scale fading is used to describe the amplitude of a radio signal over a short period of time or distance, so that the effects of large-scale path loss could be ignored. The small-scale fading based on multiple path time delay and Doppler spread. According to the time delay, it could be distinguish the flat-fading (delay spread is smaller than symbol period) and the frequency selective fading (delay spread is larger than symbol period). The fast-fading (coherence time is smaller than symbol time) and slow-fading (coherence time is larger than symbol period) are distinguished by Doppler spread [9]. The Doppler spread and coherence time are parameters which describe the time varying characteristic of the channel in a small-scale domain.

Under the idealized assumption of independent and identically distributed (i.i.d.) spatial channel correlation coefficients, the solution is relatively straightforward due to the i.i.d. nature of the coefficients [10], [11]. However, this idealized assumption could be not acceptable in real complex environment such as multiple path fading. We assume the correlated fading channel model, where the channel is small-scale fading. The antenna array shape is applied by a uniform linear array at both Tx and Rx part. The correlation of channel in the Tx and Rx antenna arrays induce correlation in the rows and columns of the channel matrix [2]. We consider a general model for correlated MIMO channels that exposes the true degree of freedom of the channel. The degrees of freedom are governed by the scattering geometry, the antenna spacing, and the number of antenna elements. In this paper, an analysis of channel capacity for 2×2 MIMO antenna systems is described. The channel capacity is simultaneously analyzed by arbitrary channel matrix with singular value decomposition (SVD).

II. MIMO CHANNEL MODEL

A wireless communication system comprising N_T transmit and N_R receive antennas are considered. Fig. 1 shows the modeling of the $N_R \times N_T$ MIMO antenna system with numerical analysis. It consists of Tx, Rx antenna part and correlated fading channel. The MIMO antenna technique is recover to transmitted signals by using the received one.

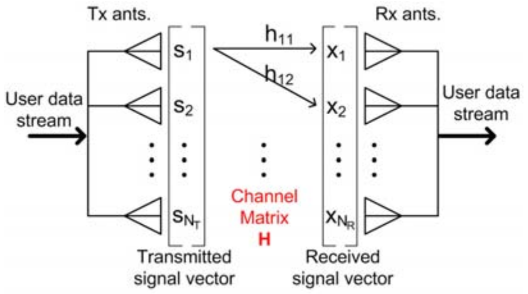

Fig. 1 The MIMO antenna channel modeling.

The received signal at the Rx antennas in the MIMO systems can be express by

$$\mathbf{x} = \mathbf{Hs} + \mathbf{n} \qquad (1)$$

where channel matrix **H** is an $N_R \times N_T$ channel matrix coupling the Tx and Rx antenna element. The **x**, **s** and **n** are Rx complex signal vector, Tx complex signal vector, and noise signal vector at radio channel, respectively. The vector **n** represents zero mean, complex additive white Gaussian noise (AWGN) with covariance matrix $E[\mathbf{nn}^H] = \sigma_n^2 \boldsymbol{I}$, where $[\cdot]^H$ denotes the conjugate transpose of the corresponding vector or matrix and $\boldsymbol{I}$ represents the identity matrix. The channel correlation is defined as $\mathbf{R} = E(\mathbf{hh}^H)$, where $\mathbf{h} = \mathrm{vec}(\mathbf{H})$. In this paper, we assume that the channel matrix can be expressed by the following canonical statistical model [12].

$$\mathbf{H} = \mathbf{U}_R \mathbf{H}_V \mathbf{U}_T^H \tag{2}$$

where $\mathbf{U}_T$ and $\mathbf{U}_R$ are the transmit and receive unitary matrices, and the elements of H_V are uncorrelated but not necessarily i.i.d. $\mathbf{U}_T$ and $\mathbf{U}_R$ are the eigenvectors of $E(\mathbf{H}^H\mathbf{H})$ and $E(\mathbf{HH}^H)$, respectively. The correlation of the elements of $\mathbf{H}_V$ is given by a diagonal matrix $\mathbf{R}_V = \mathrm{E}(\mathbf{h}_V\mathbf{h}_V^H)$, where $\mathbf{h}_V = \mathrm{vec}(\mathbf{H}_V)$. It can write $\mathbf{h} = \mathrm{vec}(\mathbf{H}) = (\mathbf{U}_T^* \otimes \mathbf{U}_R)\mathbf{h}_V$. $\mathbf{R}$ and $\mathbf{R}_V$ are related as

$$\mathbf{R} = (\mathbf{U}_T^* \otimes \mathbf{U}_R)\mathbf{R}_V (\mathbf{U}_T^* \otimes \mathbf{U}_R)^H \tag{3}$$

In the Tx and Rx correlated channel model, the channel matrix could be written as

$$\mathbf{H} = \mathbf{R}_R^{1/2}\mathbf{H}_W\mathbf{R}_T^{1/2} \tag{4}$$

where the elements of $\mathbf{H}_W$ are i.i.d. The matrix $\mathbf{R}_T$ and $\mathbf{R}_R$ are the Tx and Rx array correlation matrix. The SVD of $\mathbf{R}_T$ and $\mathbf{R}_R$ are $\mathbf{U}_T\mathbf{Q}_T\mathbf{U}_T^H$ and $\mathbf{U}_R\mathbf{Q}_R\mathbf{U}_R^H$, respectively. Using (4), the channel matrix could be write as

$$\begin{aligned}\mathbf{H} &= \mathbf{U}_R\mathbf{Q}_R^{1/2}\mathbf{U}_R^H\mathbf{H}_W\mathbf{U}_T\mathbf{Q}_T^{1/2}\mathbf{U}_T^H \\ &= \mathbf{U}_R\mathbf{H}_V\mathbf{U}_T^H\end{aligned} \tag{5}$$

where the second equality arises from the following two observations. First, the elements of $\mathbf{U}_R^H\mathbf{H}_W\mathbf{U}_T$ are still i.i.d., and second, the pre- and post-multiplication of diagonal matrix $\mathbf{Q}_R^{1/2}$ and $\mathbf{Q}_T^{1/2}$, respectively, makes the elements of $\mathbf{H}_V = \mathbf{Q}_R^{1/2}\mathbf{U}_R^H\mathbf{H}_W\mathbf{U}_T\mathbf{Q}_T^{1/2}$ uncorrelated with diagonal covariance matrix given by

$$\mathbf{R}_V = \mathbf{Q}_T \otimes \mathbf{Q}_R \tag{6}$$

where $\otimes$ is Kronecker product which is an operation on two matrix of arbitrary size resulting in a block matrix.

The channel matrix with complex number can be expressed the product of nonnegative number, r, and a number $e^{j\theta}$ by De Moivre's equation. The r is corresponds to a positive semi-definite matrix and $e^{j\theta}$ corresponds to an orthogonal matrix. Since $e^{j\theta}$ is complex and satisfies $e^{-j\theta}e^{j\theta} = 1$, it forms a unitary matrix, $\mathbf{U}^H\mathbf{U} = \boldsymbol{I}$. The SVD extends this factorization to matrix. Insert $\mathbf{U}_2^H\mathbf{U}_2 = \boldsymbol{I}$ into the middle of the $\mathbf{H}_P = \mathbf{U}_1\mathbf{Q}_h\mathbf{U}_2^H$ with SVD.

$$\begin{aligned}\mathbf{H}_P &= \mathbf{U}_1\mathbf{Q}_h\mathbf{U}_2^H \\ &= (\mathbf{U}_1\mathbf{U}_2^H)(\mathbf{U}_2\mathbf{Q}_h\mathbf{U}_2^H)\end{aligned} \tag{7}$$

where the factor $\mathbf{S} = \mathbf{U}_2\mathbf{Q}_h\mathbf{U}_2^H$ is symmetric and semi-definite. The factor $\mathbf{B} = \mathbf{U}_1\mathbf{U}_2^H$ is an orthogonal matrix, because $\mathbf{U}^H\mathbf{U} = \mathbf{U}_2\mathbf{U}_1^H\ \mathbf{U}_1\mathbf{U}_2^H = \boldsymbol{I}$. Therefore polar decomposition of channel matrix is (8).

$$\mathbf{H}_P = \mathbf{BS} \tag{8}$$

III. CHANNEL CAPACITY AND HARACTERISTICs

Channel capacity means the amount of discrete information bits that a defined area or segment in a wireless communication medium can hold. The correlated channel capacity of the narrowband MIMO antenna system is given as following.

$$C = \log_2 \det\left(\mathbf{I} + \frac{\mathrm{SNR}_R}{N_T}\mathbf{H_P H_P^H}\right) \quad \mathrm{bps/Hz} \tag{9}$$

where N_T, SNR and $\mathbf{I}$ are number of Tx antenna elements, the average SNR of Rx antenna part, and N×N identity matrix, respectively. $[\cdot]^H$ denotes the conjugate transpose of the corresponding matrix. In the (7), $\mathbf{H}_P\mathbf{H_P}^H$ can write as follow.

$$\mathbf{H}_P\mathbf{H_P}^H = \mathbf{U}_1\mathbf{Q}_h\mathbf{U}_2^H \tag{10}$$

where (12) is satisfied by unitary matrix characteristics. From (12) $\mathbf{U}_2^H\mathbf{U}_1 = \boldsymbol{I}$ is also satisfied. Therefore (11) can be write as

$$C = \sum_{i=1}^{K} \log_2\left(1 + \frac{\mathrm{SNR}_R}{N_T}\Phi_i^2\right) \quad \mathrm{bps/Hz} \tag{11}$$

where K is rank of channel matrix, Φ_i is singular value of matrix and i = 1, 2, …, K. The channel capacity can get by using rank and singular values of correlated channel matrix. However, it has different capacity with correlated fading channel state. The MIMO channels could be modeled with independent channel K by using SVD and channel gain is expressed by Φ_i. In the special case, channel gain given by inverse of null vector norm of channel matrix. Correlated channel capacity with minimized symbol error can be realize by using inverse of norm of matrix.

The relative movement between the Tx and Rx causes Doppler shifts, known as Doppler spread or Doppler frequency. In the realistic channel, parameters for analysis of channel characteristics are path delays, average path gain, and maximum Doppler spread. In the mobile communication system, Doppler frequency is occurred in terms of the speed of the mobile station. If the mobile moves at speed v m/s, then the maximum Doppler frequency, f_d, is given

$$f_d = \frac{1}{2\pi}\frac{\Delta\varphi}{\Delta t} = \frac{v}{\lambda}\cos\theta \tag{12}$$

where Δt, $\Delta\varphi$, and λ are the time required for the movement at constant distance, the phase difference in the path length, and the transmission carrier frequency in Hz and wavelength of *f*, respectively. The term of $\cos\theta$ in (12) is considered to 1 for simple calculation of algorithm. In case of Tx and Rx theta, we assume that array antennas are uniform linear array antennas and have a omni-directional pattern, then varying of theta is less important than another parameters such as antenna spacing, correlation, and SNR. The root mean square (rms) delay defines the standard deviation value of the delay of reflections, weighted proportional to the energy in the reflected waves. The rms delay spread is the average over the entire angular interval. The rms delay value is 0.25 μs at 1×2 Rx antenna array. It is similar to urban at 900 MHz band. While the delay spread is a natural phenomenon caused by reflected and scattered propagation paths in the radio channel, the coherence bandwidth, $\mathbf{B}_C$, is a defined relation derived from the rms delay spread. Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered flat-fading[9]. In the MIMO system with diversity technique, frequency correlation is below 0.5, therefore the coherence bandwidth is approximately

$$\mathbf{B}_C \approx 1/(5\tau_{rms}) \text{ Hz} \tag{13}$$

where τ_{rms} is rms delay spread. From (13), the coherence bandwidth is 800 kHz. It is shown that the assumption for MIMO antenna system is narrowband. It is important that an exact relation between coherence bandwidth and rms delay spread does not exist. The spectral analysis techniques and simulation are required to determine the exact impact that time varying multipath has on a particular transmitted signal [13]. Therefore accurate multipath channel models must be used in the design of specific modems for wireless applications.

IV.SIMULATION RESULTS AND DISCUSS

Fig. 2 shows the velocity of movement versus Doppler frequency with application service frequency band. 2 m/s, 16.7 m/s, and 83.3 m/s are velocities at pedestrian, highway, and rapid-transit railway, respectively. From the results, we confirm that the Doppler spreads are increased by increase of velocity. Especially, in case of high frequency, i.e. 5.5 GHz band, the Doppler frequency is increased rapidly.

Fig. 3 shows the channel capacity versus Rx SNR. It assumed that highway of 100 Km/h is applied to several service frequency each other. The Doppler frequencies at 900 MHz band cellular or global system for mobile communication (GSM-900), 1.8 GHz band personal service system (PCS) or GSM-1800, 2.4 GHz band WLAN (IEEE 802.11b), and 5.5 GHz band WLAN (IEEE 802.11a) are 83.4 Hz, 166.8 Hz, 222.4 Hz, and 509.7 Hz, respectively. Depending on the SNR, capacity is increased rapidly. According to the Doppler spread, the channel capacity is not changed much. It is occurred by application frequency band in mobile communication system. In the real environment system, Rx SNR is critical parameter and decoding as well as antenna and RF analog circuits are needed for achievement of high quality data.

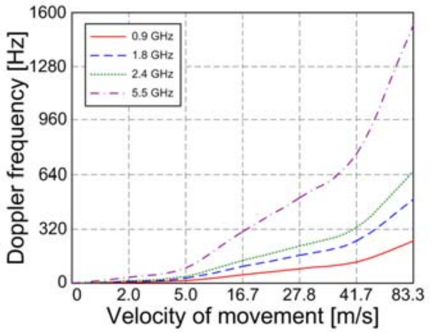

Fig. 2 The Doppler spread vs. velocity of movements.

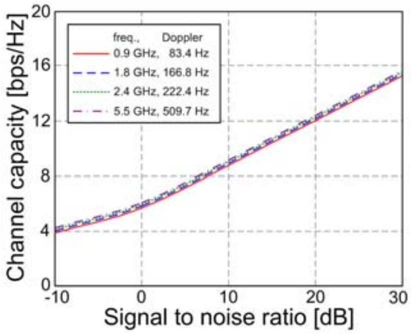

Fig. 3 Channel capacity vs. Rx SNR.

Fig. 4 shows the channel capacity versus Rx signal angle at antennas. It is assumed that the signal of Tx antenna transmitted to 0 degree with 30 dB Tx SNR and Rx signal is received between -90 degree and 90 degree while the Rx SNR is changing from -10 dB to 30 dB. The 0 degree of Rx signal means that is transmitted from Tx antenna array through flat-fading with AWGN. The results show symmetric characteristics for received signal direction. The capacities of another received direction are smaller than 0 degree's one, because the radio wave is propagated delay signal by reflection, scattering, and etc.

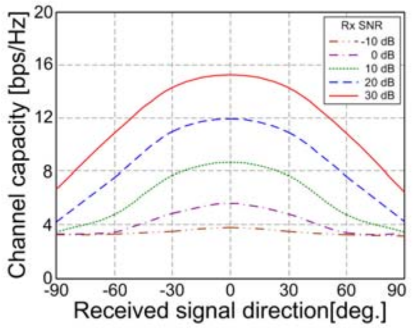

Fig. 4 Channel capacity vs. Rx signal angle

In the 2×2 MIMO channel modeling with polar decomposition is

$$\mathbf{H_p} = \begin{bmatrix} r_{11}e^{j\theta} & r_{12}e^{j\theta} \\ r_{21}e^{j\theta} & r_{22}e^{j\theta} \end{bmatrix} \quad (14)$$

where diagonal elements of matrix are only considered for confirmation of the channel characteristics with changing of $e^{j\theta}$. In (14), the component of $r_{12}e^{j\theta}$ and $r_{21}e^{j\theta}$ are assumed to be 0. The eigenvalues of matrix $\mathbf{H_P}$ are

$$\mathbf{H_p} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad (15)$$

where eigenvalue $\lambda_1 \gg \lambda_2$.

Fig. 5 shows the channel capacity and percentage of largest eigenvalue of channel matrix versus theta changing at polar decomposition matrix $\mathbf{H_P}$. It is considered that the signal direction of Tx and Rx are 0 degree and Rx SNR is 10 dB with flat-fading. The channel capacity depends on the theta of channel matrix with polar decomposition. If the percentages of eigenvalue λ_1 are increased, then channel capacity is decreased. In other hand, if eigenvalue λ_1 and λ_2 have a similar value, then the channel capacity could be increase. In addition, Tx and Rx signal is designed according to channel matrix characteristics, if maximum channel capacity is achieved by control of theta of channel matrix.

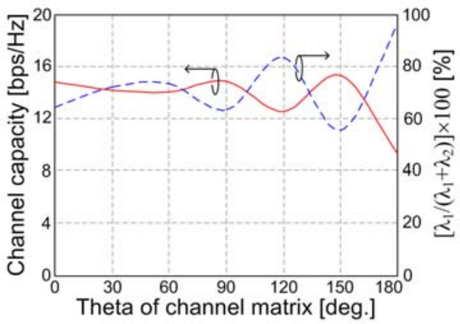

Fig. 5 Channel capacity and eigenvalue vs. theta of channel matrix $\mathbf{H_P}$

The rms delay spreads of 2×2 and 4×4 MIMO antenna arrays are 0.25 μ and 0.56 μs, respectively. Therefore coherence bandwidths are 800 kHz and 357 kHz, respectively. These values present that the MIMO system modeling is narrowband analysis. In practice, base station has a space enough for multiple antenna arrays. Because mobile station is not enough the space for multiple antenna, however, 4×4 MIMO antenna systems are not compatible.

V. CONCLUSION

This paper presents relation between channel state and correlation matrix with Tx and Rx. The channel capacity is simultaneously analyzed by arbitrary Tx and Rx antenna correlation matrix with correlated fading channel response matrix. In MIMO channel modeling, the relations are presented with SVD of channel matrix. In order to achieve maximum channel capacity and optimized Tx and Rx signal design, the channel matrix is decomposed by SVD. In addition, it is considered the Doppler spread, rms delay spread, and coherence bandwidth at several frequency bands. From the simulation results, we confirm that the maximum channel capacity is achieved by control of theta of channel matrix with polar decomposition.

REFERENCES

[1] G. J. Foschini and M. J. Gans, “On Limits of Wireless Communications in a Fading Environment When Using Multiple Antennas,” Wireless Personal Communications, Vol. 6, No. 3, pp. 311-335, March 1998.

[2] Da-Shan Shiu, G. J. Foschini, M. J. Gans and J. M. Kahn, “Fading Correlation and its Effect on the Capacity of Multielement Antenna Systems,” IEEE Transactions on Communications, Vol. 48, No. 3 pp. 502-513. March, 2000.

[3] Mehmet Kemal Ozdemir, Ercument Arvas, and Huseyin Arslan, “Dynamics of Spatial Correlation and Implications on MIMO Systems,” IEEE Radio Communication, pp. S14-S19, June 2004.

[4] Dmitry Chizhik, Gerard J. Foschini, Michael J. Gans, and Reinaldo A. Valenzuela, “Keyholes, Correlations, and Capacities of Multielement Transmit and Receive Antennas,” IEEE Transactions on Wireless Communications, Vol. 1, No. 2, pp. 361-368, April 2002.

[5] Snezana Krusevac, Predrag B. Rapajic, and Rodney A. Kennedy, “Mutual Coupling Effect on Thermal Noise in Multi-Element Antenna systems.” PIERS 2005, pp. 53-57, August 2005.

[6] Zukang Shen, Robert W. Heath, Jr., Jeffrey G. Andrews, and Brian L. Evans, “Comparison of Space-Time Water-Filling with Spatial Water-Filling for MIMO Fading Channels,” IEEE Global Communications conference, November 2004.

[7] S. K. Yong, and J. S. Thompson, “A Three-Dimensional Spatial Fading Correlation Model for Uniform Rectangular Arrays,” IEEE Antennas and Wireless Propagation Letters, Vol. 2, August 2003.

[8] Gregory D. Durgin, and Theodore S. Rappaport, “Thoery of Multipath Shape Factors for Small-Scale Fading Wireless Channels,” IEEE Transactions on Antennas and Propagation, Vol. 48, No. 5, pp. 682-693, May 2000.

[9] Theodore S. Rappaport, “Wireless Communications : Principles and Practice,” Prentice-Hall, Inc., 1996.

[10] A. J. Paulraj and C. B. Papadias, “Space-time processing for wireless communications,” IEEE Signal Processing Mag., pp. 49-83, Nov. 1997.

[11] S. M. Kay, Fundamentals of Statistical Signal Processing : Estimation Theory, Upper Saddle River, Prentice-Hall PTR, 1998.

[12] J. H. Kotecha and A. M. Sayeed, “Canonical statistical model for correlated MIMO fading channels and capacity analysis,” in preparation, preprint : http://www.ece.wisc.edu/~jkotecha.

[13] Fung, V., Rappaport, T. S., and Thoma, B., “Bit Error Simulation for $\pi/4$ DQPSK Mobile Radio Communication using Two-ray and Measurement-based Impulse Response Models,” IEEE Journal on Selected Areas in Communication, Vol. 11, No. 3, pp. 393-405, April 1993.