A Layer Structured Algorithm for Azimuth/Elevation Angle Estimation

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***Abstract*— A one dimensional (1D) based tree structure algorithm is proposed for the estimation of azimuth and elevation angles of the signals received by a uniform rectangular array (URA). The proposed algorithm successively apply several times of 1D-MUSIC algorithm, in tree structure, to estimate the azimuth and the elevation angles independently. Subspace projectors are exploited in conjunction with the 1D MUSIC algorithms to decompose the received signal into several signals each coordinated by its own 2D-DOA. The pairing of the azimuth estimates and the associated elevation estimates is naturally determined due to the tree structure of the algorithm.**

I. Introduction

Joint estimation of the azimuth and elevation angles, referred to as the two-dimensional (2-D) direction of arrivals (DOAs), of propagating waveforms has received significant attention over last several decades. In wireless communication systems, many applications using antenna arrays have been found to exploit the spatial information of the wireless channel to improve coverage, reduce co-channel interference, and increase system capacity.

Many algorithms have been proposed for solving the 2D-DOA estimation problem [1]-[4]. Most of these algorithms can provide precise 2-D DOAs estimates at the expense of high computational complexity in dealing with the high dimensional signal received by the rectangular antenna array. To effectively reduce the computation cost, we propose a one dimensional (1-D) based tree structure algorithm for the 2D-DOA estimation problem. The basic idea of the proposed algorithm is to successively apply several times of 1-D MUSIC algorithm [5], in tree structure, to estimate the azimuth and the elevation independently. Subspace projectors are exploited in conjunction with the 1-D MUSIC algorithms to decompose the received signal into a set of independent signals each coordinated by its own 2D-DOA. The pairing of the azimuth estimates and the associated elevation estimates is automatically determined due to the tree structure of the algorithm.

II. Problem Formulation

Consider L narrowband, far-field and uncorrelated signals impinged on an $M \times N$ URA with MN identical and omnidirectional antenna elements as shown in Fig. 1. The signal received by the antenna element at the m^{th} row and n^{th} column of the URA can be expressed as

$$y_{m,n}(i) = \frac{1}{\sqrt{MN}} \sum_{l=1}^{L} s_l(i)\, e^{j\frac{2\pi d_x}{\lambda}(m-1)u_l} e^{j\frac{2\pi d_y}{\lambda}(n-1)v_l} + z_{m,n}(i) \quad (1)$$

where $s_l(i)$ is the l^{th} signal sampled at time instant i, λ denotes the wavelength of the carrier, and d_x and d_y represent the interelement distance of the antennas in the column (x-domain) and in the row (y-domain) of the URA, respectively. $u_l = \sin\phi_l \cos\theta_l$, in which ϕ_l and θ_l denote the elevation and the azimuth angles of $s_l(i)$, referred to as the X-DOA, represents the direction component of $s_l(i)$ propagating in the x-domain while $v_l = \sin\phi_l \sin\theta_l$ being the Y-DOA in the y-domain and $z_{m,n}(i)$ is the additive white Gaussian noise with variance σ_z^2. Writing the signal received by the URA in matrix form, we have

$$\mathbf{Y}(i) = \sum_{l=1}^{L} s_l(i)\, \mathbf{a}_{\mathrm{x}}(u_l)\, \mathbf{a}_{\mathrm{y}}^T(v_l) + \mathbf{Z}(i) \quad (2)$$

where $\mathbf{a}_{\mathrm{x}}(u_l) = \frac{1}{\sqrt{M}}\left[1, e^{j\frac{2\pi d_x}{\lambda}u_l}, \cdots, e^{j\frac{2\pi d_x}{\lambda}(M-1)u_l}\right]^T$ and $\mathbf{a}_{\mathrm{y}}(v_l) = \frac{1}{\sqrt{N}}\left[1, e^{j\frac{2\pi d_y}{\lambda}v_l}, \cdots, e^{j\frac{2\pi d_y}{\lambda}(N-1)v_l}\right]^T$ represent the spatial signatures of $s_l(i)$ in the x-domain (the x-signature) and the y-domain (the y-signature), respectively. $\mathbf{Z}(i)$ is the AWGN matrix.

III. The Proposed Approach

From (2), it is obvious that the information of u_l and v_l are in the column space and the row space of $\mathbf{Y}(i)$, respectively. We thus can apply the 1-D MUSIC algorithm [5] along the columns and the rows of (2) for the X-DOA and the Y-DOA estimation, respectively.

A. The MUSIC Algorithm

The MUSIC algorithm [5] is a subspace-based algorithm which uses the eigen structure of the received signal for parameter estimation. In the proposed algorithm, we apply the MUSIC algorithm to the columns and the rows of $\mathbf{Y}(i)$ for the X-DOA and the Y-DOA estimation, respectively. To see this, notice that the covariance matrix of the columns and the rows of $\mathbf{Y}(i)$ in (2) are

$$\mathbf{R}_{\mathrm{x}} = \frac{1}{N}E\left\{\mathbf{Y}(i)\mathbf{Y}^{H}(i)\right\} = \mathbf{A}_{\mathrm{x}}(\mathbf{u})\mathbf{R}_{\mathbf{s}}\mathbf{A}_{\mathrm{x}}^{H}(\mathbf{u}) + \sigma_{\mathrm{z}}^{2}\mathbf{I}, \tag{3}$$

and

$$\mathbf{R}_{\mathrm{y}} = \frac{1}{M}E\left\{\mathbf{Y}^{T}(i)\mathbf{Y}^{*}(i)\right\} = \mathbf{A}_{\mathrm{y}}(\mathbf{v})\mathbf{R}_{\mathbf{s}}\mathbf{A}_{\mathrm{y}}^{H}(\mathbf{v}) + \sigma_{\mathrm{z}}^{2}\mathbf{I}, \tag{4}$$

where $\mathbf{A}_{\mathrm{x}}(\mathbf{u}) = [\mathbf{a}_{\mathrm{x}}(u_1), \cdots, \mathbf{a}_{\mathrm{x}}(u_L)]_{M\times L}$ and $\mathbf{A}_{\mathrm{y}}(\mathbf{v}) = [\mathbf{a}_{\mathrm{y}}(v_1), \cdots, \mathbf{a}_{\mathrm{y}}(v_L)]_{N\times L}$. The eigen decomposition of $\mathbf{R}_{\mathrm{x}}$ and $\mathbf{R}_{\mathrm{y}}$ can be expressed as

$$\mathbf{R}_{\mathrm{x}} = \mathbf{V}_{\mathrm{x}}^{\mathrm{s}}\mathbf{\Lambda}_{\mathrm{x}}^{\mathrm{s}}(\mathbf{V}_{\mathrm{x}}^{\mathrm{s}})^{H} + \sigma_{\mathrm{z}}^{2}\mathbf{V}_{\mathrm{x}}^{\mathrm{n}}(\mathbf{V}_{\mathrm{x}}^{\mathrm{n}})^{H} \tag{5}$$

and

$$\mathbf{R}_{\mathrm{y}} = \mathbf{V}_{\mathrm{y}}^{\mathrm{s}}\mathbf{\Lambda}_{\mathrm{y}}^{\mathrm{s}}\left(\mathbf{V}_{\mathrm{y}}^{\mathrm{s}}\right)^{H} + \sigma_{\mathrm{z}}^{2}\mathbf{V}_{\mathrm{y}}^{\mathrm{n}}\left(\mathbf{V}_{\mathrm{y}}^{\mathrm{n}}\right)^{H}, \tag{6}$$

where $\mathbf{V}_{\mathrm{x}}^{\mathrm{s}}$ and $\mathbf{V}_{\mathrm{y}}^{\mathrm{s}}$ consisting of the L largest eigen vectors of $\mathbf{R}_{\mathrm{x}}$ and $\mathbf{R}_{\mathrm{y}}$, respectively, are referred to as the signal subspace matrices, $\mathbf{\Lambda}_{\mathrm{x}}^{\mathrm{s}}$ is a diagonal matrix of the eigen values corresponding to $\mathbf{V}_{\mathrm{x}}^{\mathrm{s}}$ and $\mathbf{V}_{\mathrm{y}}^{\mathrm{s}}$. $\mathbf{V}_{\mathrm{x}}^{\mathrm{n}}$ and $\mathbf{V}_{\mathrm{y}}^{\mathrm{n}}$, constituted by the rest eigen vectors of $\mathbf{R}_{\mathrm{x}}$ and $\mathbf{R}_{\mathrm{y}}$, are referred to as the noise subspace matrices being orthonormal complementary to $\mathbf{V}_{\mathrm{x}}^{\mathrm{s}}$ and $\mathbf{V}_{\mathrm{y}}^{\mathrm{s}}$, respectively. The MUSIC algorithm uses the property that $\mathbf{A}_{\mathrm{x}}(\mathbf{u})$ shares the same column space with $\mathbf{V}_{\mathrm{x}}^{\mathrm{s}}$. By scanning over all possible values of u and v with respect to the associated manifolds $\mathbf{a}_{\mathrm{x}}(u)$ and $\mathbf{a}_{\mathrm{y}}(v)$, respectively, the pseudo spectra of the MUSIC algorithm are defined as

$$S_{\mathrm{x}}(u) = \frac{1}{\mathbf{a}_{\mathrm{x}}^{H}(u)\mathbf{V}_{\mathrm{x}}^{\mathrm{n}}\mathbf{V}_{\mathrm{x}}^{\mathrm{n}}\mathbf{a}_{\mathrm{x}}(u)}, \tag{7}$$

and

$$S_{\mathrm{y}}(v) = \frac{1}{\mathbf{a}_{\mathrm{y}}^{H}(u)\mathbf{V}_{\mathrm{y}}^{\mathrm{n}}\mathbf{V}_{\mathrm{y}}^{\mathrm{n}}\mathbf{a}_{\mathrm{y}}(v)}. \tag{8}$$

The u_l estimate and the v_l estimate can thus be obtained from the ones producing peaks in the related psudo spectrum. We referred the MUSIC algorithm for the X-DOA estimation as the X-MUSIC algorithm whereas the one for the Y-DOA estimation as the Y-MUSIC algorithm.

The MUSIC algorithm is attractive for its accuracy and immunity to noise. In the cases where the incoming signals contain only mild contamination and are neither close in the X-domain nor in the Y-domain, the MUSIC algorithm can obtain a fairly precise estimates of the X-DOAs and Y-DOAs. However, if the signal-gathering scenarios happen in the X-domain or in the Y-domain, a hump, instead of several peaks occur in the MUSIC spectrum around each signal-gathering area and thus only an average DOA can be resulted. In this paper, we use the estimates of the group X-DOAs to identify the signal group as will be shown in next subsection.

B. The Proposed Tree Structure Algorithm

In this paper, signals with close-by $u_l's$ are regarded as a group which blurs the pseudo spectrum of the X-MUSIC and thus only a average X-DOA can be obtained for the group signals. However, even though the signals within a group have similar X-DOAs, they generally posess very different Y-DOAs. With this understanding, the basic idea behind the proposed algorithm is to use the geometric characteristic of the signal received by a URA to group and then to isolate the received signal successively in the x-domain and the y-domain. In conjunction with the signal isolation technique, the 2-D parameter (u_l, v_l) can be estimated independently in the x-domain and the y-domain, respectively, and thus effectly mitigate the computational complexity of the proposed algorithm.

The proposed algorithm consists of three layers of signal processing: 1) The grouping layer (the G-layer) for the X-DOA rough estimation and the signal grouping, 2) The isolation layer (I-layer) for the Y-DOA estimation and the signal isolation, and 3) The parameter pairing layer (P-layer) for the X-DOA fine estimation and pairing each signal's X-DOA estimate with its associated Y-DOA estimate. In conjunction with the subspace projectors in the x-domain and the y-domain, two X-MUSIC algorithms and one Y-MUSIC algorithm are invoked in interlace to provide the information required in each layer.

1) The G-layer: Shown in Fig. 2 is the structure of the propsoed algorithm. In the G-layer of the proposed approach, we intend to identify each group of the received signal and then separate it in the X-domain. To achieve this, the X-MUSIC is first applied to $\mathbf{Y}(i)$ to obtain the X-DOA estimates of each group, denoted as $\hat{u}_1, \ldots, \hat{u}_K$ with $\hat{u}_k \approx u_{k,l}$; $l = 1, \cdots, N_k$. Based on these X-DOA estimates, the group separation can be carried out through K X-domain projectors. For separating the k^{th} group, we form the self-exclusive matrix $\mathbf{E}_k^{\mathrm{x}} = \left[\{\mathbf{a}_{\mathrm{x}}(\hat{u}_{k'})\}_{k'\neq k}\right]$, which is constructed by the x-signatures of all other groups excluding that of the k^{th} group. For instance, the self-exclusive matrix for the first group is $\mathbf{E}_1^{\mathrm{x}} = \left[\ \mathbf{a}_{\mathrm{x}}(\hat{u}_2),\ \cdots,\ \mathbf{a}_{\mathrm{x}}(\hat{u}_M)\ \right]$. Thereby, the subspace projector, referred to as the X-projector, of the k^{th} group can be expressed as

$$\mathbf{P}_k^{\mathrm{x}} = \mathbf{I} - \mathbf{E}_k^{\mathrm{x}}\mathbf{E}_k^{\mathrm{x}\dagger}, \tag{9}$$

where $(\cdot)^{\dagger}$ denotes the Moore-Penrose pseudo inverse operation. Since $\mathbf{P}_k^{\mathrm{x}}$ maps to the subspace orthogonal to the columns of $\mathbf{E}_k^{\mathrm{x}}$, the signals of k^{th} group can be separated by applying (9) to $\mathbf{Y}(i)$ as

$$\mathbf{Y}_k(i) = \mathbf{P}_k^{\mathrm{x}}\mathbf{Y}(i) = \sum_{l=1}^{N_k} s_{k,l}(i)\,\tilde{\mathbf{a}}_{\mathrm{x}}(u_{k,l})\,\mathbf{a}_{\mathrm{y}}^{T}(v_{k,l}) + \mathbf{Z}_k(i), \tag{10}$$

where $\tilde{\mathbf{a}}_{\mathrm{x}}(u_{k,l}) = \mathbf{P}_k^{\mathrm{x}}\mathbf{a}_{\mathrm{x}}(u_{k,l})$, and $\mathbf{Z}_k(i) = \mathbf{P}_k^{\mathrm{x}}\mathbf{Z}(i)$ is the projected noise matrix.

2) The I-layer: Taking over the outputs of the G-layer, the I-layer in Fig. 2 aims at Y-DOA estimation and signal isolation. The Y-MUSIC algorithm is first applied to matrix $\mathbf{Y}_k(i)$ of each group for Y-DOA estimation and obtain the exact Y-DOA estimates $\{\hat{v}_{k,1}, \cdots, \hat{v}_{k,N_k}\}$. For those signals with distinct X-DOAs but with close-by Y-DOAs, after the grouping procedure conducted in the previous G-layer, the signals with close-by Y-DOAs are now in different groups. Since the Y-MUSIC is applied to each group separately, the close-by Y-DOAs of the signals can be estimated independently and accurately without much mutual interference, which is impossible without cooperation between the G-layer and the I-layer. With the estimate $\{\hat{v}_{k,l}\}$ available, we can regenerate the signatures in Y-domain as $\{\mathbf{a}_y(\hat{v}_{k,l})\}$. According to these signature estimates, the proposed algorithm is now ready to isolate each individual signal in the Y-domain. The Y-projector for isolating the l^{th} signal of the k^{th} group can be expressed as

$$\mathbf{P}^{y}_{k,l} = \mathbf{I} - \mathbf{E}^{y}_{k,l}\mathbf{E}^{y\dagger}_{k,l}, \tag{11}$$

where $\mathbf{E}^{y}_{k,l} = \left[\{\mathbf{a}_y(\hat{v}_{k,l'})\}_{l'\neq l}\right]$ denotes the self exclusive matrix for the l^{th} signal of the k^{th} group. Post-multipling $\mathbf{P}^{y}_{k,l}$ to $\mathbf{Y}_k(i)$ yields

$$\begin{aligned} \mathbf{Y}_{k,l}(i) &= \mathbf{Y}_k(i)\,\mathbf{P}^{y}_{k,l} && (12)\\ &= s_{k,l}(i)\,\tilde{\mathbf{a}}_x(u_{k,l})\,\bar{\mathbf{a}}^T_y(v_{k,l}) + \mathbf{Z}_{k,l}(i). && (13) \end{aligned}$$

where $\bar{\mathbf{a}}^T_y(v_{k,l}) = \mathbf{a}^T_y(v_{k,l})\,\mathbf{P}^{y}_{k,l}$ is the projected array vector. In (13), we use the orthogonalties of $\mathbf{a}^T_y(v_{k,l'})\,\mathbf{P}^{y}_{k,l} \approx \mathbf{0}^T; \forall l' \neq l, l = 1, \cdots, N_k$.

3) The P-layer: From (13), it is apparent that the output of the Y-projector contains only a single signal. The X-MUSIC can be used again to precisely estimate the X-DOA of each signal. This is similar to the cases of Y-DOA estimation in the I-layer of the proposed algorithm. With the cooperation between the I-layer and the P-layer, X-DOA estimation for the signals which have distinct Y-DOAs but close-by X-DOAs is made much easier and more accurate, since these signals have been separated by the Y-projector in the I-layer and the X-MUSIC is applied to each signal separately.

In the P-layer of the proposed algorithm, by applying the X-MUSIC to (13), $\hat{u}_{k,l}$ can be obtained and automatically paired with $\hat{v}_{k,l}$, which is already known in the I-layer of the proposed algorithm. Notably, in implementing the X-MUSIC, due to the X-projector implemented in the first G-layer, the X-MUSIC algorithm at this time uses $\tilde{\mathbf{a}}_x(u)$ as its spatial manifold when searching for the optimum u.

The overall procedures of the proposed algorithm are concluded as follows:

G-layer:

1. Apply the X-MUSIC to columns of $\mathbf{Y}(i)$ to estimate the averaged X-DOAs, $\hat{u}_1, \ldots, \hat{u}_K$
2. Form the self-exclusive matrices $\mathbf{E}^{x}_{k} = \left[\{\mathbf{a}_x(\hat{u}_{k'})\}_{k'\neq k; k'=1,\cdots,K}\right]$, and then construct the X-projector $\mathbf{P}^{x}_{k} = \mathbf{I} - \mathbf{E}^{x}_{k}\mathbf{E}^{x\dagger}_{k}$,
3. The output of the k^{th} X-projector is $\mathbf{Y}_k(i) = \mathbf{P}^{x}_{k}\mathbf{Y}(i), k = 1, \cdots, K$ $\approx \sum_{l=1}^{N_k} s_{k,l}(i)\,\tilde{\mathbf{a}}_x(u_{k,l})\,\mathbf{a}^T_y(v_{k,l})$,

I-layer:

4. Apply the Y-MUSIC algorithm to rows of $\mathbf{Y}_k(i)$ to estimate the Y-DOAs, $\hat{v}_{k,l}, l = 1, \cdots, N_k$.
5. Construct the self-exclusive matrix $\mathbf{E}^{y}_{k,l} = \left[\{\mathbf{a}_y(\hat{v}_{k,l'})\}_{l'\neq l; l'=1,\cdots,N_k}\right]$,and the Y-projector, $\mathbf{P}^{y}_{k,l} = \mathbf{I} - \mathbf{E}^{y}_{k,l}\mathbf{E}^{y\dagger}_{k,l}$, the output of Y-projector (k, l) is $\mathbf{Y}_{k,l}(i) = \mathbf{Y}_k(i)\,\mathbf{P}^{y}_{k,l}$ $\approx s_{k,l}(i)\,\tilde{\mathbf{a}}_x(u_{k,l})\,\bar{\mathbf{a}}^T_y(v_{k,l})$,

P-layer:

6. Apply X-MUSIC again to each $\mathbf{Y}_{k,l}(i)$ to estimate the exact $\hat{u}_{k,l}$ and directly pair it with $\hat{v}_{k,l}$ obtained in 4. With the X-DOA and the Y-DOA estimates, the estimates of the azimuth and the elevation angles are $\hat{\theta}_{k,l} = Tan^{-1}\frac{\hat{v}_{k,l}}{\hat{u}_{k,l}}$, $\hat{\phi}_{k,l} = \sin^{-1}\frac{\hat{u}_{k,l}}{\cos\hat{\theta}_{k,l}}$.

IV. Simulation Results

In this section, computer simulations are conducted to demonstrate the effectiveness of the proposed algorithm. We assume three $(L = 3)$ narrow-band signals received by a 20×20 URA. The antenna elements are of equal gains and half wavelength spaced $\left(d_x = d_y = \frac{\lambda}{2}\right)$. The received signal $\mathbf{Y}(i)$ is sampled 100 snapshots for each trial. The X-DOAs/Y-DOAs of the received signals are set $[-0.21, -0.16, 0.38] / [0.46, -0.32, -0.3]$ or the equivalent azimuth/elevation angles are $[-65^o, 63^o, -38.3^o] / [-30^o, -21^o, 29^o]$. Note, the first two of them are with close-by X-DOAs whereas the later two of them are with close-by Y-DOAs. The three signals are assumed uncorrelated Gaussian distributed with zero mean and equal variance normalized to 0 dB $\left(\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 0dB\right)$. The average power of the additive Gaussian noise is adjusted to achieve the required signal to noise power ratio (SNR).

Fig. 3 demonstrates the X-MUSIC spectra for applying the X-MUSIC algorithm to the data matrices $\mathbf{Y}(i)$, $\mathbf{Y}_{1,1}(i)$ and $\mathbf{Y}_{1,2}(i)$, respectively, in which the later two data matrices each containing a isolated single signal are produced by the Y-projectors of the proposed algorithm. As shown in the figure, applying the X-MUSIC to $\mathbf{Y}(i)$, only an averaged X-DOA can be obtained for the two close-by signals while by generating the data matrices $\mathbf{Y}_{1,1}(i)$ and $\mathbf{Y}_{1,2}(i)$, the proposed approach successfully identify these two gathering signals in the X-domain. In addition, providing the averaged X-DOA in the G-layer of the proposed algorithm, only local searches are required for forming the X-MUSIC spectra of $\mathbf{Y}_{1,1}(i)$ and $\mathbf{Y}_{1,2}(i)$, respectively. On the other hand, Fig. 4 shows the Y-MUSIC spectra of the proposed approach. As shown in the figure, the grouping process of the proposed approach separate the signals with close-by Y-DOAs to different data matrices, $\mathbf{Y}_1$ and $\mathbf{Y}_2$, and thus makes the fine Y-DOA estimation easy to be achieved.

Fig. 5 compares the root-mean-square-error (RMSE) of the proposed algorithm, the 2D-ESPRIT [3], and the 2D-MUSIC [4] algorithms with respect to various SNRs ranged from 0 dB to 30 dB. The RMSE is defined as

$$RMSE = \frac{1}{L}\sum_{l=1}^{L}\left(RMSE\left(\theta_l\right) + RMSE\left(\phi_l\right)\right)$$

where $RMSE\left(\theta_l\right) = \left[\frac{1}{N_{\mathrm{T}}}\sum_{n=1}^{N_{\mathrm{T}}}\left(\theta_l - \hat{\theta}_l\left(n\right)\right)^2\right]^{\frac{1}{2}}$, and $RMSE\left(\phi_l\right) = \left[\frac{1}{N_{\mathrm{T}}}\sum_{n=1}^{N_{\mathrm{T}}}\left(\phi_l - \hat{\phi}_l\left(n\right)\right)^2\right]^{\frac{1}{2}}$ with N_{T} being the number of monte Carlo trials, $\hat{\theta}_l\left(n\right)$ and $\hat{\phi}_l\left(n\right)$ the n^{th} estimates of the azimuth and the elevation angles of the l^{th} signal, respectively. For each specific SNR, $N_{\mathrm{T}} = 200$ trials are conducted. As shown in Fig. 5, the proposed algorithm shows similar RMSE as that of the 2-D MUSIC algorithm and is significantly superior to the 2D-ESPRIT algorithm at low SNR.

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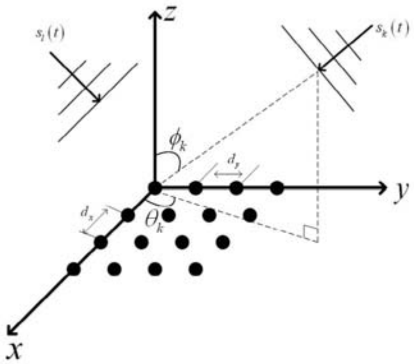

Fig. 1: The geometry of a uniform rectangular array

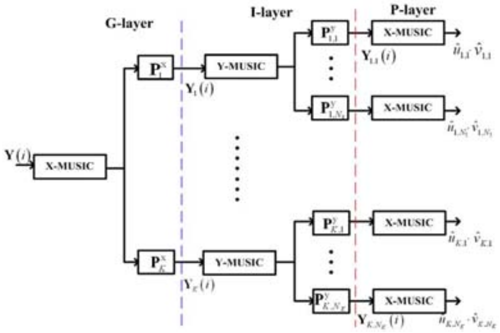

Fig. 2: The schematic of the proposed algorithm

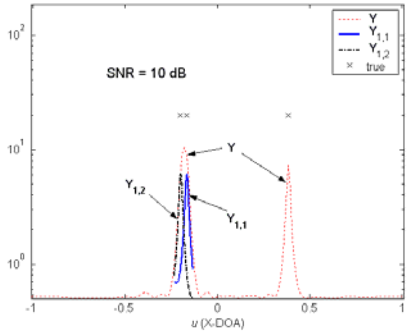

Fig. 3: The pseudo spectra of the X-MUSIC algorithm in the proposed approach.

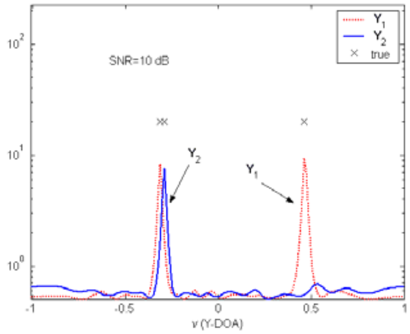

Fig. 4: The pseudo spectra of the Y-MUSIC algorithm in the proposed approach.

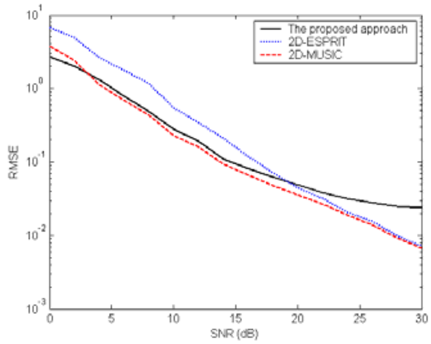

Fig. 5: Comparisons of the RMSE of the proposed algorithm to that of the 2D-MUSIC algorithm and the 2D-ESPRIT algorithm.