ANALYSIS OF THE EFFECTIVE MEDIUM PARAMETERS OF A MEDIUM CONTAINING RANDOMLY DISTRIBUTED CHIRAL PARTICLES

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1. Introduction

Recently, studies on the analysis of the effective properties of electromagnetic fields in bianisotropic and chiral mixtures[1–6]. The effective constitutive parameters of chiral mixtures containing chiral particles and background medium have been analyzed by Maxwell-Garnett(MG) method, Bruggeman effective medium approximation, and the applications of these methods[1–6]. However, these methods are suitable for low dielectric constant of chiral particles[7].

Up to now we have analyzed the effective dielectric constant ($\epsilon_{\rm eff}$) of a medium containing randomly distributed dielectric particles using the approach presented by one of authors[8–9]. This approach is an unconventional multiple scattering method by which wave scattering can be systematically treated in a medium whose dielectric particles are randomly displaced from a uniformly ordered spatial distribution. We also have shown that our method is more powerful for the analysis of $\epsilon_{\rm eff}$ than the conventional methods[7]. However, we have not so far analysed the effective properties of a medium containing anisotropic particles by this approach.

In this paper, we have analysed the effective constitutive parameters for a medium containing randomly distributed chiral particles embedded in an achiral background medium, changing the fractional volume and dielectric constant of particles. These computed results have been compared with those of conventional method: MG method.

2. Effective parameters of chiral mixture

Consider a mixture with n_0 chiral particles per unit volume and achiral background medium of dielectric constant ϵ_0 and permeability μ_0. It is assumed in this paper that particles are randomly displaced from a uniform distribution, independent of each other. For simplicity, we assume that all the particles are the same chiral sphere of radius $b_{\rm s}$, dielectric constant $\epsilon_{\rm s}\epsilon_0$, permeability $\mu_{\rm s}\mu_0$ and chirality $\kappa_{\rm s}$. In addition, we assume that the mean length between each sphere is a in all directions: $a_\nu = a$, $\nu = x, y, z$, and that the variance from the uniformity is homogeneous and isotropic in all directions: $\sigma_\nu^2 = \sigma^2$. Therefore the fractional volume of chiral particles $f = n_0 v_{\rm s} = a^{-3} v_{\rm s}$, where $v_{\rm s} = 4\pi b_{\rm s}^3/3$.

The chiral particle is of the following constitutive relations for electromagnetic fields and displacements

$$\boldsymbol{D} = \epsilon_{\rm s}\epsilon_0 \boldsymbol{E} - i\kappa_{\rm s}\sqrt{\epsilon_0\mu_0}\boldsymbol{H} \quad (1)$$

$$\boldsymbol{B} = \mu_{\rm s}\mu_0 \boldsymbol{H} + i\kappa_{\rm s}\sqrt{\epsilon_0\mu_0}\boldsymbol{E} \quad (2)$$

Define the effective constitutive parameters of the chiral mixture by coefficients in the macroscopic constitutive relations between the average flux densities and the average fields

$$\langle \boldsymbol{D} \rangle = \epsilon_{\rm eff}\epsilon_0 \langle \boldsymbol{E} \rangle - i\kappa_{\rm eff}\sqrt{\mu_0\epsilon_0}\langle \boldsymbol{H} \rangle \quad (3)$$

$$\langle \boldsymbol{B} \rangle = \mu_{\rm eff}\mu_0 \langle \boldsymbol{H} \rangle + i\kappa_{\rm eff}\sqrt{\mu_0\epsilon_0}\langle \boldsymbol{E} \rangle \quad (4)$$

On the basis of the quasistatic version of our method[7–9], we can derive $\epsilon_{\rm eff}$, $\mu_{\rm eff}$ and $\kappa_{\rm eff}$ in the low frequency approximation. Then, $\epsilon_{\rm eff}$, $\mu_{\rm eff}$ and $\kappa_{\rm eff}$ are expressed in the following forms:

$$\epsilon_{\rm eff}\epsilon_0 = \epsilon_{\rm av}\epsilon_0 + n_0(\alpha_{\rm ee1} - \alpha_{\rm ee2}) \quad (5)$$

$$\mu_{\rm eff}\mu_0 = \mu_{\rm av}\mu_0 + n_0(\alpha_{\rm mm1} - \alpha_{\rm mm2}) \quad (6)$$

$$\kappa_{\rm eff}\sqrt{\mu_0\epsilon_0} = \kappa_{\rm av}\sqrt{\mu_0\epsilon_0} + n_0(\alpha_{\rm em1} - \alpha_{\rm em2}) \quad (7)$$

where $\epsilon_{\rm av}$, $\mu_{\rm av}$ and $\kappa_{\rm av}$ are the constitutive parameters in a continuous chiral medium of which the dielectric constant, permeability and chirality are the average of each material parameter of the chiral particles and the background medium at the rate of each space occupation:

$$\epsilon_{\rm av}\epsilon_0 = \epsilon_0 + \epsilon_0\varepsilon_{\rm d}f = f\epsilon_s\epsilon_0 + (1-f)\epsilon_0 = \epsilon_0(1+f\epsilon_{\rm d}) \quad (8)$$

$$\mu_{\rm av}\mu_0 = \mu_0 + \mu_0\mu_{\rm d}f = f\mu_{\rm s}\mu_0 + (1-f)\mu_0 = \mu_0(1+f\mu_{\rm d}) \quad (9)$$

$$\kappa_{\rm av} = f\kappa_{\rm s} \quad (10)$$

Here, $\alpha_{\rm ee1}$,$\alpha_{\rm mm1}$, and $\alpha_{\rm em1}$ are each component of the polarizability of one chiral sphere with radius b_s located in an unbounded chiral medium $(\epsilon_{\rm av}\epsilon_0, \mu_{\rm av}\mu_0, \kappa_{\rm av})$. The chiral sphere is of the following medium parameters[7].

$$\text{dielectricconstant} : \epsilon_{\rm av}(1+\epsilon_{\rm d}/\epsilon_{\rm av})\epsilon_0 \quad (11)$$

$$\text{permeability} : \mu_{\rm av}(1+\mu_{\rm d}/\mu_{\rm av})\mu_0 \quad (12)$$

$$\text{chirality} : \kappa_{\rm av}(1+\kappa_{\rm s}/\kappa_{\rm av}) \quad (13)$$

Similarly, $\alpha_{\rm ee2}$, $\alpha_{\rm mm2}$, and $\alpha_{\rm em2}$ are each component of the polarizability of the other chiral sphere with radius $[W(\sigma)]^{-1/3}b_{\rm s}$[7] located in an unbounded chiral medium $(\epsilon_{\rm av}\epsilon_0, \mu_{\rm av}\mu_0, \kappa_{\rm av})$. This chiral sphere has the following medium parameters[7].

$$\text{dielectricconstant} : \epsilon_{\rm av}[1+\epsilon_{\rm d}W(\sigma)/\epsilon_{\rm av}]\epsilon_0 \quad (14)$$

$$\text{permeability} : \mu_{\rm av}[1+\mu_{\rm d}W(\sigma)/\mu_{\rm av}]\mu_0 \quad (15)$$

$$\text{chirality} : \kappa_{\rm av}[1+\kappa_{\rm s}W(\sigma)/\kappa_{\rm av}] \quad (16)$$

where $W(\sigma)$ is the following distribution function of particles[7].

$$W(\sigma) = \sqrt{\frac{2}{\pi}}\frac{1}{\sigma^3}\int_0^b \exp\left(-\frac{r_1^2}{2\sigma^2}\right) r_1^2 dr_1 \quad (17)$$

Here, $W(\sigma) = 1$ for $\sigma/a \to 0$ and $W(\sigma) \simeq 0$ for $\sigma/a \to 1$. In this paper, we assume that $\sigma/a = 1-f$.

With these specified parameters, both $\alpha_{\rm ee1}$, $\alpha_{\rm mm1}$, $\alpha_{\rm em1}$ and $\alpha_{\rm ee2}$, $\alpha_{\rm mm2}$, $\alpha_{\rm em2}$ can be calculated from the general formula given in [9]. We recapitulate the formula here for the sake of completeness. Suppose a chiral inclusion with parameters $(\epsilon_2\epsilon_0, \mu_2\mu_0, \kappa_2)$ is located in an unbounded background $(\epsilon_1\epsilon_0, \mu_1\mu_0, \kappa_1)$. Then each component of the polarizability read:

$$n_0\alpha_{\rm ee} = 3f\epsilon_0\frac{\epsilon_1(\epsilon_2-\epsilon_1)(\mu_2+2\mu_1) - \epsilon_1(\kappa_2-\kappa_1)^2 - 3(\varepsilon_2-\varepsilon_1)\kappa_1^2}{(\mu_2+2\mu_1)(\epsilon_2+2\epsilon_1)-(\kappa_2+2\kappa_1)^2} \quad (18)$$

$$n_0\alpha_{\rm mm} = 3f\mu_0\frac{\mu_1(\mu_2-\mu_1)(\epsilon_2+2\epsilon_1) - \mu_1(\kappa_2-\kappa_1)^2 - 3(\mu_2-\mu_1)\kappa_1^2}{(\mu_2+2\mu_1)(\epsilon_2+2\epsilon_1)-(\kappa_2+2\kappa_1)^2} \quad (19)$$

$$n_0\alpha_{\rm em} = 3f\sqrt{\epsilon_0\mu_0}\frac{3\mu_1\epsilon_1(\kappa_2-\kappa_1) + \kappa_1[(\mu_2-\mu_1)(\epsilon_2-\epsilon_1) - (\kappa_2-\kappa_1)(\kappa_2+2\kappa_1)]}{(\mu_2+2\mu_1)(\varepsilon_2+2\varepsilon_1)-(\kappa_2+2\kappa_1)^2} \quad (20)$$

3. Numerical results

According to [4], effective constitutive parameters of MG method are expressed in the following form on the same assumption used for getting those of our method:

$$\epsilon_{\rm eff} = 1+3f\frac{(\epsilon_{\rm s}-1)[\mu_{\rm s}+2-f(\mu_{\rm s}-1)]-\kappa_{\rm s}^2(1-f)}{[\mu_{\rm s}+2-f(\mu_{\rm s}-1)][\epsilon_{\rm s}+2-f(\epsilon_{\rm s}-1)]-\kappa_{\rm s}} \tag{21}$$

$$\mu_{\rm eff} = 1+3f\frac{(\mu_{\rm s}-1)[\epsilon_{\rm s}+2-f(\epsilon_{\rm s}-1)]-\kappa_{\rm s}^2(1-f)}{[\mu_{\rm s}+2-f(\mu_{\rm s}-1)][\epsilon_{\rm s}+2-f(\epsilon_{\rm s}-1)]-\kappa_{\rm s}} \tag{22}$$

$$\kappa_{\rm eff} = \frac{9f\kappa_{\rm s}}{[\mu_{\rm s}+2-f(\mu_{\rm s}-1)][\epsilon_{\rm s}+2-f(\epsilon_{\rm s}-1)]-\kappa_{\rm s}} \tag{23}$$

Figure 1 and 2, respectively, show $\epsilon_{\rm eff}$ and $\kappa_{\rm eff}$ as a function of f for $\epsilon_{\rm s}=3$ and $\epsilon_{\rm s}=30$. All numerical results are valid for $f\leq 0.6$, because chiral particles are completely packed at about $f=0.6$ and deformed for $f>0.6$. From figure 1, we can see that $\epsilon_{\rm eff}$ of our method becomes large for increasing the volume fraction of chiral particles, while $\epsilon_{\rm eff}$ of MG method does not. Figure 2 shows that $\kappa_{\rm eff}$ of our method changes only a little with $\epsilon_{\rm s}$, while $\kappa_{\rm eff}$ of MG method does not.

4. Conclusion

Using our method[7–9] we have analyzed the effective constitutive parameters of a medium containing randomly distributed many chiral particles embedded in an achiral background medium. Consequently, our method is shown to be more valid for the analysis of a chiral mixture containing chiral particles than MG method; in particular, the difference between both methods becomes remarkable for chiral particles with high dielectric constant.

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References

[1] A. Sihvola and I. V. Lindell, "Analysis on chiral mixtures", Journal of Electromagnetic Waves and Applications. vol.6, pp.553–572, 1992.

[2] A. Sihvola, "Electromagnetic modeling of biisotropic media", Progress in Electromagnetic Research, vol.PIER 9, pp.45–86, 1994.

[3] W. Ren, "Bi-isotropic layered mixtures", Progress in Electromagnetic Research, vol.PIER 9, pp.103–116, 1994.

[4] N. P. Zhuck and A. S. Omaru, "Calculation of the effective constitutive parameters of a disordered bi-isotropic medium using renormalization method", IEEE Trans. Antennas Propagat., vol.AP-44, pp.1142–1149, 1996.

[5] I. V. Lindell, A. Sihvola, S. A. Tretyakov and A. J. Viitanen, Electromagnetic Waves in Chiral and Bi-Isotropic Media, Artech House, Boston·London, 1994.

[6] D. Cheng, "Homogenisation of densely-distributed chiral spheres", Electronics Letters, vol.32, pp.2326–2327, 1994.

[7] Y. Nanbu and M. Tateiba, "A comparative study of the effective dielectric constant of a medium containing randomly distributed dielectric spheres embedded in a homogeneous background medium", Waves in Random Media, vol.6, pp.347-360, 1996.

[8] M. Tateiba, "A new approach to the problem of wave scattering by many particles", Radio Science, vol.22, pp.881-884, 1987.

[9] M. Tateiba, "Electromagnetic wave scattering in media whose particles are randomly displaced from a uniformly ordered spatial distribution", IEICE Trans. Electron., vol.E78-C, pp.1357-1365, 1995.

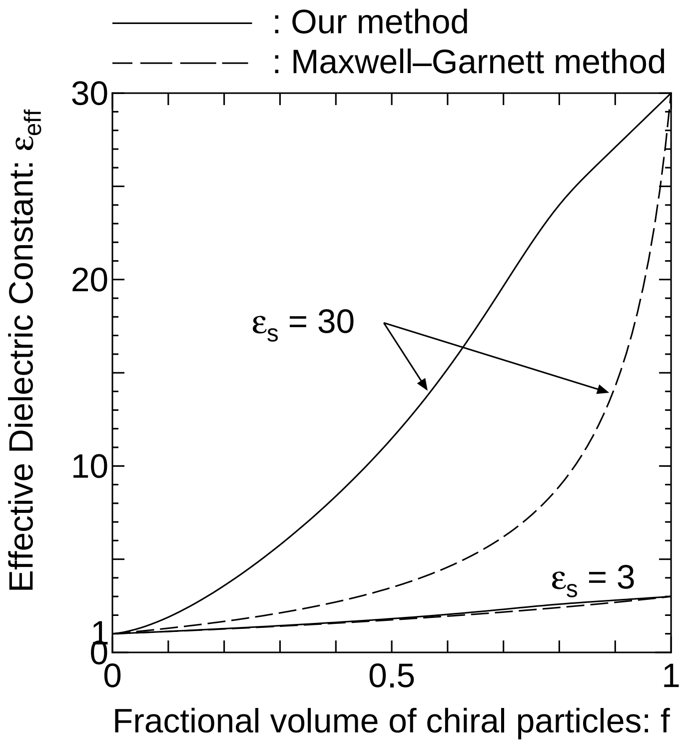

Figure 1: Effective dielectric constant: ϵ_{eff} as a function of the volume fraction of chiral particles($\epsilon_{\mathrm{s}} = 3$ and 30, $\mu_{\mathrm{s}} = 1$ and $\kappa_{\mathrm{s}} = 0.1$)

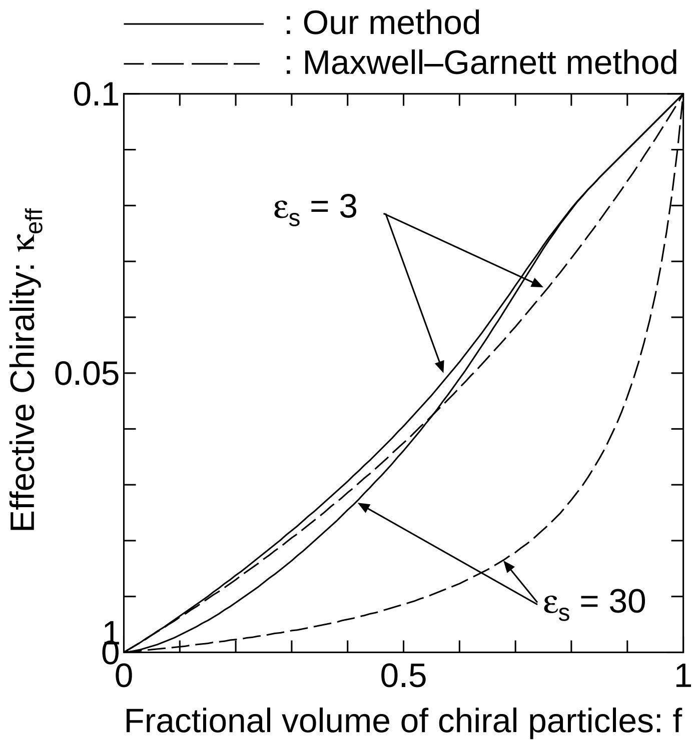

Figure 2: Effective chirality: κ_{eff} as a function of the volume fraction of chiral particles($\epsilon_{\mathrm{s}} = 3$ and 30, $\mu_{\mathrm{s}} = 1$ and $\kappa_{\mathrm{s}} = 0.1$)