NUMERICAL ANALYSIS OF THE EM SCATTERED POWER BY A LAYER OF RANDOM MEDIUM FOR APPLICATION TO WATER DETECTION OF MOIST SOIL

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1 Introduction

The detection of a water content of moist soil is an important problem in civil and agricultural engineering. Moist soil is composed of air, soil particles, bound water and free water[1] and may be regarded as a dense random medium from a theoretical point of view. A radiative transfer equation called dense medium radiative transfer equation(DMRT) has been derived from a wave equation with the Quasi Crystalline Approximation with Coherent Potential(QCA-CP) and the ladder approximation[2] to analyze the wave propagation and scattering in a dense random medium. Under these approximations, the random medium is assumed to be a homogeneous medium with the effective dielectric constant evaluated by QCA-CP in Rayleigh scattering region, and the scattering coefficient and the extinction rate in the DMRT are closely related to the effective dielectric constant. Another method for evaluating the effective dielectric constant has been presented by one of the authors [3], which method is called "our method" in this paper. It has been shown that our method is physically valid for scatterers of high dielectric constant like water drops where QCA-CP becomes invalid[4]. We have shown that the scattering cross section of a random medium is fairly dependent on the effective dielectric constant when using a radiative transfer equation[5].

This paper considers a three layer model, composed of air, moist soil layer and bottom layer for developing a method for detecting a water content of soil by active remote sensing. The moist soil layer is assumed to be a random medium with spherical water drops embedded in a homogeneous background medium. A radiative transfer equation with the parameters evaluated by our method is used in the medium to calculate the scattering cross section of the moist soil layer by changing the fractional volume of water drops and the incident angle and polarization of incident waves. From the numerical results, we discuss the detection possibility of the water content in this approach.

2 Formulation

Let us consider a layer of $\varepsilon_g\varepsilon_0$ and thickness d (region 1) where identical dielectric spheres of $\varepsilon_s\varepsilon_0$ and radius a are embedded, layer which is over a semi-infinite layer of $\varepsilon_2\varepsilon_0$(region 2) and under air ε_0(region 0), as shown in Fig.1. A polarized electromagnetic plane-wave is incident on region 1 from region 0 in the direction of $(\pi - \theta_{0i}, \phi_{0i})$. The second moment of waves in a random medium obeys the Bethe-Salpeter equation in general. By applying the ladder approximation, the Bethe-Salpeter equation can be reduced to a DMRT. The DMRT in region

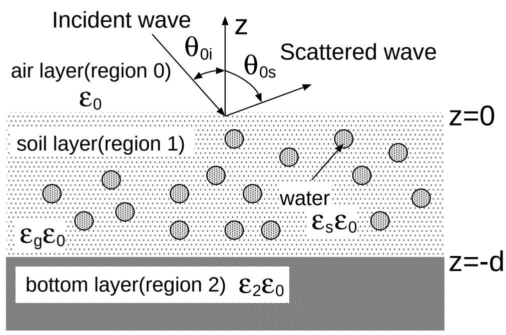

Figure 1: Geometry of the problem of wave scattering from a layer of discrete random medium.

1 is written as, for $0 \le \theta \le \pi$,

$$\cos\theta \frac{\partial \boldsymbol{I}(\theta,\phi;z)}{\partial z} = -\kappa_{\rm e}\boldsymbol{I}(\theta,\phi;z) + \frac{\kappa_{\rm s}}{4\pi}\int_0^{\pi} {\rm d}\theta' \sin\theta' \int_0^{2\pi} {\rm d}\phi' \bar{\boldsymbol{P}}(\theta,\phi;\theta',\phi') \cdot \boldsymbol{I}(\theta',\phi';z), \quad (1)$$

where $\boldsymbol{I}(\theta,\phi;z)=[I_{\rm v}, I_{\rm h}, U, V]^t$ is the Stokes vector, in which the superscript t denotes transposition. $\bar{\boldsymbol{P}}(\theta,\phi;\theta',\phi')$ is a 4×4 matrix, called phase matrix which describes the relation between the incident Stokes vector in the direction (θ',ϕ') and the scattered one in the direction (θ,ϕ), and is assumed to be the Rayleigh phase matrix[6] in this paper. Here $\kappa_{\rm e}$ and $\kappa_{\rm s}$ are the extinction rate and the scattering coefficient, respectively, and closely related to the effective dielectric constant of a random medium $\varepsilon_{\rm eff}\varepsilon_0$.

On the other hand, the effective propagation constant $K(=K' + {\rm j}K'')$ in the random medium can be expressed as

$$K^2 = k^2\varepsilon_{\rm eff} = k_{\rm g}^2 + n_0 c \quad (2)$$

where k and $k_{\rm g}$, respectively, are the wave number in free space and of the background medium, and n_0 denotes the distribution density of spheres. The parameter c, which shows the multiple scattering effects on K, has been given by our method[4]:

$$c = k_{\rm d}^2 V_0 + \frac{3k_{\rm d}^2(k_{\rm g}^2 + k_{\rm d}^2 f)(k_{\rm e0}^2 - k_{\rm d}^2)V_0}{[k_{\rm d}^2 + 3(k_{\rm g}^2 + k_{\rm d}^2 f)][k_{\rm e0}^2 - k_{\rm g}^2 + 3(k_{\rm g}^2 + k_{\rm d}^2 f)]} + {\rm j}\frac{2a^3 k_{\rm d}^4 (k_{\rm g}^2 + k_{\rm d}^2 f)^{5/2}}{[k_{\rm d}^2 + 3(k_{\rm g}^2 + k_{\rm d}^2 f)]^2}\frac{(1-f)^4}{(1+2f)^2}V_0. \quad (3)$$

Here $k_{\rm d}^2 = k_{\rm s}^2 - k_{\rm g}^2$, and V_0 is the volume of a sphere and, $k_{\rm s}$ the wave number of a sphere, f the fractional volume of spheres and $k_{\rm e0}$ is defined by the following equation[7]:

$$k_{\rm e0} = \sqrt{\frac{2}{\pi}}\frac{k_{\rm d}^2}{\sigma^3}\int_0^a \exp\left[-\frac{r_1^2}{2\sigma^2}\right] r_1^2 {\rm d}r_1, \quad (4)$$

where σ is the variance of random replacement of spheres from a regular distribution. In Eq.(1), $\kappa_{\rm e}$ and $\kappa_{\rm s}$, respectively, are given as

$$\kappa_{\rm e} = 2K'', \quad \kappa_{\rm s} = \frac{n_0|c|^2}{6\pi}\frac{(1-f)^4}{(1+2f)^2}. \quad (5)$$

The reflection and transmission angles at the boundaries are assumed to obey Snell's law for K' because $K' \gg K''$. The boundary conditions for the Stokes vector at $z=0$ and $-d$ are as follows: for $0 \le \theta \le \pi/2$,

$$\begin{aligned} \boldsymbol{I}(\pi-\theta,\phi;0) &= \bar{\boldsymbol{T}}_{01}(\theta_0)\cdot\boldsymbol{I}_{0{\rm i}}(\pi-\theta_0,\phi_0;0) + \bar{\boldsymbol{R}}_{10}(\theta)\cdot\boldsymbol{I}(\theta,\phi;0) \\ \boldsymbol{I}(\theta,\phi;-d) &= \bar{\boldsymbol{R}}_{12}(\theta)\cdot\boldsymbol{I}(\pi-\theta,\phi;-d) \end{aligned} \quad (6)$$

where $\boldsymbol{I}_{0{\rm i}}(\pi-\theta_0,\phi_0;0)$ is the Stokes vector of the incident plane-wave from region 0. $\bar{\boldsymbol{R}}_{\rm ij}(\theta)$ and $\bar{\boldsymbol{T}}_{\rm ij}(\theta)$, i, j $= 0,1,2$ denote the reflection and transmission matrices for the Stokes vector at the propagation from region i to j with incident angle θ.

The scattered Stokes vector $\boldsymbol{I}_{0{\rm s}}(\theta_{0{\rm s}},\phi_{0{\rm s}};0) = [I_{\rm v0s}, I_{\rm h0s}, U_{\rm 0s}, V_{\rm 0s}]^t$ in the direction $(\theta_{0{\rm s}},\phi_{0{\rm s}})$ is expressed as

$$\boldsymbol{I}_{0{\rm s}}(\theta_{0{\rm s}},\phi_{\rm os};0) = \bar{\boldsymbol{T}}_{10}(\theta_{\rm s})\cdot\boldsymbol{I}(\theta_{\rm s},\phi_{\rm s};0), \quad (7)$$

When we assume that an α-polarized wave intensity $I_{\alpha0{\rm i}}(\pi-\theta_{0{\rm i}},\phi_{0{\rm i}};0)$ is incident on the random medium in the direction $(\pi-\theta_{0{\rm i}},\phi_{0{\rm i}})$ and a β-polarized wave intensity $I_{\beta0{\rm s}}(\theta_{0{\rm i}},\pi+\phi_{0{\rm i}};0)$ is scattered in the backward direction $(\theta_{0{\rm i}},\pi+\phi_{0{\rm i}})$, then the backscattering cross section $\sigma_{\beta\alpha}(\theta_{0{\rm i}})$ is defined as

$$\sigma_{\beta\alpha}(\theta_{0{\rm i}}) = 4\pi\frac{\cos\theta_{0{\rm i}} I_{\beta0{\rm i}}(\theta_{0{\rm i}},\pi+\phi_{0{\rm i}};0)}{I_{\alpha0{\rm i}}}, \quad (8)$$

where α,β =vertically(v) or horizontally(h).

3 Numerical Results

Water drops are lossy in microwave region[1]. Therefore the scattering effect in the random medium becomes small, which means that the ladder approximation used here is valid and then the backscattering enhancement can be neglected. Moreover we can use the iterative solution up to the second order as an efficient solution. The validity of the solution was also proved by the directly numerical analysis of Eq.(1). The physical parameters are assumed to be the operating frequency $\nu = 2$ GHz, $\varepsilon_{\rm g} = 3.0$ and $a = 1$mm, and the dielectric constant of water drops $\varepsilon_{\rm s}$ is calculated from Debye's equation[1]. Fig.2 shows $\sigma_{\beta\alpha}$ as a function of f when $\theta_{0{\rm i}} = 13.7$ degree, ε_2=$\varepsilon_{\rm eff}$ and d=100, 200 and 500cm. The $\sigma_{\beta\alpha}$ behaves as a convex function of f for three cases of d. Fig.3 depicts the effect of bottom layer on $\sigma_{\beta\alpha}$ by assuming $\varepsilon_2 = \varepsilon_{\rm eff}$(no reflection) and $\varepsilon_2 = \infty$(total reflection). For $f < 0.05$, the multi-reflection of waves due to upper and lower boundaries increases the $\sigma_{\beta\alpha}$ for total reflection case. The contribution of bottom layer to $\sigma_{\rm vv}$ and $\sigma_{\rm hh}$ is smaller than that to $\sigma_{\rm hv}$ and $\sigma_{\rm vh}$ because the co-polarized wave intensity depends mainly on the first order solution. From these results, $\sigma_{\beta\alpha}$ has a common property that it first increases to a certain level and then decreases as f becomes large. It means that we cannot determine f directly from the measurement of $\sigma_{\beta\alpha}$ because there are two values of f corresponding to one value of $\sigma_{\beta\alpha}$.

The ratio of $\sigma_{\rm vv}$ to $\sigma_{\rm hh}$ is illustrated in Fig.4 when $\theta_{0{\rm i}}$=13.7, 33.7 and 53.7 degree and $\varepsilon_2 = \varepsilon_{\rm eff}$. Fig.4 shows that $\sigma_{\rm vv}/\sigma_{\rm hh}$ is more sensitive to larger incident angles. Fig.5 shows the $\sigma_{\rm vv}/\sigma_{\rm hh}$ as a function of f when $d = 100$, 200 and 500 cm, $\varepsilon_2 = \varepsilon_{\rm eff}$ and ∞ and $\theta_{0{\rm i}} = 53.7$ degree. We can observe that there is a one-to-one correspondence between $\sigma_{\rm vv}/\sigma_{\rm hh}$ and f at $f > 0.05$, which means that the measurement of $\sigma_{\rm vv}/\sigma_{\rm hh}$ is applicable to sensing of the soil moisture at $f > 0.05$. The threshold value of f, at which $\sigma_{\rm vv}/\sigma_{\rm hh}$ becomes independent of the bottom layer, becomes small for large d.

4 Conclusion

We have assumed a moist soil model as a random medium layer sandwiched between free space and a homogeneous bottom space and numerically evaluated scattering cross sections of the layer by using a dense medium radiative transfer equation(DMRT). A multiple scattering method applicable to scatterers of high dielectric constant has been used to estimate the parameters in the DMRT. The detection possibility of a water content of soil has been discussed by using the characteristics of the scattering cross section as functions of the incident angle and polarization of incident waves and the water content. We found out that the ratio of co-polarized backscattering cross section between both cases of vertical and horizontal polarization incidence has a one-to-one correspondence to a volumetric water content larger than 0.05.

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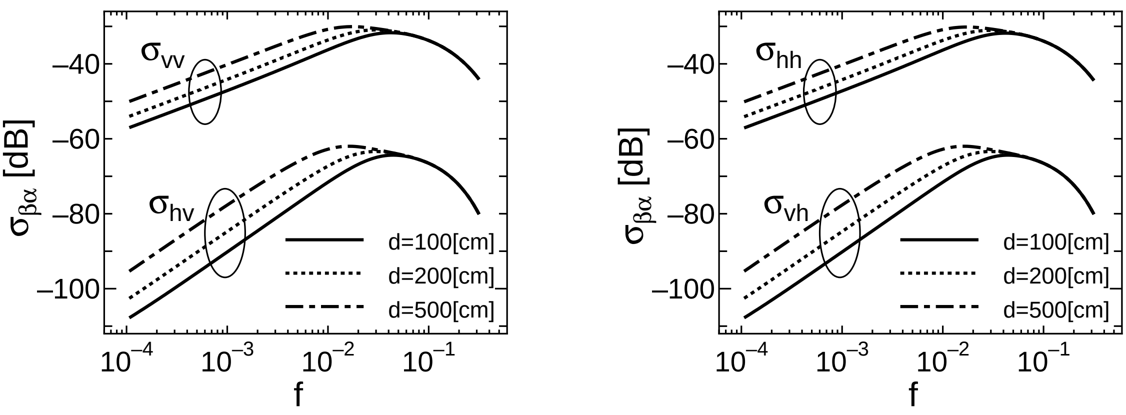
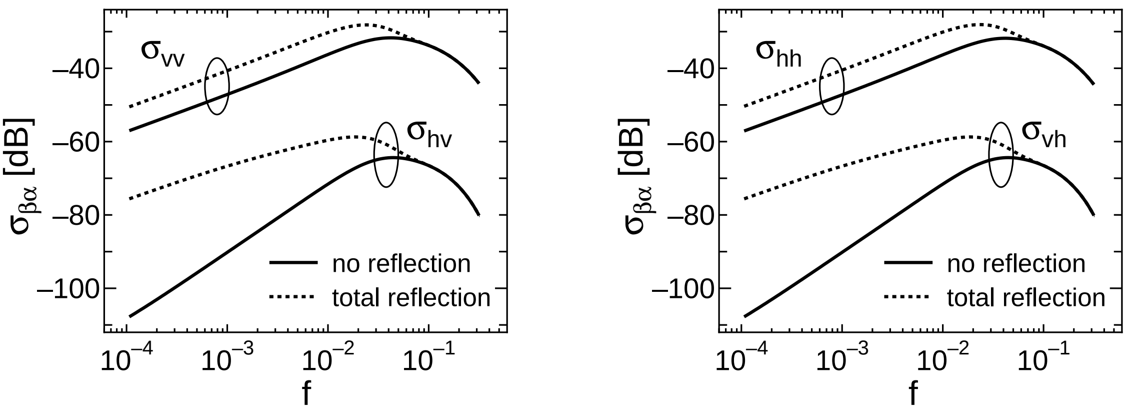


(a) (b)

Figure 2: The backscattering cross section $\sigma_{\beta\alpha}$ as a function of the fractional volume f, when ν=2 GHz, a=1 mm, $\varepsilon_{\rm g}$= 3.0, $\varepsilon_2 = \varepsilon_{\rm eff}$, $d = 100, 200, 500$ cm, $\theta_{0\rm i}$=13.7 degree, for both cases: (a) v-polarized wave intensity incidence and (b) h-polarized wave intensity incidence.



(a) (b)

Figure 3: The backscattering cross section $\sigma_{\beta\alpha}$ as a function of the fractional volume f, when ν=2 GHz, a=1 mm, $\varepsilon_{\rm g}$= 3.0, $\varepsilon_2 = \varepsilon_{\rm eff}$ and ∞, d=100cm, $\theta_{0\rm i}$ = 13.7 degree, for both cases: (a) v-polarized wave intensity incidence and (b) h-polarized wave intensity incidence.

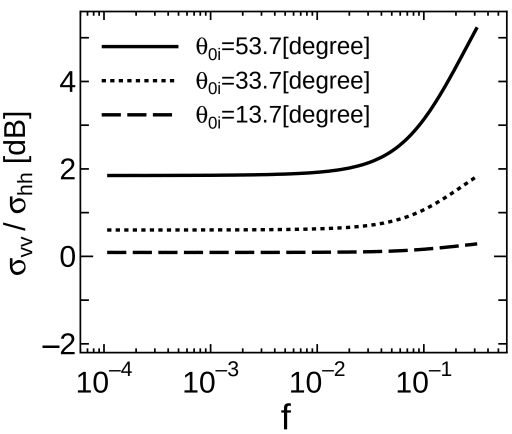


Figure 4: The ratio of $\sigma_{\rm vv}$ to $\sigma_{\rm hh}$ as a function of the fractional volume f, when ν=2 GHz, a= 1 mm, $\varepsilon_{\rm g}$ = 3.0, $\varepsilon_2 = \varepsilon_{\rm eff}$, d = 100 cm, $\theta_{0\rm i}$ = 13.7, 33.7, 53.7 degree

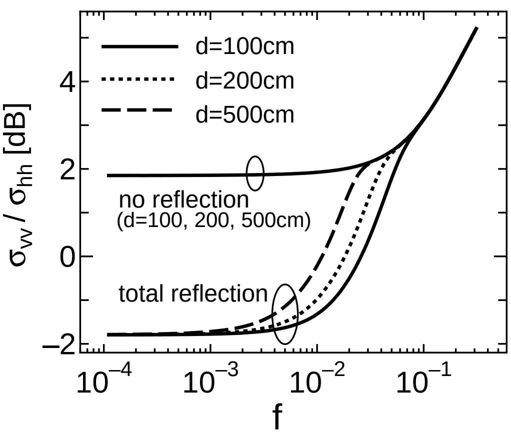


Figure 5: As Fig.4, but with $\varepsilon_2 = \varepsilon_{\rm eff}, \infty$, d=100, 200, 500cm and $\theta_{0\rm i}$ = 53.7 degree.