Bistatic cross-sections of conducting circular cylinders embedded in continuous random media

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1. Introduction

The backscattering enhancement was found out in theory and experiment as a fundamental phenomenon in a random medium, which is produced by statistical coupling of incident and backscattered waves due to the effect of double passage. In general, the problem of wave scattering by a body in a random medium needs to be treated by taking account of the boundary conditions of incident and scattered waves on the body. However conventional methods developed in free space are not directly applicable to the analysis of this problem, because the incident and scattered waves are random functions. Recently, a method has been presented for solving the problem as a boundary value problem by introducing an operator named current generator, which transforms incident waves into surface currents on the body surface and depends only on the body[1]. The scattered waves are obtained with Green's function in a random medium and the current generator. Note that the Green's function is a random function and that the statistical coupling of incident and scattered waves means a statistical coupling of the Green's functions. In other words, under the condition that the forward scattering approximation is valid, a second moment of Green's functions is necessary for analyzing the average of scattered waves, and the fourth moment of Green's functions is needed to analyze the average intensity of scattered waves. It is not easy to obtain the fourth moment. An approximation of the fourth moment is given by a product of the second moments for backscattering in some cases, and some numerical analyses show that the monostatic cross-sections of conducting cylinders in random media become nearly twice as large as that in free space under the condition that the spatial coherence length of an incident wave on the cylinder is larger enough than the radius of the cylinder[1, 2].

When the average scattered intensity is enhanced in the backward direction, it is predicted to decrease in some other directions from the law of energy conservation for a point body scattering[3]. To make clear numerically the prediction as well as the scattering characteristics for a practical body scattering, we need to analyze the bistatic cross-section (BCS) and therefore need a more general fourth moment, not only valid for backscattering.

Several approximation methods have been presented for getting the fourth moment. One of them gives a solution for propagator of a paired field measure in a random medium as a two-point random function, by using a two-scale asymptotic procedure[5, 6]. And the fourth moment of Green's functions can be obtained approximatively with the second moment of the propagators. We apply the approximate fourth moment to the analysis of BCS of conducting cylinders in continuous random media for E-wave incidence case and discuss numerically the effects of random media on the BCS.

Time factor $\exp(-j\omega t)$ is assumed and suppressed in the following.

2. Formulation

Consider the problem of electromagnetic wave scattering from a perfectly conducting circular cylinder embedded in a continuous random medium, as shown in Fig.1.

The random medium is assumed to be described by the dielectric constant ε, the magnetic permeability μ and the electric conductivity σ, which are expressed as

$$\varepsilon = \varepsilon_0[1+\delta\varepsilon(\boldsymbol{r})]\ ,\quad \mu=\mu_0\ ,\quad \sigma=0 \tag{1}$$

where ε_0, μ_0 are constant and $\delta\varepsilon(\boldsymbol{r})$ is a random function with

$$\langle\delta\varepsilon(\boldsymbol{r})\rangle = 0\ ,\quad \langle\delta\varepsilon(\boldsymbol{r}_1)\cdot\delta\varepsilon(\boldsymbol{r}_2)\rangle = B(\boldsymbol{r}_1,\boldsymbol{r}_2) \tag{2}$$

Here the angular brackets denote the ensemble average and $B(\boldsymbol{r}_1,\boldsymbol{r}_2)$ is a correlation function of the random function. For many cases, B can be approximated as

$$B(\boldsymbol{r}_1,\boldsymbol{r}_2) = B_{\mathrm{t}}(\boldsymbol{r}_{\mathrm{t}1}-\boldsymbol{r}_{\mathrm{t}2})\delta(z_1-z_2) \tag{3}$$

where $\boldsymbol{r}_{\mathrm{t}}$ is a two-dimensional position vector in a plane transverse to the z direction. The $B_{\mathbf{t}}$ is assumed to be the Gaussian function

$$B_{\mathrm{t}}(\boldsymbol{r}_{\mathrm{t}}) = B_0\exp\left(-\frac{|\boldsymbol{r}_{\mathrm{t}}|^2}{l^2}\right) \tag{4}$$

where B_0, l are the intensity and scale-size of the random medium fluctuation, respectively. For practical turbulence, the following conditions may be satisfied:

$$B_0 \ll 1,\qquad kl \gg 1 \tag{5}$$

where $k=\omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber in free space.

Let us now consider the E-wave incidence of an electromagnetic wave radiated from a line source which is far from the cylinder and parallel to the y axis. Under (5), the scalar wave approximation and the forward scattering approximation are valid. Then the average intensity of scattered waves is given by the following equation[1]:

$$\begin{aligned}\langle|u_{\mathrm{s}}|^2\rangle &= \int_S \mathrm{d}\boldsymbol{r}_{01}\int_S \mathrm{d}\boldsymbol{r}_{02}\int_S \mathrm{d}\boldsymbol{r}_1'\int_S \mathrm{d}\boldsymbol{r}_2'\,[Y_{\mathrm{E}}(\boldsymbol{r}_{01}|\boldsymbol{r}_1')Y_{\mathrm{E}}^*(\boldsymbol{r}_{02}|\boldsymbol{r}_2')\\ &\quad\times\langle G(\boldsymbol{r}|\boldsymbol{r}_{01})G(\boldsymbol{r}_1'|\boldsymbol{r}_{\mathrm{T}})G^*(\boldsymbol{r}|\boldsymbol{r}_{02})G^*(\boldsymbol{r}_2'|\boldsymbol{r}_{\mathrm{T}})\rangle]\end{aligned} \tag{6}$$

where the asterisk denotes the complex conjugate. Here Y_{E} is the current generator in E wave case, and can be obtained in a simple form for a conducting circular cylinder[1].

$$Y_{\mathrm{E}}(\boldsymbol{r}|\boldsymbol{r}') = \frac{\mathrm{j}}{\pi^2 a^2}\sum_{n=-\infty}^{\infty}\frac{\exp[\mathrm{j}n(\theta-\theta')]}{J_n(ka)H_n^{(1)}(ka)} \tag{7}$$

where J_n is the Bessel function of order n and $J_n(ka)\neq 0$; that is, the internal resonance frequencies are excepted. The $H_n^{(1)}$ is the Hankel function of first kind and the $G(\boldsymbol{r}|\boldsymbol{r}')$ is Green's function in the random medium. The Fourth moment of Green's functions can be written as a product[4]:

$$\begin{aligned}&\langle G(\boldsymbol{r}|\boldsymbol{r}_{01})G(\boldsymbol{r}_1'|\boldsymbol{r}_T)G^*(\boldsymbol{r}|\boldsymbol{r}_{02})G^*(\boldsymbol{r}_2'|\boldsymbol{r}_T)\rangle\\ &\quad= G_0(\boldsymbol{r}|\boldsymbol{r}_{01})G_0^*(\boldsymbol{r}|\boldsymbol{r}_{02})G_0(\boldsymbol{r}_1'|\boldsymbol{r}_{1T})G_0^*(\boldsymbol{r}_2'|\boldsymbol{r}_{2T})m_{\mathrm{s}}\end{aligned} \tag{8}$$

where G_0 is Green's function in free space. The m_{s} includes multiple-scattering effects of random medium.

Let $u(\boldsymbol{r}_\mathrm{t}, z)$ denotes a random wave function in the random medium and $R(\boldsymbol{r}_{\mathrm{t}1}, \boldsymbol{r}_{\mathrm{t}2}, z)$ be a two-point random function (TPRF) defined by $R(\boldsymbol{r}_{\mathrm{t}1}, \boldsymbol{r}_{\mathrm{t}2}, z) = u(\boldsymbol{r}_{\mathrm{t}1}, z)u^*(\boldsymbol{r}_{\mathrm{t}2}, z)$. The $R(\boldsymbol{r}_{\mathrm{t}1}, \boldsymbol{r}_{\mathrm{t}2}, z)$ at a range plane $z = z'$ can be related to $R(\boldsymbol{r}_{\mathrm{t}10}, \boldsymbol{r}_{\mathrm{t}20}, z_0)$ at the excitation plane $z = z_0$ by

$$R(\boldsymbol{r}_{\mathrm{t}1}, \boldsymbol{r}_{\mathrm{t}2}, z') = \int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{r}_{\mathrm{t}10} \int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{r}_{\mathrm{t}20} g_2(\boldsymbol{r}_{\mathrm{t}1}, \boldsymbol{r}_{\mathrm{t}2}, z'|\boldsymbol{r}_{\mathrm{t}10}, \boldsymbol{r}_{\mathrm{t}20}, z_0) R(\boldsymbol{r}_{\mathrm{t}10}, \boldsymbol{r}_{\mathrm{t}20}, z_0) \tag{9}$$

where $g_2(\boldsymbol{r}_{\mathrm{t}1}, \boldsymbol{r}_{\mathrm{t}2}, z'|\boldsymbol{r}_{\mathrm{t}10}, \boldsymbol{r}_{\mathrm{t}20}, z_0)$ is a TPRF propagator. Then we can write

$$m_\mathrm{s} = \langle g_2(\boldsymbol{r}_{\mathrm{t}1}, \boldsymbol{r}_{\mathrm{t}2}, z|\boldsymbol{r}_{\mathrm{t}10}, \boldsymbol{r}_{\mathrm{t}20}, z_0) g_2(\boldsymbol{r}_{\mathrm{t}3}, \boldsymbol{r}_{\mathrm{t}4}, z|\boldsymbol{r}_{\mathrm{t}30}, \boldsymbol{r}_{\mathrm{t}40}, z_0)\rangle \, . \tag{10}$$

By referring to papers [5] and [6], for our case we have

$$m_\mathrm{s} = \frac{k}{2\pi L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}\eta \mathrm{d}\rho \exp\left\{ -\frac{\mathrm{j}k}{L} \eta [\rho - (x - x_\mathrm{T})] \right\} P(\rho, \eta) \tag{11}$$

$$\begin{aligned} P(\rho, \eta) = \exp\Bigg(-\frac{k^2 L}{8} \int_0^1 \mathrm{d}t \Big\{ & D_\mathrm{t}[a(\sin\theta_1' - \sin\theta_2')t + \eta t] \\ & + D_\mathrm{t}[a(\sin\theta_{01} - \sin\theta_{02})t + \eta t] \\ & - D_\mathrm{t}[a(\sin\theta_1' - \sin\theta_{01})t - \rho(1-t) + \eta t] \\ & - D_\mathrm{t}[a(\sin\theta_2' - \sin\theta_{02})t - \rho(1-t) - \eta t] \\ & + D_\mathrm{t}[a(\sin\theta_1' - \sin\theta_{02})t - \rho(1-t)] \\ & + D_\mathrm{t}[a(\sin\theta_2' - \sin\theta_{01})t - \rho(1-t)] \Big\} \Bigg) \end{aligned} \tag{12}$$

$$D_\mathrm{t}(r) = 2[B_\mathrm{t}(0) - B_\mathrm{t}(r)] \tag{13}$$

where L is the distance between the source (or receiver) and the circular cylinder.

3. Numerical results

As an example, parameters of the random medium are assumed as $kl = 100\pi$ and $B_0 = 2.5 \times 10^{-7}$. The BCS in the random medium σ normalized to that in free space σ_0 is shown in Fig.2 in three cases of $ka = 1, 3, 5$, where ϕ denotes the angle between a line from the source to the cylinder and a line from the cylinder to the receiver. The BCSs in the three cases have almost the same value, because the spatial coherence length of the incident wave on the cylinders l_c is larger enough than the radius of the cylinders ($kl_c \simeq 270 \gg ka$) and the effects of the random medium on σ/σ_0 are the same for the three cases. We note that there is a scattering enhancement peak at $\phi = 0$ and two valleys on both sides of the peak where the scattering intensity is diminished. The normalized BCS at the peak is about 2, which means that radar cross-sections (RCS) of those circular cylinders in the random medium become nearly twice as large as that in free space. The same result for RCS has been obtained in [1] with a different fourth moment approach. The integration value of the normalized BCS with respect to θ in the whole range becomes 1.0006, which shows that the result agrees with the law of energy conservation.

Next, let us see the effects of the scale size and intensity of random media on the BCS. The radius of the cylinder is fixed on $ka = 5$ for convenience of discussion. Figure 3 shows changes in the normalized BCS with different values of kl. As decreasing the value of kl, the backscattering enhancement peak becomes sharp and high on the one hand, and the valleys become deep and wide on the other. The change of BCS is physically reasonable.

Figure 4 shows the BCSs with different values of B_0. The changes of BCS is similar to that in Fig.3.

4. Conclusion

The bistatic cross-section (BCS) of a conducting circular cylinder in a continuous random medium has been analyzed numerically by using an approximation of the fourth moment of Green's functions in the random medium. The approximation is obtained by two-scale asymptotic procedure. The numerical results of the BCS agree well with the law of energy conservation. The effects of spatial correlation and intensity of the random medium are also discussed numerically, and shown to be physically reasonable. From this study we conclude that we are now at a stage that we may clarify the bistatic scattering of a conducting body of arbitrary shape and size in a random medium.

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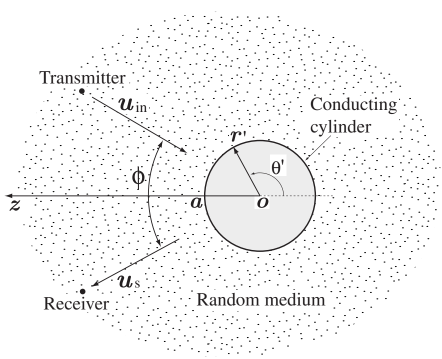

Fig.1: Geometry of the scattering problem from a conducting circular cylinder surrounded by a random medium.

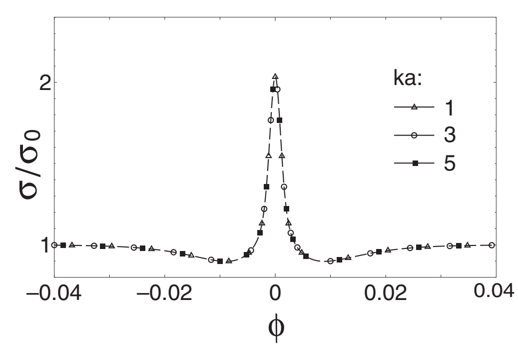

Fig.2: The normalized BCS of conducting circular cylinders.

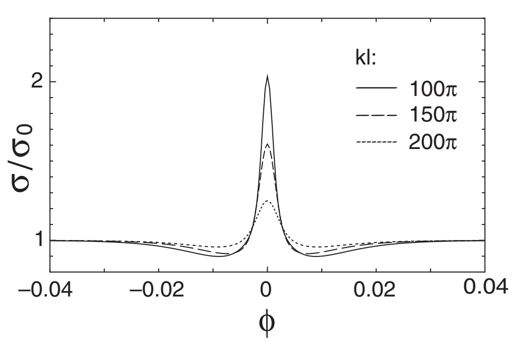

Fig.3: The effect of scale size of the random medium on the BCS.

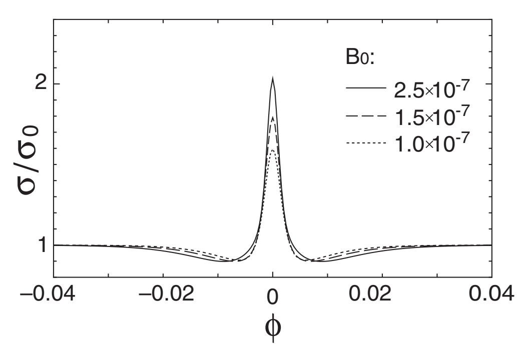

Fig.4: The effect of intensity of the random medium on the BCS.