FAR-FIELD APPROXIMATION OF THE INTERFACIAL FIELDS GENERATED BY A TINY CURRENT SOURCE ON A UNIAXIALLY ANISOTROPIC HALF-SPACE DIELECTRIC

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1. Introduction

The far-zone fields generated by interfacial dipole antennas located at the surface of a half-space lossless dielectric have been derived in [1] showing that the fields have a null along the interface, employing the inverse-distance approximation. The existence of null along the interface also holds for the case in which the lower half-space medium is uniaxially anisotropic. We are interested in the far-zone electromagnetic fields at this approximated interface null.

In this paper, we present the expressions for the interfacial far-fields generated by a tiny interfacial current source on a uniaxially anisotropic half-space dielectric with its axis normal to the interface. The physical configuration is shown in Fig. 1, where the source antenna is simply a current source and the ideal receiver antenna is for measuring the electromagnetic fields. (A practical situation involving this type of configuration may be the sea ice which contains vertically elongated pockets of brine making the icy medium approximately macroscopically anisotropic [2].) The uniaxial case gives us more physical insight into the propagation characteristics of the transverse-electric/magnetic (TE/TM) components than the isotropic case since the uniaxial anisotropy of the lower medium associates different speeds to the TE and TM lagging waves, and hence bears more generalized solutions.

For our anisotropic analysis, we should first obtain the transient interfacial-field expressions by applying the isotropic *interfacial scheme* of the *Cagniard-de-Hoop* method [3], and then we make far-field approximations extracting only the singular constituents. It can be shown that there exist full-explicit solutions for the HH and VV problems as in the isotropic problems, and that the far-field transient signals are dominated by the leading- and lagging-impulse components (i.e., the δ-function terms). Hence the fields in the frequency domain can be obtained by just Fourier-transforming these impulse components, provided that the operating frequency is sufficiently high that many cycles can fill up the interval between the leading and the lagging impulse, which is true in general for the far fields.

The HV and VH far-field expressions can be also obtained, although their exact solutions are yet unavailable. The far-fields of the HV and VH cases are acquired by going through some more analytical procedures taking into account the simple-pole singularity around the void lagging impulse. We then Fourier-transform the approximated fields to obtain the frequency-domain expressions.

The solutions in this paper may help save the calculation time in the multilayer time-domain far-field analysis using the wavenumber/frequency synthesis or the numerical Cagniard-de-Hoop method [4], as well as they give us the far-zone descriptions for the interfacial fields.

2. Theory – Derivation of the Interfacial Fields

As an exemplary case, we consider the x-directional electric field generated by a tiny x-

directional current source of the form $\hat{\mathbf{x}}\,\delta(x)\delta(y)\delta(z)\delta(t)$. We may assume the y-directional current component to be zero without loss of generality of the horizontal direction. The interfacial x-oriented electric fields can be expressed in the Fourier/Laplace domain as

$$E_x \;=\; -s\frac{\mu_0}{\gamma_1+\gamma_{2\mathrm{TE}}}\sin^2\!\phi_k - \frac{1}{s}\,\frac{\frac{\gamma_1}{\varepsilon_1}\frac{\gamma_{2\mathrm{TM}}}{(\varepsilon_{2h}\varepsilon_{2z})^{1/2}}}{\frac{\gamma_1}{\varepsilon_1}+\frac{\gamma_{2\mathrm{TM}}}{(\varepsilon_{2h}\varepsilon_{2z})^{1/2}}}\cos^2\!\phi_k \tag{1}$$

by a derivation similar to that in [3], through the planar Fourier transform $\mathscr{F}$ defined by $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\,)\exp\big(j(k_x x+k_y y)\big)\,dxdy$ and through the Laplace transform $\mathscr{L}$ defined by $\int_0^{\infty}(\,)\times\exp(-st)\,dt$ with the transform variable s assumed real. The first term of Eq. (1) stems from the TE^z-wave, and the second term from the TM^z-wave. The vertical propagation constant γ_1 is given by $(k^2+s^2s_1^2)^{1/2}$, while $\gamma_{2\mathrm{TE}}$ and $\gamma_{2\mathrm{TM}}$ are given by $(k^2+s^2s_{2h}^2)^{1/2}$ and $(k^2+s^2s_{2z}^2)^{1/2}$, respectively; the intrinsic slownesses s_1, s_{2h}, and s_{2z} are equal to $(\mu_0\varepsilon_1)^{1/2}$, $(\mu_0\varepsilon_{2h})^{1/2}$, and $(\mu_0\varepsilon_{2z})^{1/2}$, respectively. The permittivities ε_{2h} and ε_{2z} are the horizontal and vertical ones of the anisotropic lower half-space medium. The radial wavenumber k is defined as $(k_x^2+k_y^2)^{1/2}$, and the wavenumber angle ϕ_k denotes the phase angle measured from the k_x-axis on the (k_x,k_y)-plane.

The plano-temporal version of the TE contribution in Eq. (1) can be processed as

$$\begin{aligned}\mathscr{L}^{-1}\mathscr{F}^{-1}\Big\{-s\frac{\mu_0}{\gamma_1+\gamma_{2\mathrm{TE}}}\sin^2\!\phi_k\Big\} \;&=\; -\frac{1}{\varepsilon_{2h}-\varepsilon_1}\partial_y^2\,\mathscr{L}^{-1}\Big\{\mathscr{H}_0^{-1}\Big\{\frac{\gamma_1}{sk^2}\Big\}-\mathscr{H}_0^{-1}\Big\{\frac{\gamma_{2\mathrm{TE}}}{sk^2}\Big\}\Big\}\\ &\approx\; \frac{1}{2\pi}\frac{\varepsilon_1}{\varepsilon_{2h}-\varepsilon_1}\frac{y^2}{\rho^4}\Big[-\eta_1\delta(t-s_1\rho)+\eta_{2h}\frac{\varepsilon_{2h}}{\varepsilon_1}\delta(t-s_{2h}\rho)\Big],\end{aligned} \tag{2}$$

through the *interfacial scheme* of the *Cagniard-de-Hoop* method as in [3]. The inverse Hankel transform $\mathscr{H}_0^{-1}$ is defined as $(1/4\pi)\int_{-\infty-j0^+}^{\infty-j0^+}(\,)\,H_0^{(2)}(k\rho)\,k\,dk$, and the interfacial scheme is featured by the deformation of the integration path shown in Fig. 2. The inverse Laplace transform can be performed explicitly without numerical calculation. The intrinsic impedances η_1 and η_{2h} are equal to $(\mu_0/\varepsilon_1)^{1/2}$ and $(\mu_0/\varepsilon_{2h})^{1/2}$, respectively. The omitted unit-step-function terms in Eq. (2) can be neglected for far-field approximation, since they are all proportional to the inverse cube of the radial distance ρ, whereas the δ-function terms in Eq. (2) are proportional to the inverse square of ρ. Likewise, the plano-temporal version of the TM contribution in Eq. (1) can be approximated as

$$\mathscr{L}^{-1}\mathscr{F}^{-1}\Bigg\{-\frac{1}{s}\,\frac{\frac{\gamma_1}{\varepsilon_1}\frac{\gamma_{2\mathrm{TM}}}{(\varepsilon_{2h}\varepsilon_{2z})^{1/2}}}{\frac{\gamma_1}{\varepsilon_1}+\frac{\gamma_{2\mathrm{TM}}}{(\varepsilon_{2h}\varepsilon_{2z})^{1/2}}}\cos^2\!\phi_k\Bigg\} \;\approx\; \frac{1}{2\pi}\frac{x^2}{\rho^4}\Big[\eta_1\delta(t-s_1\rho)\;+\;\eta_{2h}\delta(t-s_{2z}\rho)\Big]. \tag{3}$$

In a similar manner, the y-directional electric field can be calculated from the raw expression in the Fourier/Laplace domain

$$E_y \;=\; s\frac{\mu_0}{\gamma_1+\gamma_{2\mathrm{TE}}}\cos\phi_k\sin\phi_k - \frac{1}{s}\,\frac{\frac{\gamma_1}{\varepsilon_1}\frac{\gamma_{2\mathrm{TM}}}{(\varepsilon_{2h}\varepsilon_{2z})^{1/2}}}{\frac{\gamma_1}{\varepsilon_1}+\frac{\gamma_{2\mathrm{TM}}}{(\varepsilon_{2h}\varepsilon_{2z})^{1/2}}}\cos\phi_k\sin\phi_k\,. \tag{4}$$

Thus the approximated horizontal electric field can be written as

$$\begin{aligned}\Big(\mathsf{E}_x\,,\,\mathsf{E}_y\Big) \;&\approx\; \frac{1}{2\pi}\frac{\varepsilon_1}{\varepsilon_{2h}-\varepsilon_1}\Big(-\frac{y^2}{\rho^4}\,,\,\frac{xy}{\rho^4}\Big)\Big[\eta_1\,e^{-j\omega s_1\rho}\;-\;\eta_{2h}\frac{\varepsilon_{2h}}{\varepsilon_1}\,e^{-j\omega s_{2h}\rho}\Big]\\ &+\;\frac{1}{2\pi}\Big(\frac{x^2}{\rho^4}\,,\,\frac{xy}{\rho^4}\Big)\Big[\eta_1\,e^{-j\omega s_1\rho}+\eta_{2h}\,e^{-j\omega s_{2z}\rho}\Big].\end{aligned} \tag{5}$$

The z-directional electric field is discontinuous at the interface and is given by

$$\mathsf{E}_{1z} \approx \frac{1}{2\pi}\left(\frac{\varepsilon_{2h}\varepsilon_{2z}}{\varepsilon_1(\varepsilon_{2z}-\varepsilon_1)}\right)^{1/2}\frac{x}{\rho^3}\left[\eta_1 e^{-j\omega s_1\rho} - j\,\eta_{2h}\left(\frac{\varepsilon_1}{\varepsilon_{2h}}\right)^{1/2} e^{-j\omega s_{2z}\rho}\right], \ \mathsf{E}_{2z} = \frac{\varepsilon_1}{\varepsilon_{2z}}\mathsf{E}_{1z}, \tag{6}$$

just above and just below the interface, respectively. The imaginary terms in (6) originate from the simple-pole singularities in the transient waveforms at $t = s_{2z}\rho$. Likewise, the magnetic field can be also obtained by way of the interfacial scheme as

$$\begin{aligned}\left(\mathsf{H}_x\,,\,\mathsf{H}_y\right) \approx\ & \frac{1}{2\pi}\left(\frac{\varepsilon_1}{\varepsilon_{2h}-\varepsilon_1}\right)^{1/2}\left(\frac{xy}{\rho^4}\,,\,\frac{y^2}{\rho^4}\right)\left[e^{-j\omega s_1\rho} - j\left(\frac{\varepsilon_{2h}}{\varepsilon_1}\right)^{1/2} e^{-j\omega s_{2h}\rho}\right] \\ & - \frac{1}{2\pi}\left(\frac{\varepsilon_{2h}\varepsilon_{2z}}{\varepsilon_1(\varepsilon_{2z}-\varepsilon_1)}\right)^{1/2}\left(-\frac{xy}{\rho^4}\,,\,\frac{x^2}{\rho^4}\right)\left[e^{-j\omega s_1\rho} - j\left(\frac{\varepsilon_1}{\varepsilon_{2h}}\right)^{1/2}\frac{\varepsilon_1}{(\varepsilon_{2h}\varepsilon_{2z})^{1/2}} e^{-j\omega s_{2z}\rho}\right],\end{aligned} \tag{7}$$

$$\mathsf{H}_z \approx \frac{1}{2\pi}\frac{\varepsilon_1}{\varepsilon_{2h}-\varepsilon_1}\frac{y}{\rho^3}\left[e^{-j\omega s_1\rho} - \frac{\varepsilon_{2h}}{\varepsilon_1} e^{-j\omega s_{2h}\rho}\right]. \tag{8}$$

Similar procedure can be applied to the case in which the current source is z-directional, i.e., of the form $\hat{\mathbf{z}}\,\delta(x)\delta(y)\delta(z\pm 0^+)\delta(t)$. This vertical current source generates different amounts of fields according as it is located just above or just below the interface. The latter fields are the $(\varepsilon_1/\varepsilon_{2z})$-scaled versions of the former fields. Then the electric field excited by the current source $\hat{\mathbf{z}}\,\delta(x)\delta(y)\delta(z-0^+)\delta(t)$ is derived as

$$\left(\mathsf{E}_x\,,\,\mathsf{E}_y\right) \approx -\frac{1}{2\pi}\left(\frac{\varepsilon_{2h}\varepsilon_{2z}}{\varepsilon_1(\varepsilon_{2z}-\varepsilon_1)}\right)^{1/2}\left(\frac{x}{\rho^3}\,,\,\frac{y}{\rho^3}\right)\left[\eta_1 e^{-j\omega s_1\rho} - j\,\eta_{2h}\left(\frac{\varepsilon_1}{\varepsilon_{2h}}\right)^{1/2} e^{-j\omega s_{2z}\rho}\right], \tag{9}$$

$$\mathsf{E}_{1z} \approx -\frac{1}{2\pi}\frac{\varepsilon_{2h}\varepsilon_{2z}}{\varepsilon_1(\varepsilon_{2z}-\varepsilon_1)}\frac{1}{\rho^2}\left[\eta_1 e^{-j\omega s_1\rho} - \eta_{2h}\frac{\varepsilon_1}{\varepsilon_{2h}} e^{-j\omega s_{2z}\rho}\right], \ \mathsf{E}_{2z} = \frac{\varepsilon_1}{\varepsilon_{2z}}\mathsf{E}_{1z}. \tag{10}$$

The magnetic field is derived as

$$\left(\mathsf{H}_x\,,\,\mathsf{H}_y\right) \approx \frac{1}{2\pi}\frac{\varepsilon_{2h}\varepsilon_{2z}}{\varepsilon_1(\varepsilon_{2z}-\varepsilon_1)}\left(-\frac{y}{\rho^3}\,,\,\frac{x}{\rho^3}\right)\left[e^{-j\omega s_1\rho} - \frac{\varepsilon_1}{\varepsilon_{2h}}\frac{\varepsilon_1}{(\varepsilon_{2h}\varepsilon_{2z})^{1/2}} e^{-j\omega s_{2z}\rho}\right], \tag{11}$$

$$\mathsf{H}_z \approx 0. \tag{12}$$

Strictly, the j-terms in (6), (7), and (9) must be read as $j\,\mathrm{sgn}(\omega)$.

The uniaxial problem in this paper is a generalization of the isotropic one, and hence its solutions reduce to those of the isotropic problem if the anisotropy is removed.

3. Concluding Remark

We have presented the expressions of the interfacial far-fields generated by a tiny interfacial current source on a uniaxially anisotropic half-space dielectric with its axis normal to interface, obtainable by the interfacial scheme of the Cagniard-de-Hoop method. The methodology dealing with our problem in this paper may be also applied to other types of interfacial problem, such as the acoustic or the vibrational problem.

References

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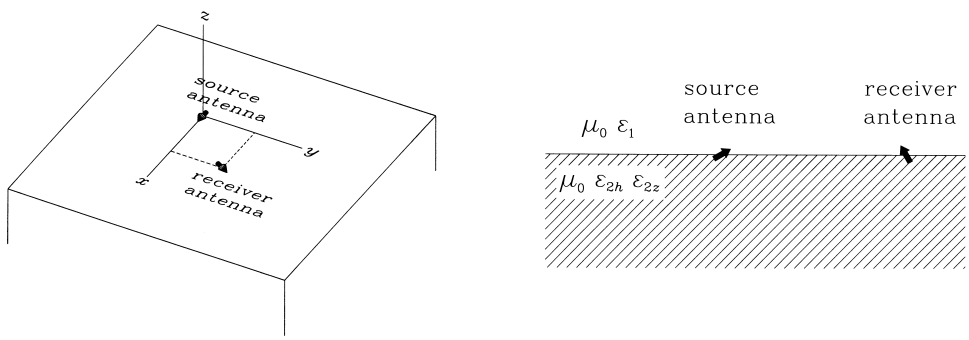

Figure 1: The half-space-interfacial problem.

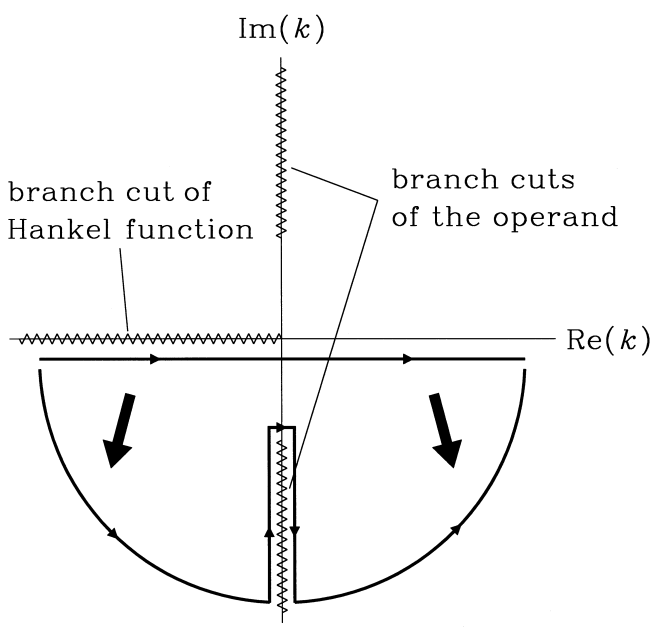

Figure 2: Deformation of the integration path of $\mathscr{H}_0^{-1}$.