COUPLED-MODE ANALYSIS FOR DISPERSION AND IMPEDANCE CHARACTERISTICS OF MICROSTRIP LINES ON FERRITE SUBSTRATES

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1. Introduction

A coupled-mode formulation for coupled microstrip lines on a magnetized ferrite substrate is presented. The formulation is an extention of the coupled-mode theory for microstrip lines on an isotropic substrate [1], [2]. Since the magnetized ferrite exhibits a biaxial anisotropy in the permeability, the guided-wave fields in such medium do not in general obey the conventional reciprocity relation for the fields in an isotropic medium. Thus, we derive first the generalized reciprocity relation for two sets of guided-wave fields propagating in the ferrite. The reciprocity relation is used to obtain the coupled-mode equations for the modal amplitudes in each isolated lines. The new formulation is applied to two coupled microstrip lines on a ferrite substrate.

2. Generalized reciprocity relation

Consider the guided-wave fields in a ferrite magnetized in the x direction. The permeability tensor of the ferrite is given as:

$$[\bar{\bar{\mu}}^{(+)}] = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu & -i\kappa \\ 0 & i\kappa & \mu \end{bmatrix}, \quad \text{with} \quad \mu = 1 - \frac{\omega_0 \omega_M}{\omega^2 - \omega_0^2} \quad \text{and} \quad \kappa = \frac{\omega \omega_M}{\omega^2 - \omega_0^2} \tag{1}$$

where $\omega_0 = -\gamma\mu_0 H_i$, $\omega_M = -\gamma\mu_0 M_i$, H_i is the internal dc magnetic field, M_i is the saturation magnetization and γ is the gyromagnetic ratio. We define the electric fields $\boldsymbol{E}^{(+)}$, the magnetic fields $\boldsymbol{H}^{(+)}$, and the current density $\boldsymbol{J}^{(+)}$ associated with the guided wave propagating in the $+z$ direction in the magnetized ferrite as follows:

$$\boldsymbol{E}^{(+)} = \boldsymbol{e}^{(+)}(x,y)e^{-i\beta z} = [\boldsymbol{e}_t(x,y) + \hat{\boldsymbol{z}} e_z(x,y)]e^{-i\beta z} \tag{2}$$

$$\boldsymbol{H}^{(+)} = \boldsymbol{h}^{(+)}(x,y)e^{-i\beta z} = [\boldsymbol{h}_t(x,y) + \hat{\boldsymbol{z}} h_z(x,y)]e^{-i\beta z} \tag{3}$$

$$\boldsymbol{J}^{(+)} = \boldsymbol{j}^{(+)}(x,y)e^{-i\beta z} = [\boldsymbol{j}_t(x,y) + \hat{\boldsymbol{z}} j_z(x,y)]e^{-i\beta z} \tag{4}$$

where $\boldsymbol{e}^{(+)}(x,y)$, $\boldsymbol{h}^{(+)}(x,y)$ and $\boldsymbol{j}^{(+)}(x,y)$ represent the eigenmode fields and current, and β denotes the mode propagation constant. Then, $\boldsymbol{E}^{(+)}$, $\boldsymbol{H}^{(+)}$ and $\boldsymbol{J}^{(+)}$ satisfy the following Maxwell's equations:

$$\nabla \times \boldsymbol{E}^{(+)} = -i\omega[\bar{\bar{\mu}}^{(+)}]\boldsymbol{H}^{(+)}, \quad \nabla \times \boldsymbol{H}^{(+)} = i\omega\varepsilon \boldsymbol{E}^{(+)} + \boldsymbol{J}^{(+)}. \tag{5}$$

Let $\boldsymbol{E}^{(-)}$, $\boldsymbol{H}^{(-)}$ and $\boldsymbol{J}^{(-)}$ be another set of modal fields and current which are defined using the respective components of $\boldsymbol{e}^{(+)}(x,y)$, $\boldsymbol{h}^{(+)}(x,y)$ and $\boldsymbol{j}^{(+)}(x,y)$ as follows:

$$\boldsymbol{E}^{(-)} = \boldsymbol{e}^{(-)}(x,y)e^{i\beta z} = [\boldsymbol{e}_t(x,y) - \hat{\boldsymbol{z}} e_z(x,y)]e^{i\beta z} \tag{6}$$

$$\boldsymbol{H}^{(-)} = \boldsymbol{h}^{(-)}(x,y)e^{i\beta z} = [-\boldsymbol{h}_t(x,y) + \hat{\boldsymbol{z}} h_z(x,y)]e^{i\beta z} \tag{7}$$

$$\boldsymbol{J}^{(-)} = \boldsymbol{j}^{(-)}(x,y)e^{i\beta z} = [\boldsymbol{j}_t(x,y) - \hat{\boldsymbol{z}} j_z(x,y)]e^{i\beta z}. \tag{8}$$

These new fields express the guided waves propagating in the $-z$ direction. If a medium supporting the guided waves is isotropic, ($\boldsymbol{E}^{(-)}$,$\boldsymbol{H}^{(-)}$,$\boldsymbol{J}^{(-)}$) also satisfy the same Maxwell's equations as those for ($\boldsymbol{E}^{(+)}$,$\boldsymbol{H}^{(+)}$,$\boldsymbol{J}^{(+)}$). However this is not the case. Substituting the transformations (6)–(8) of the field variables into Eq. (5), and comparing the equations with Eq. (5) term by term, it follows that ($\boldsymbol{E}^{(-)}$,$\boldsymbol{H}^{(-)}$,$\boldsymbol{J}^{(-)}$) satisfy the following Maxwell's equations:

$$\nabla \times \boldsymbol{E}^{(-)} = -i\omega[\bar{\bar{\mu}}^{(-)}]\boldsymbol{H}^{(-)}, \quad \nabla \times \boldsymbol{H}^{(-)} = i\omega\varepsilon\boldsymbol{E}^{(-)} + \boldsymbol{J}^{(-)} \tag{9}$$

where $[\bar{\bar{\mu}}^{(-)}]$ represents the permeability tensor of the ferrite magnetized in the $-x$ direction and is related to the original $[\bar{\bar{\mu}}^{(+)}]$ as follows:

$$[\bar{\bar{\mu}}^{(-)}] = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu & i\kappa \\ 0 & -i\kappa & \mu \end{bmatrix} = [\bar{\bar{\mu}}^{(+)}]^T. \tag{10}$$

Let $\boldsymbol{E}$ and $\boldsymbol{H}$ be the electric and magnetic fields produced by a current source $\boldsymbol{J}$ in a medium with $\varepsilon(y)$ and $[\bar{\bar{\mu}}(y)]$, and $\boldsymbol{E}'$ and $\boldsymbol{H}'$ be the electric and magnetic fields produced by a current source $\boldsymbol{J}'$ in another medium with $\varepsilon(y)$ and $[\bar{\bar{\mu}}'(y)]$. These two sets of fields satisfy the following Maxwell's equations, respectively:

$$\begin{cases} \nabla \times \boldsymbol{E} = -i\omega[\bar{\bar{\mu}}(y)] \cdot \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = i\omega\varepsilon(y)\boldsymbol{E} + \boldsymbol{J} \end{cases}, \quad \begin{cases} \nabla \times \boldsymbol{E}' = -i\omega[\bar{\bar{\mu}}'(y)] \cdot \boldsymbol{H}' \\ \nabla \times \boldsymbol{H}' = i\omega\varepsilon(y)\boldsymbol{E}' + \boldsymbol{J}'. \end{cases} \tag{11}$$

When $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ belong to the guided fields $(\boldsymbol{E}^{(+)}, \boldsymbol{H}^{(+)}, \boldsymbol{J}^{(+)})$ satisfying Eq. (5) and $(\boldsymbol{E}', \boldsymbol{H}', \boldsymbol{J}')$ belong to the guided fields $(\boldsymbol{E}^{(-)}, \boldsymbol{H}^{(-)}, \boldsymbol{J}^{(-)})$ satisfying Eq. (9), the following reciprocity relation is obtined:

$$\frac{\partial}{\partial z}\int_S (\boldsymbol{E} \times \boldsymbol{H}' - \boldsymbol{E}' \times \boldsymbol{H}) \cdot \hat{\boldsymbol{z}} dxdy = \int_S \boldsymbol{E}' \cdot \boldsymbol{J} dxdy - \int_S \boldsymbol{E} \cdot \boldsymbol{J}' dxdy \tag{12}$$

where S denotes the cross-sectional area in the transverse x-y plane.

3. Coupled-mode equations

Using the reciprocity relation (12), the coupled-mode equations for N coupled microstrip lines on the substrate of magnetized ferrite with $[\bar{\bar{\mu}}(y)]$=$[\bar{\bar{\mu}}^{(+)}]$ as shown in Fig. 1 can be formulated in the same way of the case for an isotropic substrate. For the first set of solutions $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ in Eq. (12), we adopt the eigenmode fields and current in the original coupled structure and approximate them as follows:

$$\boldsymbol{E} = \sum_{\nu=1}^{N} a_\nu(z)\boldsymbol{e}_\nu(x,y), \; \boldsymbol{H} = \sum_{\nu=1}^{N} a_\nu(z)\boldsymbol{h}_\nu(x,y), \; \boldsymbol{J} = \sum_{\nu=1}^{N} a_\nu(z)\boldsymbol{j}_\nu(x,y) \tag{13}$$

where $\boldsymbol{e}_\nu(x,y)$, $\boldsymbol{h}_\nu(x,y)$ and $\boldsymbol{j}_\nu(x,y)$ $(\nu = 1, 2, ...N)$ are eigenmode functions for the fields and current propagating in the $+z$ direction along each of the N microstrip lines in isolation, and $a_\nu(z)$ is an unknown amplitude function. As the second set of solutions $(\boldsymbol{E}', \boldsymbol{H}', \boldsymbol{J}')$ in Eq. (12), we employ each of the eigenmode fields and current propagating in the $-z$ direction along the respective N isolated microstrip lines on the ferrite substrate with $[\bar{\bar{\mu}}'(y)] = [\bar{\bar{\mu}}^{(-)}]$ as follows:

$$\begin{aligned} \boldsymbol{E}' &= \boldsymbol{e}_\nu^{(-)}(x,y)e^{i\beta_\nu^{(0)}z} = [\boldsymbol{e}_{\nu,t}(x,y) - \hat{\boldsymbol{z}}e_{\nu,z}(x,y)]e^{i\beta_\nu^{(0)}z} \\ \boldsymbol{H}' &= \boldsymbol{h}_\nu^{(-)}(x,y)e^{i\beta_\nu^{(0)}z} = [-\boldsymbol{h}_{\nu,t}(x,y) + \hat{\boldsymbol{z}}h_{\nu,z}(x,y)]e^{i\beta_\nu^{(0)}z} \\ \boldsymbol{J}' &= \boldsymbol{j}_\nu^{(-)}(x,y)e^{i\beta_\nu^{(0)}z} = [\boldsymbol{j}_{\nu,t}(x,y) - \hat{\boldsymbol{z}}j_{\nu,z}(x,y)]e^{i\beta_\nu^{(0)}z}, \; (\nu = 1, 2, ..., N) \end{aligned} \tag{14}$$

where $\beta_\nu^{(0)}$ is the propagation constant of the isolated ν-th microstrip line. Substituting Eqs. (13) and (14) for each of N isolated microstrip lines into the reciprocity relation Eq. (12), the coupled-mode equations are derived as follows:

$$\frac{d}{dz}\boldsymbol{a} = -i[\boldsymbol{C}]\boldsymbol{a}, \quad \text{with} \quad \boldsymbol{a} = [a_1 \; a_2 \; ...a_N]^T \quad \text{and} \quad [\boldsymbol{C}] = [\boldsymbol{M}]^{-1}[\boldsymbol{K}] \tag{15}$$

$$K_{\nu\mu} = \beta_\nu^{(0)} M_{\nu\mu} + Q_{\nu\mu}\,,\; M_{\nu\mu} = \frac{1}{2}(N_{\nu\mu} + N_{\mu\nu})\,,\; N_{\nu\mu} = \frac{1}{2}\int_S [\boldsymbol{e}_\nu(x,y) \times \boldsymbol{h}_\mu(x,y)] \cdot \hat{\boldsymbol{z}} dxdy \quad (16)$$

$$Q_{\nu\mu} = -\frac{i}{4}\int_{l_\mu} [e_{\nu,x}(x,h_1) j_{\mu,x}(x) - e_{\nu,z}(x,h_1) j_{\mu,z}(x)] dx\,, \quad (\nu,\mu = 1,2,...,N) \quad (17)$$

where l_μ denotes the cross-sectional contour of the μ-th line and the eigenmode fields and current in the isolated lines are normalized so that $N_{\nu\nu} = 1$. Note that $Q_{\nu\nu} = 0$ since $e_{\nu,x}(x,y) = e_{\nu,z}(x,y) = 0$ on the surface of the ν-th line. The solutions determine the forward and backward propagation constants $\beta_m^{(\pm)}$ of the coupled mode m and the modal amplitudes $a_{\nu m}^{(\pm)}$ of the current on the ν-th line.

3.1. Characteristic mode impedances

The characteristic mode impedances are calculated using the orthogonality of the eigenvoltages and eigencurrents in the coupled lines. The eigencurrents on the ν-th line for mode m is given as:

$$I_{\nu m}^{(\pm)} = a_{\nu m}^{(\pm)} e^{-i\beta_m^{(\pm)} z} \int_{x_\nu - w_\nu}^{x_\nu + w_\nu} j_{\nu,z}(x) dx, \quad (\nu, m = 1,2,...N). \quad (18)$$

The characteristic mode impedance $Z_{c,\nu m}^{(\pm)}$ of the ν-th line for mode m is defined as:

$$V_{\nu m}^{(\pm)} = Z_{c,\nu m}^{(\pm)} I_{\nu m}^{(\pm)} \quad (19)$$

where $V_{\nu m}^{(\pm)}$ represents the eigenvoltage. $I_{\nu m}^{(\pm)}$ and $V_{\nu m}^{(\pm)}$ are related to the total power $P_m^{(\pm)}$ carried by mode m in the z direction as follows:

$$\frac{1}{2}\sum_{\nu=1}^{N} V_{\nu m}^{(\pm)} I_{\nu m'}^{(\pm)*} = P_m^{(\pm)} \delta_{mm'} \quad (20)$$

$$P_m^{(\pm)} = \frac{1}{2}\int_S (\boldsymbol{E}_m^{(\pm)} \times \boldsymbol{H}_m^{(\pm)*}) \cdot \hat{\boldsymbol{z}} dxdy = \sum_{\nu=1}^{N} \left(\sum_{\mu=1}^{N} a_{\nu m}^{(\pm)} a_{\mu m}^{(\pm)} N_{\nu\mu}^{(\pm)} \right) \quad (21)$$

where $\delta_{mm'}$ is Kronecker's delta, $\boldsymbol{E}_m^{(\pm)}$ and $\boldsymbol{H}_m^{(\pm)}$ are the total electric and magnetic fields for mode m. Substituting Eqs. (19) and (21) into Eq. (20), the expression of $Z_{c,\nu m}^{(\pm)}$ in terms of the solutions to the coupled-mode equation (15) is derived.

3.2. Impedance Matrices

Consider the impedance matrix for the coupled-line 2N-port as shown in Fig. 2. The current $I_\nu(z)$ on the ν-th line is expressed in terms of solutions of the coupled-mode equation (15) as follows:

$$I_\nu(z) = \sum_{m=1}^{N} \left[A_{2m-1} R_{\nu m}^{i(+)} e^{-i\beta_m^{(+)} z} - A_{2m} R_{\nu m}^{i(-)} e^{i\beta_m^{(-)} z} \right] \quad (22)$$

where $R_{\nu m}^{i(\pm)} = I_{\nu m}^{(\pm)} / I_{1m}^{(\pm)}$ is the current ratio on the ν-th line for mode m and A_i $(i = 1,2,...,2N)$ is an arbitrary constant. The corresponding line voltage $V_\nu(z)$ is determined by using Eq. (19) as follows:

$$V_\nu(z) = \sum_{m=1}^{N} \left[A_{2m-1} Z_{c,\nu m}^{(+)} R_{\nu m}^{i(+)} e^{-i\beta_m^{(+)} z} + A_{2m} Z_{c,\nu m}^{(-)} R_{\nu m}^{i(-)} e^{i\beta_m^{(-)} z} \right]. \quad (23)$$

For the N coupled lines of length l, the current-voltage relationships of the 2N-port shown in Fig. 2 derived from Eqs. (22) and (23) are solved to obtain the impedance matrix $[\boldsymbol{Z}]$ defined as follows:

$$\boldsymbol{V} = [\boldsymbol{Z}]\boldsymbol{I}, \quad \text{with} \quad \boldsymbol{V} = [V_1 V_2 \ldots V_{2N}]^T \quad \text{and} \quad \boldsymbol{I} = [I_1 I_2 \ldots I_{2N}]^T \tag{24}$$

where I_n and V_n ($n = 1, 2, ..., 2N$) represent the port currents and voltages, respectively, as shown in Fig. 2.

4. Numerical results

As a model of the numerical computation, we considered two coupled microstrip lines on a ferrite substrate where h_1=1.0mm, w=0.5mm, d=0.5mm, ε_r=12.7, H_i=102555A/m and M_i=170925A/m. Figure 3 shows the frequency dependence of the normalized propagation constants β/k_0 of the two coupled-modes for the forward wave and the backward wave. Figure 4 shows the characteristic mode impedances as functions of frequency. Figure 5 shows the elements Z_{12}, Z_{13}, Z_{22} and Z_{23} of the impedance matrix for the two lines where l=20mm.

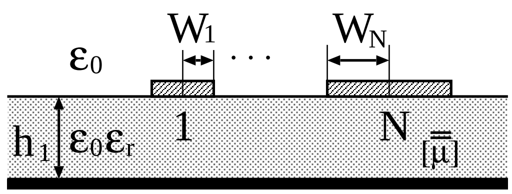

Fig.1 Cross section of N coupled microstrip lines on a magnetized ferrite substrate.

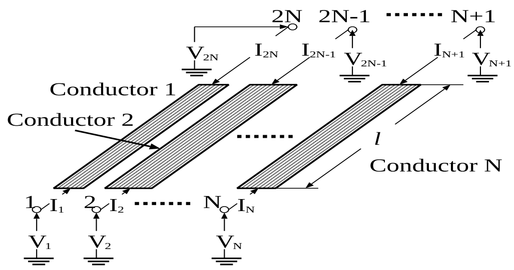

Fig.2 A 2N-port circuit of the N microstrip lines.

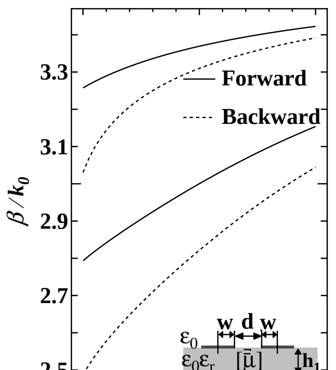

Fig.3 Normalized propagation constants of two identical microstrip lines.

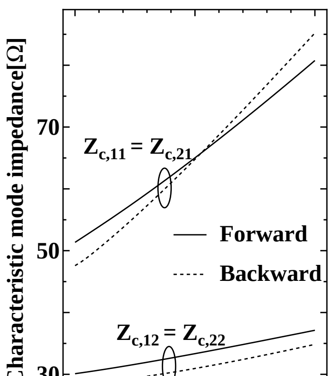

Fig.4 Characteristic mode impedances of two identical microstrip lines.

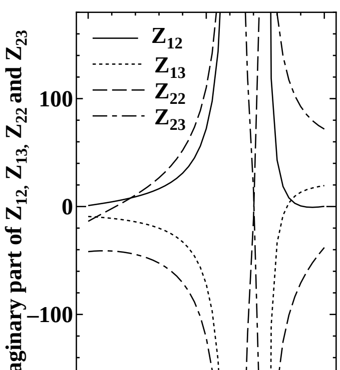

Fig.5 Elements of the impedance matrix for two identical microstrip lines.

References

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