DIRECTION FINDING METHOD USING WAVE DISTRIBUTION FUNCTION WITH THE GAUSSIAN DISTRIBUTION MODEL

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1 Introduction

Investigation of space environment around the earth has become a critical issue in designing commercial satellites. As the characteristics of plasma waves propagating in the magnetosphere reflect the plasma environment around the earth, it is important to measure the propagation (wave normal) direction of these waves. In the present study, we propose a new direction finding method applicable to plasma waves in the very low frequency (VLF) range observed by a scientific satellite based on the wave distribution function (WDF). The WDF is most suitable for the description of the plasma waves arriving from multiple directions, but its determination is an ill-posed problem. Therefore some adequate model must be assumed to solve the problem. In the present study, we introduce a model fitting method using the Gaussian distribution model.

2 Wave distribution function method

The WDF is derived from the concept that observed signals consist of a number of elementary plane waves and can be defined as a distribution of the wave energy density relative to the angular frequency ω and to the arrival directions (θ, ϕ), where θ and ϕ are the polar and azimuthal angle in the geomagnetic coordinate system, respectively. Calculating a spectral matrix composed from the auto-spectra and cross-spectra of the electric and magnetic wave field components, the WDF is related to the spectral matrix by the following equation,

$$S_{ij} = \frac{\pi}{2}\int_0^{2\pi}\int_0^{\pi} a_{ij}(\omega,\theta,\phi)F(\omega,\theta,\phi)\sin\theta \mathrm{d}\theta\mathrm{d}\phi, \quad (i,j=1,2,3,4,5,6), \tag{1}$$

where S_{ij} are the elements of spectral matrix, $a_{ij}(\omega,\theta,\phi)$ are the integration kernels [*Storey and Lefeuvre*, 1979; 1980], and $F(\omega,\theta,\phi)$ is the distribution function for a wave of frequency ω and the wave normal direction (θ,ϕ). Integration kernels a_{ij} are determined by the propagation mode, plasma frequencies, and cyclotron frequencies of electron and ions. Thus a_{ij} are derived from the refractive index of the wave in which the effects of ions are involved.

Knowing the integration kernels a_{ij} and the calculated S_{ij}, we can estimate the function F by solving the set of integral equations (1), but solving these equations is an ill-posed problem and the solution is not determined uniquely. In order to solve this kind of equations, we must assume an adequate model as a solution. In the present study we apply the Gaussian distribution model.

3 Algorithm using the Gaussian distribution model

3.1 Gaussian distribution model

The Gaussian distribution model is derived from the concept that observed signal consists of m pieces of waves whose distribution function F is represented by a combination of the Gaussian distributions as follows [*Kasahara and Kimura*, 1998];

$$F(\omega, \theta, \phi) = \sum_{l=1}^{m} \alpha_l \exp\left\{ - \left(\frac{d_{l0}(\theta, \phi)}{d_l} \right)^2 \right\}, \tag{2}$$

where α_l is the intensity at the direction (θ_l, ϕ_l) which is the center of distribution, d_l represents the angular extent of the distribution, and $d_{l0}(\theta, \phi)$ is the angle between (θ, ϕ) and (θ_l, ϕ_l). Unknown parameters $(\theta_l, \phi_l, \alpha_l, d_l)$ are determined by the non-linear least-squares fitting method. This method has an advantage of expressing the model in terms of clear physical parameters, but it requires an accurate set of initial parameters in order to find a global minimum. Exhaustive grid search for the initial parameters is a time-consuming procedure, and becomes prohibitive for $l > 2$.

In this study, we developed techniques for determination of initial values and for the integration of the WDF, which drastically improved the precision of the parameter fitting and the amount of calculation time.

3.2 Configuration of the grid points

We divide $(\theta,\ \phi)$ space uniformly on the spherical surface in the adequate interval as shown in Fig. 1. Using this configuration of the grid points, residual for the best initial values is generally improved compared with the configuration that the variables θ and ϕ are separately divided uniformly. As for the parameter d_l, grid points are selected to be exponentially uniform.

3.3 Numerical integration of the WDF

On the assumption that the wave energy is expressed by the Gaussian distribution, precision of the integration is sufficiently assured even though the integral range for the numerical integration of (1) is limited in an adequate range around the arrival direction of the wave. We adopted an adaptive method which chooses the integral range in proportion to the extent of the wave distribution (d_l) around the center of the arrival direction (θ_l, ϕ_l) and the integral interval to cover each Gaussian with the same number of samples shown as Fig. 2.

3.4 Pre-processing stage using energy function

In order to get an adequate solution by the Gaussian distribution model, the number of arrival waves m must be assumed. A pre-processing stage is introduced to determine the number m. We

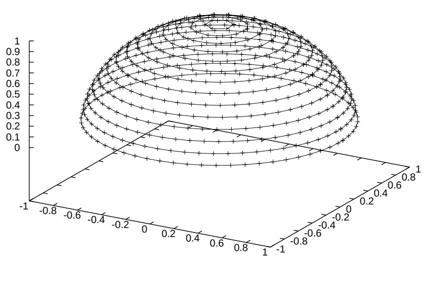

Fig. 1. Configuration of the grid points in (θ, ϕ) space.

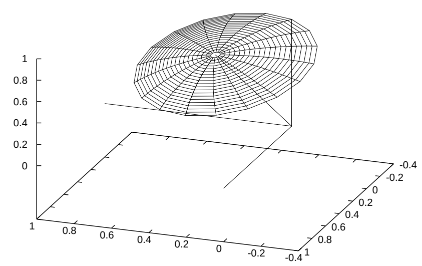

Fig. 2. Grids used for the numerical integration.

assume that the observed spectral matrix $\boldsymbol{S}_{\mathrm{obs}}$ is composed by a combination of the estimated spectral matrices $\boldsymbol{S}_{j\mathrm{est}}$,

$$\boldsymbol{S}_{\mathrm{obs}} = x_1 \boldsymbol{S}_{1\mathrm{est}} + x_2 \boldsymbol{S}_{2\mathrm{est}} + \cdots + x_n \boldsymbol{S}_{n\mathrm{est}}, \tag{3}$$

where n is the total number of grids used for the initial value determination. In the condition that no variables $(x_1 \cdots x_n)$ are negative, it is confirmed by the simplex method that there is no solution which satisfies (3). We define the following energy function E as a dispersion of the ratios of corresponding elements of both sides in (3),

$$E = \sigma^2 \left(\frac{\sum_{l=1}^{n} x_l \boldsymbol{S}_{l\mathrm{est}}}{\boldsymbol{S}_{\mathrm{obs}}} \right). \tag{4}$$

Solving the dynamics system represented by the following differential equations,

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = \eta \; x_i(1 - x_i)\frac{\partial E}{\partial x_i}, \quad \text{where} \quad \eta < 0, \tag{5}$$

the minimal value of E is obtained, when the initial values of x_i are randomly given in [0:1]. Applying the steepest descent method combined with the random search method to (5), an optimum solution of $(x_1 \cdots x_n)$, where E becomes almost minimum, is obtained. This solution is referred to as the approximate distribution of the WDF for the determination of the initial values of the parameter fitting. The merit of this method is that the calculation time is almost constant regardless of the number of arrival waves. This pre-process is also useful for examining the validity of the final solution reconstructed by the Gaussian distribution model.

4 Simulation

The performance of the WDF method with the Gaussian distribution model is evaluated using the computer-generated spectral matrices S_{ij} calculated by (1) from given wave distribution functions $F(\omega, \theta, \phi)$. In the simulation, the source wave is assumed to be whistler mode wave at 10kHz, and the plasma frequency and the cyclotron frequency of electron are 60kHz and 400kHz, respectively. Several cases are examined by varying the distribution of arrival waves.

In case (a), F is assumed to be a single point source located at (θ, ϕ)=(30°, 60°) (Fig. 3(a)). In the figure, the polar angle θ is shown by the radius of the circle, and the azimuthal angle ϕ is indicated peripherally counterclockwise from the reference radial axis of the polar angle. In case (b), F is composed of two Gaussian distribution with the parameters $(\theta_1, \phi_1, \alpha_1, d_1)$ = (30°, 60°, 3, 10°) and $(\theta_2, \phi_2, \alpha_2, d_2)$ = (60°, 200°, 1, 30°) (Fig. 3(b)). In case (c), F is assumed to be distributed along the resonance angle of whistler mode wave shown as Fig. 3(c).

Fig. 4(a) and Fig. 4(b) show the final reconstructed wave distributions of cases (a) and (b) using the parameter fitting by the Gaussian distribution model, respectively. It is found that the wave distributions are successfully reconstructed both in case (a) and (b), and the fitting errors are small enough.

Fig. 4(c) shows the approximate wave distribution of case (c) at the pre-processing stage. In this case, the wave distribution is practically reconstructed by the combination of several Gaussian distributions at the pre-processing stage, although the given distribution is non-Gaussian.

5 Results

The direction finding method using the wave distribution function with the Gaussian distribution model is proposed. It is found that the wave distribution is well reconstructed by a combination of the Gaussian distributions. It is remarkable that the proposed method is applicable to all considered cases where the arrival wave is a point source or a combination of extended sources. The amount of the calculation time is small enough for the practical use.

This method is applied to the data observed by the Akebono satellite, such as Omega signals, whistlers, chorus emissions. The derived wave normal directions are in the acceptable range from theoretical viewpoints. It is also found that the proposed method is also utilized for the calibration of the electric and magnetic sensors onboard the satellite.

References

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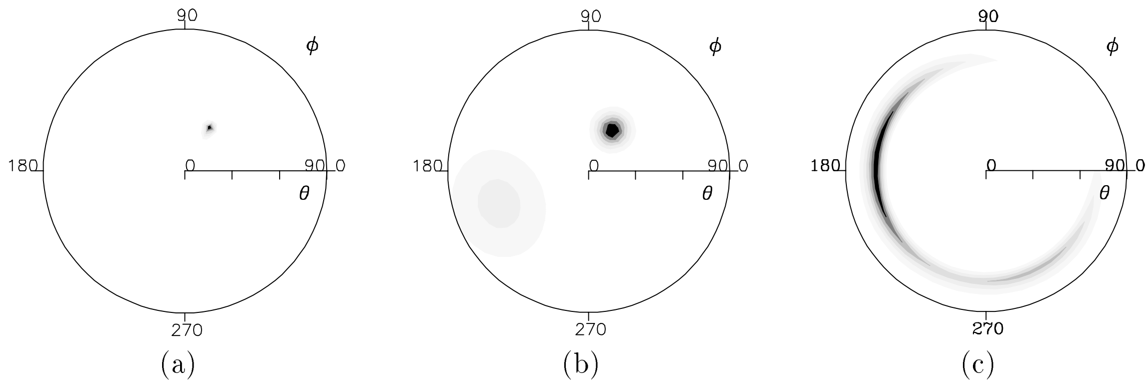

Fig. 3. Simulated wave distributions; (a) a point source, (b) two Gaussian distributed sources, and (c) the distribution along the resonance angle of whistler wave.

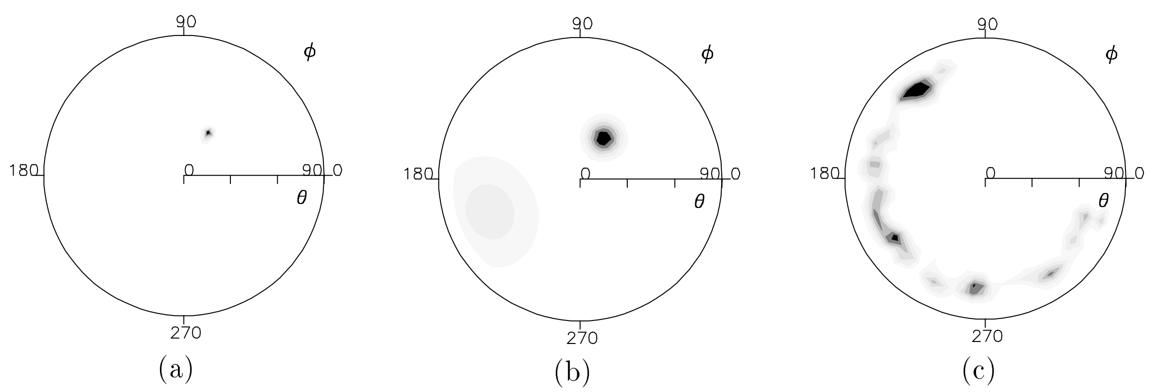

Fig. 4. Reconstructed wave distribution; (a) and (b) the final solutions determined by the proposed method, and (c) an approximate solution obtained by the pre-processing stage.