A Hybrid Solution for Radiation from Wire Antennas near a Curved Smooth Surface

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Abstract

A current-based hybrid formulation is developed for radiation from wire antennas near a curved smooth surface. The electric field integral equation (EFIE) is combined with the Fock currents on smooth surface in the method of moments (MoM) solution. The Fock solution is valid in both illuminated and shadowed regions, and is continuous across the shadow boundary. This provides a better approximation than the physical optics (PO) currents on smooth surface. Since the Fock solution is a high frequency asymptotic result, the hybrid method works well on large surfaces.

1. Introduction

The radiation from sources on or near perfectly conducting convex surfaces is critical to the design of antennas mounted on aircraft, and the antenna-antenna coupling issues in the context of electromagnetic compatibility (EMC). Although the MoM technique is suitable for this highly complex problem, it is inefficient for electrically large radiators due to high computational complexity and memory requirement. In the past decade, great progress has been made to reduce the complexity for the integral equation (IE) method. Some fast integral equation solvers such as the fast multipole method (FMM) and adaptive integral method (AIM) had successfully solved several problems that can only be handled by asymptotic methods previously. However, there remain many practical problems that lie beyond these fast integral equation solvers, even with the advanced computational power on modern computers. Furthermore, although specific problems are solvable with fast integral equation solver, it is preferred to use a more efficient method that is capable of achieving results with good engineering accuracy, in view of the pressing short lead time that is encountered in practical environment. Hence, hybrid techniques combining the IE method with asymptotic methods, which are attractive for the reduced memory requirement and CPU time, are important analysis and design tools in real engineering.

In this paper, the Fock theory [1] is combined with the IE method to obtain a hybrid solution for radiation from wire antennas near a large curved smooth surface. Compared to the PO method, the Fock theory provides a better approximation for induced currents on smooth surface. Since the Fock theory is an asymptotic method, the hybrid technique is suitable for large surfaces. The use of IE method facilitates modelling of detail features in general. The combination of these two methods in the hybrid solution provides an accurate, robust and computationally efficient tool. The results for the directive pattern of a dipole array near a sphere will be compared to that obtained via the MoM technique. Good agreement is demonstrated.

2. Hybrid Technique

We begin the analysis by sub-dividing the object under investigation into electrically large smooth convex surfaces and electrically small irregular surfaces. Irregular surfaces refer to geometries not amenable to asymptotic analyses, and will be treated using MoM for the surface currents. The surface currents for regular geometries can be obtained from Fock theory.

A. EFIE

Applying the boundary condition on the conducting surface, an EFIE can be formulated as

$$\mathrm{E}^{\mathrm{i}}(\mathrm{r})\big|_{\tan} = \left[\mathrm{L}_{\mathrm{lit}}\mathrm{J}^{\mathrm{F}}(\mathrm{r}') + \mathrm{L}_{\mathrm{sh}}\mathrm{J}^{\mathrm{F}}(\mathrm{r}') + \mathrm{L}_{\mathrm{ir}}\mathrm{J}^{\mathrm{M}}(\mathrm{r}')\right]_{\tan} \tag{1}$$

where the subscript "tan" refers to the tangential components; $\mathbf{E}^{\mathrm{i}}$ is the electric field due to the excitation; L is the integro-differential operator, subscripts "lit" and "sh" refer to the lit and shadow regions of smooth surfaces, while "ir" refers to irregular surfaces; **J** is the equivalent electric surface current, superscripts "F" and "M" are for Fock theory and MoM approach from which the surface currents are obtained. **r** and $\mathbf{r}'$ are position vectors of the field point and a point on surfaces, respectively. The surface integral operator $\mathrm{L}_{\mathrm{lit}}$, L_{sh} and L_{ir} are defined as

$$\mathrm{L}_{\mathrm{lit,sh,ir}}\mathrm{J}(\mathrm{r}) = \mathrm{j}\omega\mu \int_{\mathrm{S}_{\mathrm{lit,sh,ir}}} [\mathrm{J}(\mathrm{r}') + \frac{1}{\mathrm{k}^2}\nabla\nabla'\cdot\mathrm{J}(\mathrm{r}')]\mathrm{G}(\mathrm{r},\mathrm{r}')\mathrm{dS}' \tag{2}$$

where k is the wave-number of the incident wave; $\mathrm{G}(\mathbf{r},\mathbf{r}')$ is the three-dimensional Green's function in free space.

B. Fock-Ansatz Current

The PO method provides the simplest approximation for induced currents on smooth convex surface due to illumination by a plane wave or radiation from sources. A better approximation for the currents on smooth surface can be obtained via Fock theory [1]. Fock theory is based on the basic assumption that induced currents or fields vary slowly in amplitude on the surface of a conducting body with large radius of curvature. Fock showed that the Helmholtz equation can be approximated by a reduced-wave equation for a smooth surface when the above conditions are satisfied. The solution of the reduced-wave equation was expressed in universal functions (often termed Fock functions) of the surface curvature and conductivity. An important feature of the Fock theory is that it provides the Ansatz solution for currents near shadow boundary and in the shadow region. The Ansatz current from Fock theory, with a distinction between illuminated and shadowed regions, can be expressed as [1-3]

$$\mathbf{J}^{F}(Q) = e^{-j\xi^{3}/3}\{[\mathbf{H}^{i}(Q)\cdot\hat{\mathbf{b}}]\hat{\mathbf{t}}g(\xi) + [\mathbf{H}^{i}(Q)\cdot\hat{\mathbf{n}}']\hat{\mathbf{b}}f(\xi)\}, \quad (3a)$$

when the observation point Q is in the illuminated region and

$$\begin{aligned}\mathbf{J}^{F}(Q) = {}& [\mathbf{H}^{i}(Q')\cdot\hat{\mathbf{b}}_{Q'}]\hat{\mathbf{t}}_{Q}[F_1 + \frac{j\Lambda}{k\rho_g(Q)}D^2F_2] \\ & + [\mathbf{H}^{i}(Q')\cdot\hat{\mathbf{n}}_{Q'}]\hat{\mathbf{b}}_{Q}[F_2 - \frac{j\Lambda}{k\rho_g(Q)}D^2F_1], \quad (3b) \\ & + [\mathbf{H}^{i}(Q')\cdot\hat{\mathbf{n}}_{Q'}]\hat{\mathbf{t}}_{Q}T(Q)\rho_g(Q)F_2\end{aligned}$$

for Q in the shadow region. $\mathbf{H}^{i}$ is the incident magnetic field; $\hat{\mathbf{b}}$ is a unit vector and can be determined by the cross product of the incident wave direction $\hat{\mathbf{k}}^{i}$ and the surface normal vector $\hat{\mathbf{n}}$ at Q; $\hat{\mathbf{t}}$ and $\hat{\mathbf{n}}'$ are also unit vectors that can be obtained from the cross product of $\hat{\mathbf{n}}$ and $\hat{\mathbf{b}}$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{k}}^{i}$, respectively; *f(x)* and *g(x)* are the radiation Fock functions given by

$$f(x) = \frac{1}{\sqrt{\pi}}\int_{\infty e^{-j2\pi/3}}^{\infty}\frac{e^{-jx\tau}}{w_2(\tau)}d\tau \quad ; \quad (4a)$$

$$g(x) = \frac{1}{\sqrt{\pi}}\int_{\infty e^{-j2\pi/3}}^{\infty}\frac{e^{-jx\tau}}{w_2'(\tau)}d\tau \quad . \quad (4b)$$

In Eq. (3b), Q' is the intersection of the shadow boundary and a geodesic path leading to point Q where $\mathbf{J}^{F}$ is evaluated; $\hat{\mathbf{t}}_{Q,Q'}$ and $\hat{\mathbf{b}}_{Q,Q'}$ are unit vectors along and bi-normal to the geodesic path for each of the points Q and Q'; the functions F_1 and F_2 are defined as

$$\left.\begin{matrix}F_1\\F_2\end{matrix}\right\} = e^{-jkt}[\frac{\rho_g(Q')}{\rho_g(Q)}]^{1/6}D\left\{\begin{matrix}g(\xi)\\ jm^{-1}(Q)f(\xi)\end{matrix}\right. . \quad (5)$$

Other parameters introduced above can be found in [3, 4].

C. Hybrid IE-Fock Solution

By substituting Eq. (3a)-(3b) into Eq. (1), the only unknown in Eq. (1) is $\mathbf{J}^{M}$. This can be obtained by solving the IE with the MoM technique. Since Fock theory is applied to large smooth surface, the number of unknowns required, except for that defined on the surface of antenna, is significantly reduced. This leads to substantial reduction in computational resources.

3. NUMERICAL RESULTS

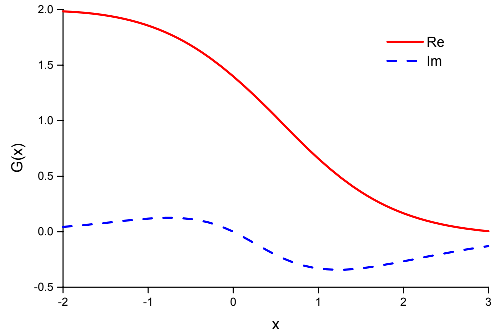
In the Fock currents given by Eq. (3a)-(3b), the two radiation Fock functions must be computed. When the absolute value of the argument is large, asymptotic expansions for these functions can be used. For moderate values of the argument, it is difficult to find an analytical expression. A possible way is to interpolate the tabulated values of these functions for this range. In our calculation, a scheme proposed by Pearson [5] is adopted to evaluate these functions. The integrals are evaluated by a simple extension of a Fourier quadrature method related to the discrete Fourier transform. The scheme renders the evaluation of Fock-type integrals practical for computation in an as-need fashion, rather than being called out of a table. Furthermore, the scheme allows one to compute Fock-type integrals for arbitrary lossy impedance boundary conditions. Graphs of the universal Fock radiation functions are shown in Fig. 1. The universal Fock radiation functions G(x) and F(x) related to g(x) and f(x), respectively, by

$$G(x) = \begin{cases} e^{-jx^3/3}g(x), & x \le 0 \\ g(x), & x > 0 \end{cases} ; \quad (6a)$$

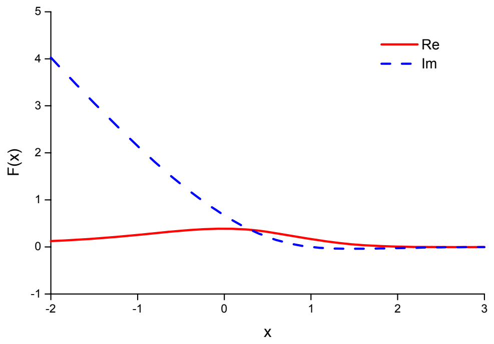
$$F(x) = \begin{cases} e^{-jx^3/3}f(x), & x \le 0 \\ f(x), & x > 0 \end{cases} . \quad (6b)$$

The results are in an excellent agreement with those presented in [6, Appendix B].

For verification of the hybrid solution, we consider the radiation from a 3-element λ/2-dipole collinear array located at 1λ from the surface of a perfectly conducting sphere of radius 1λ. The geometry is shown in Fig. 2. The φ angle is in the XOY-plane, with φ=0° referencing the X-axis. All dipoles are center-fed and oriented along the Z-axis. The dipoles are spaced at 0.6λ along the Z-direction. Fig. 3 and Fig. 4 show the co-polarised directive gain patterns in E-plane (φ=0°) and H-plane (θ=90°), respectively. Fig. 5 and Fig. 6 show the directive gain patterns for the plane φ=45° for co-polarization and cross-polarization, respectively. The results obtained via MoM technique and another hybrid technique combining integral equation method and PO method (called IE-PO method in this paper) are also plotted for comparisons. From the results, very good agreement is obtained between the MoM results and that of the hybrid technique based on the IE method and Fock theory. Hence, the hybrid IE-Fock method provides more accurate solutions as compared to the hybrid IE-PO method. This is most significant in thee cross-polarization case as shown in Fig. 6.



(a) G(x)



(b) F(x)
Fig. 1: Hard and soft universal radiation Fock Functions.

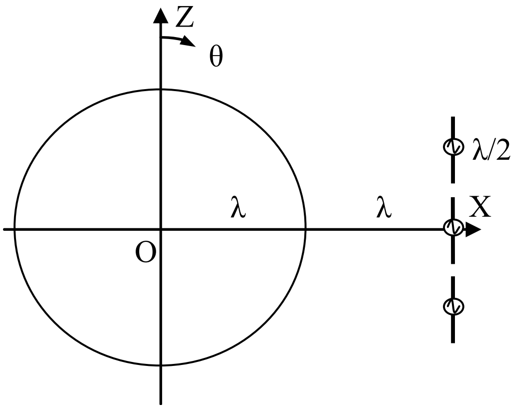


Fig. 2: A 3-element λ/2-dipole array at 1λ from the surface of a perfectly conducting sphere of radius 1λ. Side view.

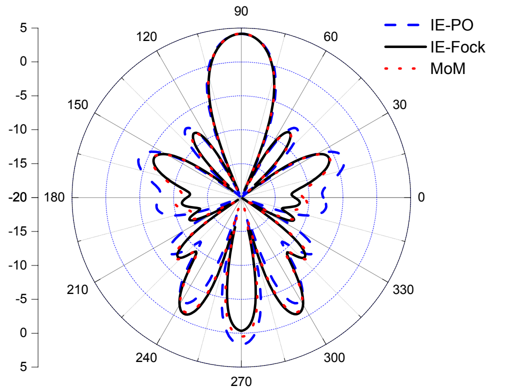


Fig. 3: Co-polarised E-plane (φ=0°) directive gain pattern of a 3-element λ/2-dipole collinear array at 1λ from the surface of a perfectly conducting sphere of radius 1λ.

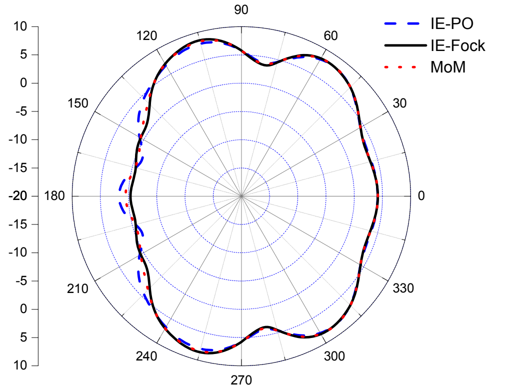


Fig. 4: Co-polarised H-plane (θ=90°) directive gain pattern of a 3-element λ/2-dipole collinear array at 1λ from the surface of a perfectly conducting sphere of radius 1λ.

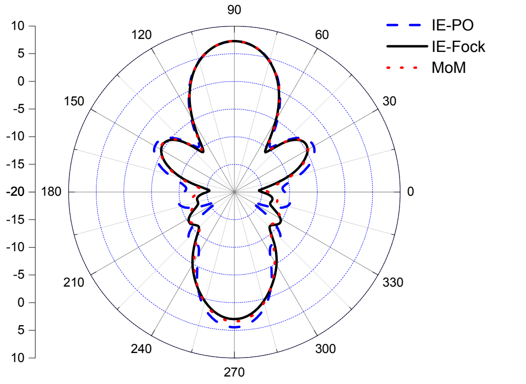


Fig. 5: Co-polarised directive gain pattern for the plane φ=45°, of a 3-element λ/2-dipole collinear array at 1λ from the surface of a perfectly conducting sphere of radius 1λ.

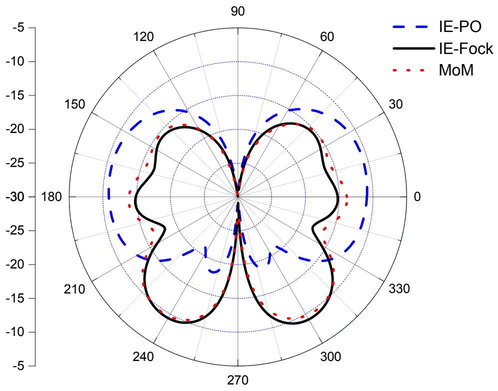


Fig. 6: Cross-polarised directive gain pattern for the plane φ=45°, of a 3-element λ/2-dipole collinear array at 1λ from the surface of a perfectly conducting sphere of radius 1λ.

4. Conclusion

A hybrid formulation of integral equation technique and Fock theory has been developed for radiation from sources near a curved smooth surface. The hybrid approach yields accurate results for induced currents on spherical surface with radius of only 1λ. As the size of the surface increases, the Fock solution becomes more accurate. This should further improve the accuracy of the hybrid technique. Future works will investigate the implementation of the hybrid IE-Fock technique on faceted objects.

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