

# THREE-DIMENSIONAL INVERSE SCATTERING FROM CONDUCTING PLATES MODELED BY RATIONAL B-SPLINE SURFACES USING THE GENETIC ALGORITHM

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## I. Introduction

Inverse scattering in electromagnetic waves has found numerous applications in various industrial and medical aspects. In this paper, we discuss a new method for the shape reconstruction of perfectly conducting three-dimensional plates from simulated radar cross-section (RCS) data using Non-Uniform Rational B-spline (NURBS) modeling and optimization algorithms. The two-dimensional case was investigated before in [1]. A three-dimensional body can be modeled by some NURBS surfaces efficiently which are able to manipulate both free-form and primitive quadratic surfaces with a low number of patches and therefore with a small amount of information([2]). The proposed method for solving the direct problem is mostly based on [3]. The inverse problem is solved by optimizing the geometrical parameters of NURBS surfaces corresponding to the optimal shape matching the RCS data. A modified genetic algorithm is applied for this purpose.

## II. The Forward Problem

The forward problem is formulated by computation of monostatic radar cross-section (RCS) of the target using the physical optics approximation and then calculating the backscattered fields in the far-field region. The RCS is a function of frequency, polarization and angle of incidence wave as well as the shape composition and texture of the target. The RCS for a three-dimensional target under a plane wave illumination in the far field region is given by:

$$\sigma = \lim_{r \rightarrow \infty} 4\pi r^2 \frac{|\bar{\mathbf{E}}^s|^2}{|\bar{\mathbf{E}}^i|^2} \quad (1)$$

where  $\bar{\mathbf{E}}^i$  and  $\bar{\mathbf{E}}^s$  are the incident and scattered field respectively. The proposed scattering problem considers just simple large bodies in comparison with operating wavelength.

Therefore the scattering regime is high-frequency and the physical optics approximation is quite applicable. Under these assumptions, given an incident monochromatic plane wave using far-field approximation, the backscattered field of an arbitrary conducting body is given by:

$$\bar{\mathbf{E}}^s = j\lambda^{-1} \frac{e^{-jk_0r}}{r} (\hat{\mathbf{k}} \cdot \bar{\mathbf{I}}) \bar{\mathbf{E}}_0, \quad \bar{\mathbf{I}} = \int_{S'} \hat{\mathbf{n}}(\bar{\mathbf{r}}') e^{-j^2 \bar{\mathbf{K}} \cdot \bar{\mathbf{r}}'} ds' \quad (2)$$

$\bar{\mathbf{I}}$  is called the physical optics(PO) integral. In (2)  $S'$  is the illuminated surface of the body,  $\lambda$  is the operating wavelength,  $\bar{\mathbf{E}}_0$  is the polarization vector of electric incident field,  $\hat{\mathbf{k}}$  is the normalized wave vector,  $\bar{\mathbf{r}}'$  is the surface point vector corresponding to  $ds'$ , while  $\hat{\mathbf{n}}(\bar{\mathbf{r}}')$  is the orthonormal vector to the surface at this point. Consequently, according to the definition of RCS, we can easily write:

$$\sigma = \frac{4\pi}{\lambda^2} |\hat{\mathbf{k}} \cdot \bar{\mathbf{I}}|^2 \quad (3)$$

The next step is target modeling with the NURBS patches. NURBS modeling provides important advantages in the description of bodies for RCS computation. It is a rational piecewise polynomial parametric surface, defined by an ordered set of control points and corresponding weights ([2]). It can also be written as a combination of rational Bezier patches that is more suitable for numerical computation of parameters associated with geometry of the patch. A Bezier patch is also a polynomial parametric surface normalized with a weight function which is defined by two degrees, a mesh of control points, and a set of associated weights ([2]). The surface point of a rational Bezier patch is given by:

$$\bar{\mathbf{r}}(u, v) = \frac{\sum_{i=0}^m \sum_{j=0}^n \omega_{ij} \bar{\mathbf{b}}_{ij} B_i^m(u) B_j^n(v)}{\sum_{i=0}^m \sum_{j=0}^n \omega_{ij} B_i^m(u) B_j^n(v)} \quad (4)$$

where  $\bar{\mathbf{b}}_{ij}$  are the control points,  $\omega_{ij}$  are the control points weights,  $m, n$  are the surface degrees and  $B_i^m(u), B_j^n(v)$  are the Bernstein polynomials, given by:

$$B_k^s(t) = \binom{s}{k} t^k (1-t)^{s-k}, \quad 0 \leq t \leq 1 \quad (5)$$

In three-dimensional space, a circular cylinder by four Bezier patches and 16 control points and a sphere by eight Bezier patches and 26 control points can be well approximated.

The main difficulty for predicting the RCS of complex bodies is the PO integral computation over an arbitrary shape. According to NURBS modeling, the PO integral can be expressed on the parametric coordinates of the Bezier patch writing the orthonormal vector function and the surface differential element in terms of partial derivatives of the surface point vector. Therefore the physical optics integral can be expressed by the following expression:

$$\bar{\mathbf{I}} = \int_0^1 \int_0^1 \bar{\mathbf{r}}_u(u, v) \times \bar{\mathbf{r}}_v(u, v) e^{-2j \bar{\mathbf{K}} \cdot \bar{\mathbf{r}}(u, v)} dudv \quad (6)$$

The method for computing the above PO integral on a Bezier patch depends on the topology of the surface. The patches are classified into three main types: 1) plane patches 2) singly curved patches 3) doubly curved patches. If the Bezier patch is a plane surface, the normal vector on the surface is constant and the PO integral can be calculated by expanding the Gordon's idea ([4]) for three-dimensional space. However, if a patch is from the second or third types, it is not possible to calculate the PO integral analytically and therefore the stationary phase method (SPM) and asymptotic integration are utilized ([3]). In fact, according to the physical assumptions in PO approximation, the SPM and its mathematical considerations are compatible with PO features. Also it is mentionable that the complex systems of equations which occur in stationary phase method for finding the critical points and segments in asymptotic calculations, are solved by a non-linear least squares optimization algorithm.

### III. The Inverse Scattering Problem

For electromagnetic inversion, the inverse problem is cast into optimizing a set of parameters that affect the forward solution. Each iteration of the optimization process consists of tuning a set of parameters according to some rules and then solving the forward problem with the new parameters set and iterating the procedure. In all optimization methods used in inverse scattering, the forward problem is formulated in terms of unknown geometry or medium parameters to be optimized. In the proposed shape reconstruction algorithm, the values of the control points of a Bezier patch are optimized to find the optimal shape of the target matching the RCS data. The iterative nature of the algorithms and the ill-conditioned nature of problems make the algorithms somehow sensitive to the initial guess. These problems and also the local minima effects are solved by advanced optimization techniques such as the Genetic Algorithms (GA's), at the expense of tremendous increase in the computation time ([5]). In this work, we use a genetic algorithm in which the objective function for a given generation is defined as follow:

$$f_i^K = \sum_f \sum_\theta \sum_\varphi |\sigma_{\theta,\varphi,f} - \sigma'_{\theta,\varphi,f}| \quad (7)$$

where  $\sigma$  and  $\sigma'$  are the goal and estimated RCS and  $K$  and  $i$  are generation and population indices respectively. Also  $\varphi, \theta$  are the angles of spherical coordinates and  $f$  is the incident frequency. It is remarkable that the probability quantity of the mutation operator in designed genetic algorithm dynamically varies for better convergence and this algorithm can reconstruct the shape of the target efficiently without any initial assumption about the target geometrical characteristics.

### IV. Results

Here we illustrate the performance of the proposed algorithm by first considering perfectly conducting  $40^\circ$  cylindrical sector with height of  $1.5 m$  and radius of  $1m$  represented by 6

control points at  $f = 1\text{GHz}$  illumination (Fig. 1). As a second example, we consider the perfectly conducting spherically curved plate with radius of  $1\text{m}$  over a beam of  $90^\circ$  for  $\varphi, \theta$  angles, represented with 9 control points at  $f = 3\text{GHz}$  illumination (Fig. 2).

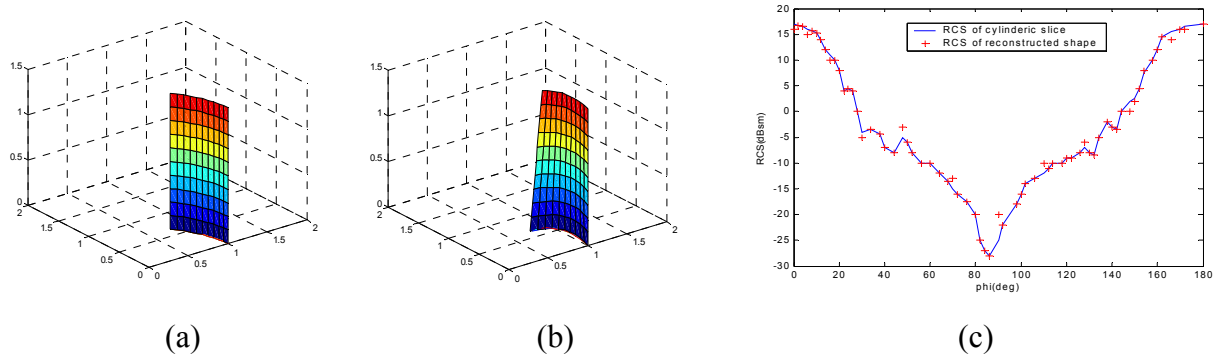


Fig. 1- (a) original shape (b) reconstructed shape (c) RCS comparison of original and reconstructed shape

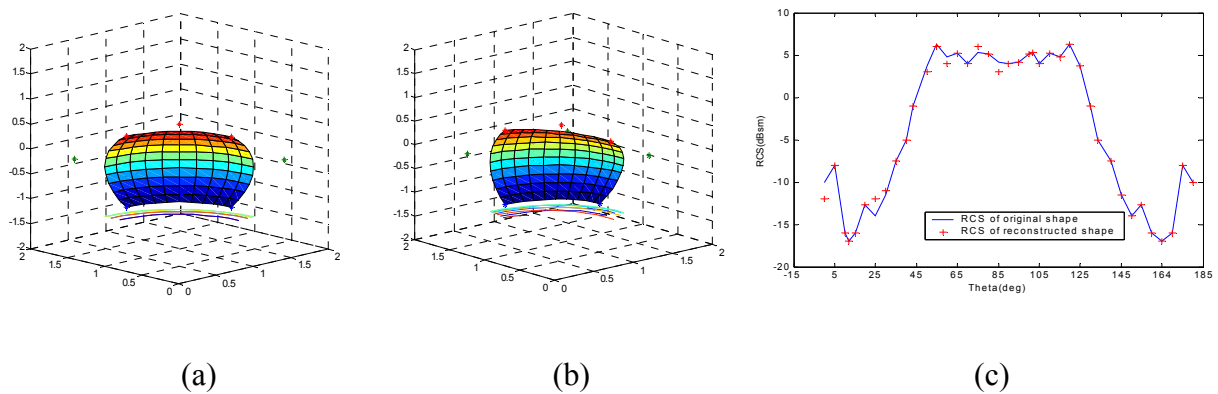


Fig.2- (a) original shape (b) reconstructed shape (c) RCS comparison of original and reconstructed shape

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## References

- [1].K. Barkeshli, M. Mokhtari and N. Mahdavi Amiri, "Image reconstruction of impenetrable cylinders using cubic B-splines and genetic algorithms," Proc. of IEEE AP-S Int Sym. 2001
- [2].Leslie Piegl, Wayne Tiller,"Curve, and Surface Constructions using Rational B-splines" Computer-Aided Design, Vol. 19, No. 9, November 1987
- [3].J. Perez, M. F. Catedra, "Application of Physical Optics to the RCS Computation of Bodies Modeled with NURBS Surfaces", IEEE Trans. , AP- 42, No.10, October 1994
- [4].W. B. Gordon, "Far-Field Approximations to the Kirchhoff-Helmholtz Representation of Scattered Fields" IEEE Trans., AP-23, July 1975
- [5].D.S.Weile and E.Michielsen"Genetic Algorithm Optimization Applied to Electromagnetics: A Review", IEEE Trans. AP-45, No.3, March 1997