Proceedings of ISAP'04, Sendai, JAPAN

RECONSTRUCTION OF A DIELECTRIC CYLINDER WITH THE BARZILAI-BORWEIN STEEPEST DESCENT METHOD

Akimichi YADA and Takato KUDOU
Department of Electrical and Electronic Engineering, Oita University
700 Dannoharu, Oita 870-1192, Japan
E-mail: ayada@eee.oita-u.ac.jp, tkudou@eee.oita-u.ac.jp

1. Introduction

The inverse scattering problem estimating unknown properties of scattering objects from the measured electromagnetic-field data outside the objects has gained much interest in a large area of applications such as medical imaging, geophysical exploration and nondestructive testing. Since the relation between the properties and the scattered field is generally nonlinear, an iterative numerical method is often required to solve the problem. During the last few decades, many iterative methods have been presented in frequency domain [1]-[5] or time domain[6], [7]. Most of them have applied a gradient method using the derivatives of the objective function to the estimation procedure.

In this article, we discuss a frequency-domain iterative method for the reconstruction of a lossless dielectric cylinder. A cost functional is defined as the error between the measured scattered field and calculated one, and minimized by using the Barzilai-Borwein steepest descent (BB) method [8]-[10]. The BB method does not guarantee a descent in the objective function at each iteration, but achieves better performance than the classical steepest descent (SD) method. Also, it has the feature that the stepsize in the reconstruction procedure can be found with simplicity and computational efficiency. Numerical results show the validity of the presented method by comparing with the conjugate gradient (CG) and the SD methods.

2. Formulation

Let us consider a dielectric cylinder with a relative permittivity $\varepsilon_r(\boldsymbol{\rho})$ and an arbitrary cross section S which is invariant in the z axis, as illustrated in Fig. 1. The position vector is denoted by $\boldsymbol{\rho}$. The cylinder is illuminated by TM incident waves due to a line source located at transmitter points $\boldsymbol{\rho}_\ell\,(\ell = 1, 2, \cdots L)$. The scattered field is measured at receiver points $\boldsymbol{\rho}_m\,(m = 1, 2, \cdots M)$. We suppress the time-harmonic dependence $\exp(j\omega t)$ in this article.

The contrast $\chi(\boldsymbol{\rho})$, with respect to the relative permittivity of free space, is defined as

$$\chi(\boldsymbol{\rho}) = \varepsilon_r(\boldsymbol{\rho}) - 1. \tag{1}$$

Then the total field E_z^{tot} within the domain S satisfies the following integral equation

$$E_z^{tot}(\boldsymbol{\rho}, \boldsymbol{\rho}_\ell) = E_z^{inc}(\boldsymbol{\rho}, \boldsymbol{\rho}_\ell) + k_0^2 \iint_S \chi(\boldsymbol{\rho}')G(\boldsymbol{\rho}, \boldsymbol{\rho}')E_z^{tot}(\boldsymbol{\rho}', \boldsymbol{\rho}_\ell)d\boldsymbol{\rho}', \tag{2}$$

where E_z^{inc} is the incident field and k_0 is the wave number of free space. The 2-D Green's function G is denoted by

$$G(\boldsymbol{\rho}, \boldsymbol{\rho}') = -\frac{j}{4}H_0^{(2)}(k_0|\boldsymbol{\rho} - \boldsymbol{\rho}'|), \tag{3}$$

in which $H_0^{(2)}$ is the zeroth-order Hankel function of the second kind. The scattered field E_z^{sct} measured at $\boldsymbol{\rho}_m$ is given by

$$E_z^{sct}(\boldsymbol{\rho}_m, \boldsymbol{\rho}_\ell) = k_0^2 \iint_S \chi(\boldsymbol{\rho}')G(\boldsymbol{\rho}', \boldsymbol{\rho}_m)E_z^{tot}(\boldsymbol{\rho}', \boldsymbol{\rho}_\ell)d\boldsymbol{\rho}'. \tag{4}$$

ISBN: 4-88552-206-4 C3055©IEICE

Let us divide the domain S into N elementary square cells. Then the moment method is employed in solving the integral equation (2) for the total field.

In order to reconstruct the relative permittivity of the cylinder, we minimize the cost functional defined as

$$F(\boldsymbol{\chi}) = \frac{\displaystyle\sum_{\ell=1}^{L}\sum_{m=1}^{M}\left|\tilde{E}_z^{sct}(\boldsymbol{\rho}_m,\boldsymbol{\rho}_\ell) - E_z^{sct}(\boldsymbol{\chi},\boldsymbol{\rho}_m,\boldsymbol{\rho}_\ell)\right|^2}{\displaystyle\sum_{\ell=1}^{L}\sum_{m=1}^{M}\left|\tilde{E}_z^{sct}(\boldsymbol{\rho}_m,\boldsymbol{\rho}_\ell)\right|^2}, \tag{5}$$

in which $\tilde{E}_z^{sct}$ and E_z^{sct} are, respectively, the measured scattered field and the calculated one from the estimated value of the relative permittivity. The contrast $\boldsymbol{\chi}$ is the $N \times 1$ column vector. Since the minimization of the cost functional (5) is a nonlinear optimization problem, one usually introduces an iterative method to solve the problem. In this article, we apply the BB method to the nonlinear optimization problem.

3. Barzilai-Borwein steepest descent method

Now suppose iteration kth contrast $\boldsymbol{\chi}_k$ are known, we may update $\boldsymbol{\chi}_k$ by

$$\boldsymbol{\chi}_{k+1} = \boldsymbol{\chi}_k + \alpha_k \boldsymbol{d}_k, \tag{6}$$

where α_k is a stepsize and $\boldsymbol{d}_k$ is the $N \times 1$ update direction vector. The update direction vector is chosen by the following formula same as the SD method

$$\boldsymbol{d}_k = -\boldsymbol{g}_k. \tag{7}$$

The gradient column vector $\boldsymbol{g}$ of the cost functional is given by

$$g_n = k_0^2 \sum_{\ell=1}^{L}\sum_{m=1}^{M} \overline{E_z^{tot}(\boldsymbol{\rho}_n,\boldsymbol{\rho}_m) E_z^{tot}(\boldsymbol{\rho}_n,\boldsymbol{\rho}_\ell)} \cdot \left[E_z^{sct}(\boldsymbol{\chi},\boldsymbol{\rho}_m,\boldsymbol{\rho}_\ell) - \tilde{E}_z^{sct}(\boldsymbol{\rho}_m,\boldsymbol{\rho}_\ell)\right], \quad n = 1, 2, \cdots, N, \tag{8}$$

where $\overline{\cdot}$ indicates the complex conjugate and $\boldsymbol{\rho}_n$ denotes the center of the nth cell. The stepsize α_k is generally chosen by minimizing $F(\boldsymbol{\chi}_k + \alpha_k \boldsymbol{d}_k)$ with a line search. Consequently, it takes expensive computational cost to determine α_k since one has to solve the forward problem many times at each iteration. On the other hand, in the BB method [8]-[10], the stepsize is derived by regarding $H_k = \alpha_k I$ as an approximation to the Hessian inverse of F at $\boldsymbol{\chi}_k$ and imposing some quasi-Newton property on H_k. By minimizing $||\Delta\boldsymbol{\chi}_k - \alpha_k \Delta\boldsymbol{g}_k||^2$, in which we define $\Delta\boldsymbol{\chi}_k = \boldsymbol{\chi}_k - \boldsymbol{\chi}_{k-1}$ and $\Delta\boldsymbol{g}_k = \boldsymbol{g}_k - \boldsymbol{g}_{k-1}$, the stepsize is obtained by

$$\alpha_k = \frac{\langle \Delta\boldsymbol{\chi}_k, \Delta\boldsymbol{g}_k \rangle}{\langle \Delta\boldsymbol{g}_k, \Delta\boldsymbol{g}_k \rangle}, \tag{9}$$

in which $\langle \cdot, \cdot \rangle$ is the scalar product of the vectors. By symmetry, another stepsize may be determined by minimizing $||\alpha_k^{-1}\Delta\boldsymbol{\chi}_k - \Delta\boldsymbol{g}_k||^2$. The corresponding stepsize is given the following equation

$$\alpha_k = \frac{\langle \Delta\boldsymbol{\chi}_k, \Delta\boldsymbol{\chi}_k \rangle}{\langle \Delta\boldsymbol{\chi}_k, \Delta\boldsymbol{g}_k \rangle}. \tag{10}$$

We are concerned with the stepsize (9) in this article, although it is not clear that the properties of those are all similar.

4. Numerical Results

As a numerical example, we apply the BB method to the reconstruction of a lossless homogeneous cylinder with the relative permittivity 1.8 and radius 0.8λ, where λ is a wavelength of free space. The performance of the method is compared with the CG (Polak-Ribière-Polyak) and the SD methods in which the stepsize is determined by a line search at each iteration. The domain S is assumed to be the $2\lambda \times 2\lambda$ square domain, and divided by 24×24 elementary cells. The transmitter and receiver points are, respectively, 36 points equally spaced along a circle with radius 2λ. All the initial values of the contrast are chosen equal to zero as the relative permittivity is the same as the value of free space, and all the estimation procedures are terminated when the cost functional is less than 10^{-5}. In order to compare the accuracy of the reconstruction results, we introduce the relative permittivity error defined as

$$\eta = \frac{\iint_S |\tilde{\varepsilon}_r(\boldsymbol{\rho}) - \varepsilon_r(\boldsymbol{\rho})| d\boldsymbol{\rho}}{\iint_S |\tilde{\varepsilon}_r(\boldsymbol{\rho})| d\boldsymbol{\rho}}, \tag{11}$$

where $\tilde{\varepsilon}_r$ and ε_r are exact relative permittivity and estimated one, respectively.

Figure 2 shows the evolution of the cost functional as a function of iteration. The solid, dotted and long-and-short dashed lines present the BB, CG and SD methods, respectively. In the BB, CG and SD methods, respectively, the estimation procedures are terminated after 63, 98 and 187 iterations and the total computation times are approximately 10, 210 and 384 minutes on a personal computer with an Atholon XP 1900+ (1.6-GHz CPU). It is seen from these results that the convergence speed and the computation time of the BB method is much faster than those of the CG and the SD methods.

Figures 3(a) and (b) illustrate the exact distribution and the cross sectional view of the final reconstruction results sliced along the axis $y = 0$, respectively. The thin solid line shows the exact distribution. The relative permittivity errors η are 5.68×10^{-2}, 5.98×10^{-2} and 5.45×10^{-2} for the BB, CG and SD methods, respectively. From Fig. 3 and the values of η, we can comprehend that any method is obtained correspondingly the good estimated results in this case.

From these results, it is confirmed that the BB method has faster convergence and less computation time than those of the CG and the SD methods in order to gain a desirable accuracy of the reconstruction.

5. Conclusion

In this article, we have applied the BB method to a frequency-domain inverse scattering problem. The stepsize is simply derived by imposing some quasi-Newton property in the BB method. Numerical results for the reconstruction of a lossless dielectric cylinder showed that the method is more effective than the CG and the SD methods in terms of the convergence speed and the computational cost. In the future studies, we will evaluate the perfomance of the method for the case that the measured scattered field contains noises and multifrequency scattering data is utilized.

Acknowledgement

The authors wish to thank Prof. Mitsuru Tanaka, Oita University, for his encouragement and helpful suggestions.

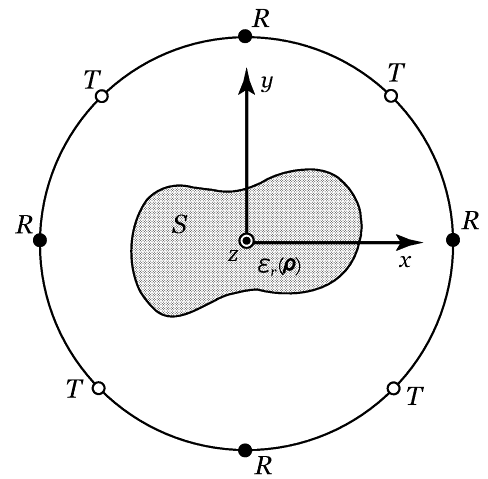


Fig. 1 Geometry of the problem.

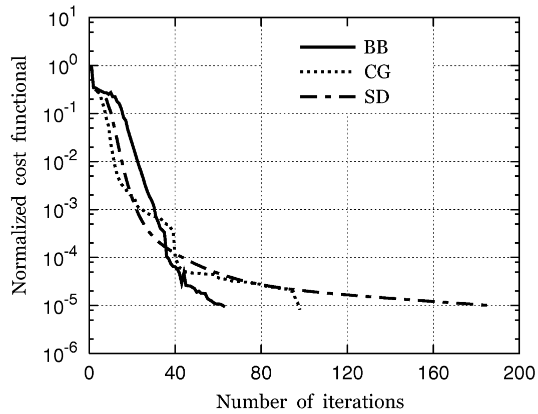
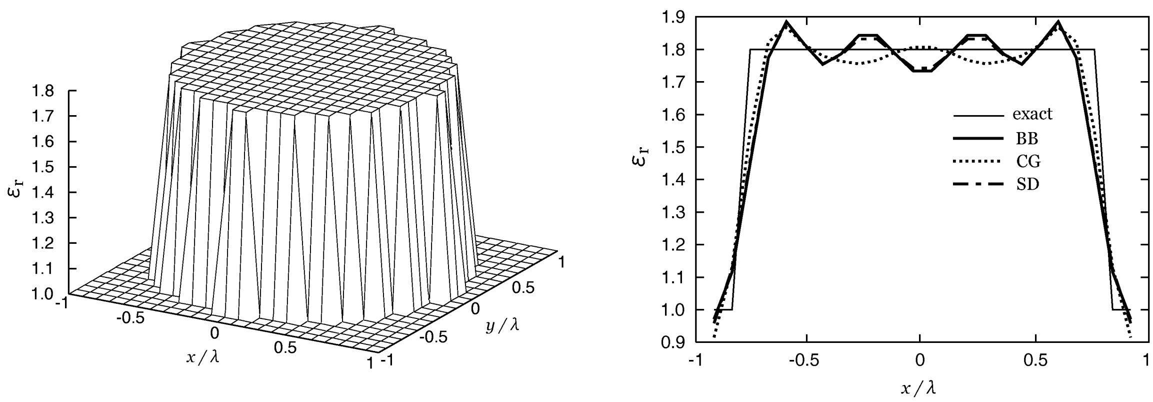


Fig. 2 Evolution of the cost functional as a function of iteration.



(a) (b)

Fig. 3 Reconstruction of a lossless dielectric cylinder : (a) exact distribution, (b) reconstruction results.

References

[1] R. E. Kleinman and P. M. van den Berg, *J. Comp. Appl. Math.*, vol. 42, pp. 17-35, 1992.

[2] H. Harada, D. J. N. Wall, T. Takenaka and M. Tanaka, *IEEE Trans. Antennas Propagat.*, vol. 43, no. 8, pp. 784-792, Aug. 1995.

[3] A. Franchois and C. Pichot, *IEEE Trans. Antennas Propagat.*, vol. 45, no. 2, pp. 203-215, Feb. 1997.

[4] A. Abubakar, P. M. van den Berg, and J. J. Mallorqui, *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 7, pp. 1761-1770, Jul. 2002.

[5] A. Franchois and A. G. Tijhuis, *Radio Sci.*, vol. 38, no. 2, pp. VIC12 1-13, 2003.

[6] I. T. Rekanos, T. V. Yioultsis, T. D. Tsiboukis, *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 3, pp. 336-344, Mar. 1999.

[7] T. Takenaka, H. Jia and T. Tanaka, *J. Electromagn. Waves. Appl.*, vol. 14, pp. 1609-1626, 2000.

[8] J. Barzilai and J. M. Borwein, *IMA J. Numer. Anal.*, vol. 8, pp. 141-148, 1988.

[9] M. Raydan, *SIAM J. Optim.*, vol. 7, pp. 26-33, 1997.

[10] Y. H. Dai, J. Y. Yuan and Y. Yuan, *Comp. Opt. Appl.*, vol. 22, pp. 103-109, 2002.