

# A Comparative Study of Radio Wave Propagation Over the Earth Due to a Vertical Electric Dipole

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## 1. Introduction

The radio wave propagation excited by a vertical electric dipole propagation over the earth has been a subject of many years. Generally, there are two analytical methods for obtaining the electromagnetic fields. The first method is to model the earth surface as an infinite plane and solve the Sommerfeld integrals either by asymptotic expansions or by numerical integrations [2], [3]. The second method is to treat the earth to be a radially stratified sphere and sum up the spherical harmonics in spherical coordinates system [4], [5]. By inspection, the solution to the second problem should be the same as that to the first one when the radius of the earth is electrically very large. Both of the planar and spherical earth models can be simplified under certain approximations. Therefore, their accuracy and validation range need to be studied in detail. In this paper, both of these planar models and the approximate spherical model are re-visited and three sets of results are compared with the results recently obtained in [6]. For the convenience of the comparison, it is assumed that the earth is a perfect conductor coated with one layer of dielectric. The accelerated spherical harmonics series in the spherical model by full-wave method is compared with the residue series. The accuracy of the residue series is confirmed for the perfect conducting sphere. For the layered sphere, the results show that the hybrid effect due to the trapped surface wave and the lateral wave was ignored in the residue series; and according to the full-wave method, the hybrid effect becomes stronger with the increase of the permittivity of the dielectric layer.

## 2. Planar Earth Model and Formulation

The field generated by a vertical electric dipole in the air with a unit electric dipole moment in the three-layered medium was given in [2], where a time dependence of  $e^{-i\omega t}$  was assumed and it is used subsequently but will be suppressed. The height of the dipole is  $z_s$  while the height of the observation point is  $z_r$ , both measured from the surface of the dielectric layer. The medium consists of a half-space of air (region 0,  $z \leq 0$ ), a dielectric layer with thickness  $l$  (region 1,  $0 \leq z \leq l$ ), and a conducting or dielectric medium (region 2,  $l \leq z$ ). The height of the dipole is  $d$  from the surface of the dielectric layer.

The total electric field is expressed as a sum of the direct wave  $E_z^{(1)}$ , the reflected wave  $E_z^{(2)}$ , and the contributions from the lateral and surface waves  $E_z^{(3)}$  as follows:

$$E_z^{(1)} = \frac{\omega\mu_0}{4\pi k_0} e^{ik_0 r_1} \left[ \frac{ik_0}{r_1} - \frac{1}{r_1^2} - \frac{i}{k_0 r_1^3} - \left( \frac{z_r - z_s}{r_1} \right)^2 \cdot \left( \frac{ik_0}{r_1} - \frac{3}{r_1^2} - \frac{3i}{k_0 r_1^3} \right) \right], \quad (1a)$$

$$E_z^{(2)} = \frac{\omega\mu_0}{4\pi k_0} e^{ik_0 r_2} \left[ \frac{ik_0}{r_2} - \frac{1}{r_2^2} - \frac{i}{k_0 r_2^3} - \left( \frac{z_r + z_s}{r_2} \right)^2 \cdot \left( \frac{ik_0}{r_2} - \frac{3}{r_2^2} - \frac{3i}{k_0 r_2^3} \right) \right], \quad (1b)$$

$$E_z^{(3)} = -\frac{\omega\mu_0}{4\pi k_0^2} \cdot \int_{-\infty}^{\infty} \frac{A(\lambda) H_0^{(1)}(\lambda \rho) \cdot e^{i\gamma_0(z_r+z_s)} \lambda^3}{q(\lambda) \gamma_0} d\lambda, \quad (1c)$$

King and Sandler [2], and Zhang and Pan [3] applied two approaches to solve the integral in (1c) for  $E_z^{(3)}$ . In the former paper, this modeling has a limited validation range as mentioned in [2]:

$$k_0^2 \ll k_1^2 \ll |k_2|^2, \quad (2a)$$

$$k_1^2 l^2 \ll 1 \quad \text{or} \quad k_1 l \leq 0.6. \quad (2b)$$

The trapped surface wave was proved to have a decay factor of  $\rho^{-1/2}$  in the  $\rho$  direction. It has been also shown that when the inequalities in (2) are satisfied, the lateral wave of  $E_z^l$  is approximately the same as that in [2]. However, the properties of the lateral wave were not discussed in detail. In [1], a physical interpretation of the lateral wave was given and the amplitude of the lateral wave decays as  $\rho^{-2}$  in the  $\rho$  direction. And it is evanescent in region 0 and propagates in region 1. In the expressions of the lateral wave, the phase velocity and amplitude attenuation factor were not explicitly derived.

### 3. Spherical Earth Model and Formulation

For the spherical earth model, the earth is treated as a layered sphere. The center of the sphere is assumed to be at the origin of the spherical coordinate system  $(r, \theta, \phi)$ . The radius of the earth is  $a$ . The dipole is located at a distance  $b = a + z_s$  from the center of the sphere. The radial distance of the observation point to the center of the sphere is  $r = a + z_r$ .

For  $r \gtrsim b$ , the electric field in the outer space is expressed as [6]

$$E(\mathbf{r}) = -\frac{\omega\mu_0 I_0}{4\pi b} \sum_{n=0}^{\infty} (2n+1) \cdot \begin{cases} [j_n(k_0 b) + \mathcal{B}_N^{11} h_n^{(1)}(k_0 b)] N_{e0n}^{(1)}(k_0), \\ h_n^{(1)}(k_0 b) [N_{e0n}(k_0) + \mathcal{B}_N^{11} N_{e0n}^{(1)}(k_0)], \end{cases} \quad (3)$$

where  $N_{e0n}^{(1)}(k)$  and  $N_{e0n}(k)$  denote the even vector eigenfunctions at  $m = 0$  and  $\mathcal{B}_N^{11}$  standing for the reflection coefficient.

Equation (3) represents the exact summation expressions for the electrical fields. However, it is known that the series converges very slowly for an electrically large sphere. Convergence acceleration methods can be used to obtain a faster convergent series as stated in [6] and [?, ].

Various asymptotic methods are explored by many researchers to find an efficient solution. Fock [4] and Wait [5] derived the field expressions in terms of residue series. The electric field radial component  $E_r$  is given by

$$E_r = E_0 e^{i\pi/4} \sqrt{\pi x} \sum_{s=1}^{\infty} \frac{1}{t_s - q^2} \frac{w_1(t_s - y_s)}{w_1(t_s)} \frac{w_1(t_s - y_r)}{w_1(t_s)} e^{it_s x}, \quad (4)$$

where  $E_0 = \frac{iI_0 \sqrt{\mu_0}}{2\pi \sqrt{\epsilon_0}} \frac{k_0 e^{ik_0 a \theta}}{a \sqrt{\theta \sin \theta}}$ ,  $y_s = \left(\frac{2}{k_0 a}\right)^{1/3} k_0 z_s$ ,  $y_r = \left(\frac{2}{k_0 a}\right)^{1/3} k_0 z_r$ ,  $x = \left(\frac{k_0 a}{a}\right)^{1/3} \theta$ ,  $q = \frac{k_0 \sqrt{k_1^2 - k_0^2}}{k_1} \left(\frac{k_0 a}{2}\right)^{1/3} \tan(\sqrt{k_1^2 - k_0^2} l)$ , while  $w_1(t)$  denotes the Fock notation of the Airy function, and  $t_s$  represents the roots of the equation  $w_1'(t) - q w_1(t) = 0$ . The first term in (4) is the trapped surface wave, which is dominant.

### 4. Numerical Results

Figures 1 and 2 show the vertical electric field amplitudes of these three asymptotic formulas varying with the propagation distance  $\rho$ . The thickness of the dielectric layer is taken to be  $l = 100$  m, and 30 m, respectively. The earth radius is assumed to be  $a = 6370$  km for the spherical earth model.

For the results in Fig. 1,  $l = 100$  m,  $k_1 l \approx 0.8$ , so the inequality conditions in (2) are not satisfied. Therefore the results produced by King and Sandler are expected to be inaccurate. When both the source point and the observation point are located on the surface of the dielectric layer as illustrated in Fig. 1(a), the curves obtained using the formulas of [3] are closer to those of spherical model. Furthermore, the oscillation shows the hybrid of the trapped surface wave and the lateral wave which was not predicted in the curves by King and the residue series [3].

When  $l = 30$  m, the inequality conditions in (2) are satisfied. When both the source point and the observation point are located on the surface of the dielectric layer as shown in Fig. 2(a), in the short range around the source ( $\rho \leq 50$  km) where the earth curvature can be ignored, the formulas by [2] are more accurate than the ones by [3], which is evident from their comparison with the curve of the spherical earth model. When the dielectric thickness is not too thin, the curve by [3] is closer to the one by spherical

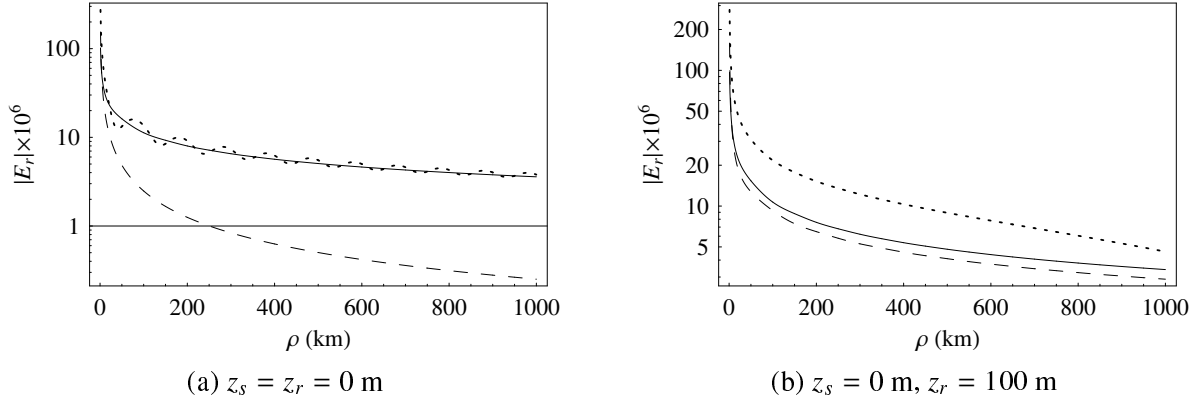


Figure 1: Amplitudes of  $E_z$  varying with horizontal distance  $\rho$ , computed using King's formula (- -) of planar model, formula in [3] ( $\cdots$ ) of planar model, and residue series formula (—) of approximate spherical model with dielectric layer thickness  $l = 100$  m, at a frequency of  $f = 100$  kHz, and  $\epsilon_r = 15$ .

model as shown in Fig. 2(a). When either the observation point or the source point moves away from the surface, we can see from Fig. 2(b), that the curves produced by the formulas of [2] are closer to those of the spherical model. This is because in this case the strength of the trapped surface wave is not as significant as the one along the surface of the dielectric layer. In other words, this may demonstrate that the trapped surface wave decays very fast in the vertical direction.

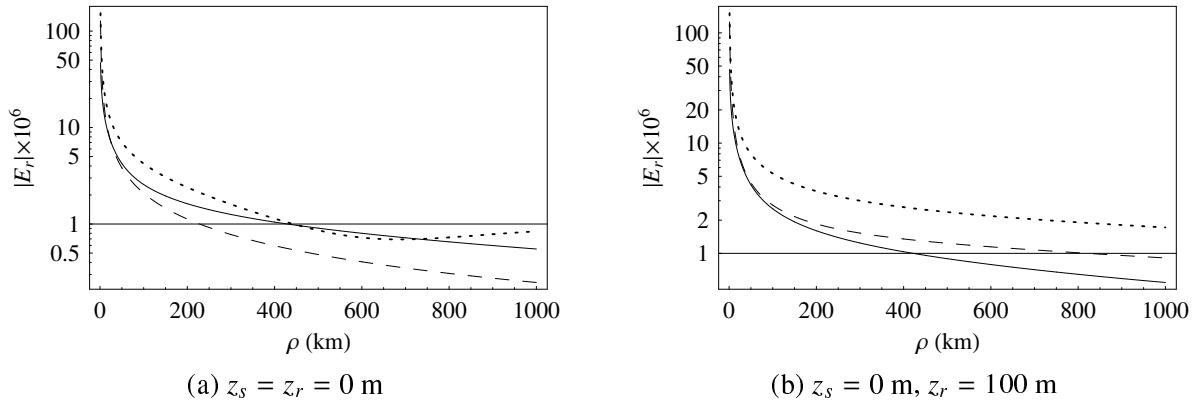


Figure 2: Amplitudes of  $E_z$  varying with horizontal distance  $\rho$ , computed using King's formula (- -) of planar model, formula in [3] ( $\cdots$ ) of planar model, and residue series formula (—) of approximate spherical model with dielectric layer thickness  $l = 30$  m, at a frequency of  $f = 100$  kHz, and  $\epsilon_r = 15$ .

#### 4.1 Asymptotic Computation Compared with Exact Computation

Fig. 3 shows the electric field in the spherical earth model which is assumed to be perfectly conducting. Results of the exact series summation are compared with those using the residue series approximation. For this case, the lateral wave vanishes and these two methods lead to exactly the same results. It confirms the accuracy of the residue series for the perfect conducting spherical earth model. In Fig.1(b), when  $\epsilon_r$  of the dielectric layer is close to 1, the hybrid modes of the trapped surface wave and the lateral wave are not strong, so the curves obtained using these two formulas are still smooth and close to each other.

In Fig. 4(a) where  $\epsilon_r = 2.0$ , the curve obtained using the series summation begins showing small oscillations. In Fig. 4(b) where  $\epsilon_r = 15$ , the oscillation of the curve obtained using the series summation becomes stronger with the increase of the permittivity of the dielectric layer while the curve obtained using the residue series still keeps to be smooth. It is shown that the curves obtained using the residue series are always very smooth while the oscillations in the curves obtained using the series summation exist due to the hybrid modes of the trapped surface wave and the lateral wave. Such surface waves can

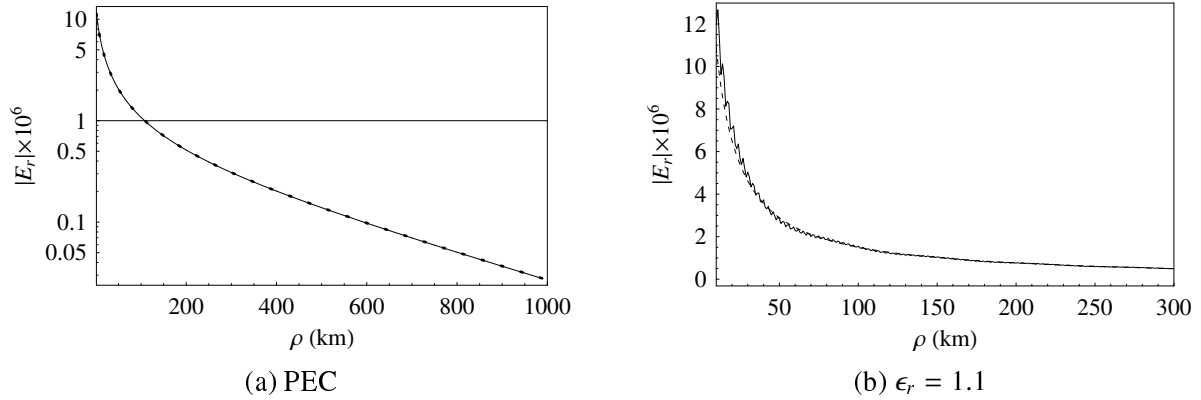


Figure 3: Amplitudes of  $E_z$  varying with  $\rho$ , compared by exact series(—), and residue series (- - -) for PEC and  $\epsilon_r = 1.1$ ,  $z_s = 10$  m,  $z_r = 500$  m, at a frequency of  $f = 100$  kHz.

be considered to be the contribution of the multiply reflected guided waves among the dielectric slab, and the oscillation can be considered as the dielectric resonance between the upper and lower dielectric interfaces. It implies that the lateral wave travels along the interface, at the same time it is multiply reflected by the upper and lower dielectric surfaces when the dielectric constant is large. When the relative permittivity of the coating layer is close to unity, then the multiple reflections by the upper and lower dielectric surfaces will be very weak and not so visible from the field curves.

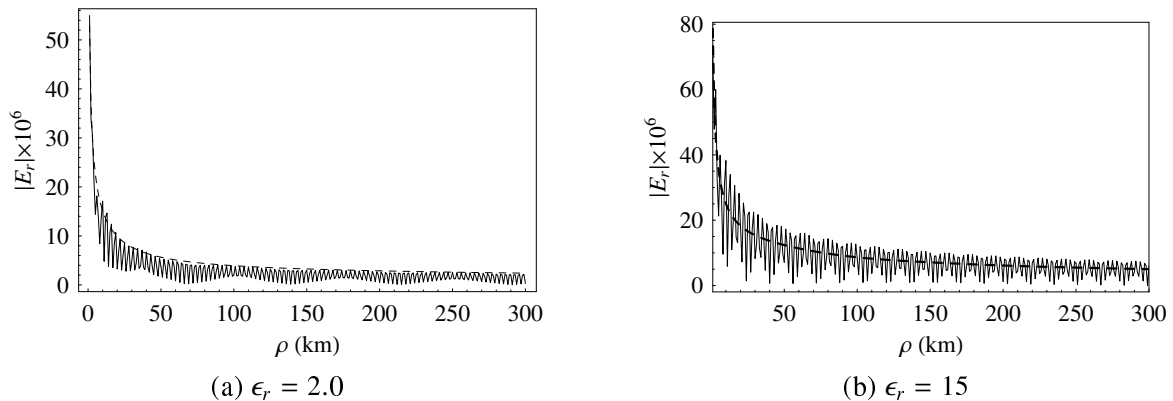


Figure 4: Amplitudes of  $E_z$  varying with  $\rho$ , compared by exact series(—), and residue series (- - -) for  $\epsilon_r = 2.0$  and  $\epsilon_r = 15$ ,  $z_s = 10$  m,  $z_r = 500$  m, at a frequency of  $f = 100$  kHz.

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