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ANALYTICAL-NUMERICAL STUDY ON PROPAGATION PHENOMENA OF BEAM FIELD OVER A CONCAVE-TO-CONVEX BOUNDARY

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1. INTRODUCTION

When the curvature of the boundary changes smoothly from concave to convex through the inflection point, conventional methods for separable problems, e.g., uncoupled mode expansions, are no longer applicable. For perfectly conducting smooth boundaries with variable radius of curvature, intermode coupling can be neglected in a lowest order of approximation, and the problem in a concave boundary can then be reduced to determining adiabatic whispering gallery (WG) modes [1],[2] , which propagate without coupling by smoothly adapting to the slowly changing curvature. The adiabatic WG mode formalism breaks down in the concave-to-convex transition region surrounding the inflection point [3] ~[5].

However, by tracking modal ray congruences, ray methods can be employed even for modal fields [6]. Modal ray congruences are confined between the modal caustic and the boundary on the concave side, and between the reflection boundary and the shadow boundary of the modal rays on the convex side. This permits an insight into the evolution of a WG mode as it approaches and passes the inflection point, and is converted into a beam-like wave and a creeping wave. The concave-to-convex conversion is schematized in Fig.1 by tracing modal ray congruences.

The characteristics of an initially well confined WG mode propagating from the concave toward the convex side have been explored by two methods. The first, based on the boundary layer near the surface, leads to a parabolic equation (PE) that is solved numerically [3]. This PE method gives sufficiently high accuracy in carrying out the computations near the boundary. The second assumes the Kirchhoff's surface currents to be approximated by the analytic continuation of the WG mode from positive to negative radius of curvature, and then treats the resulting Kirchhoff integral asymptotically [4]. The results obtained from the PE method have been used to estimate the accuracy of the results obtained by the Kirchhoff method [7]. The relative magnitude of the difference in the value of the wave fields computed by the two methods varies from 1% to 20% [7].Therefore, one may prefer to use the PE method to obtain the radiated beam accurately. However, because validity of the PE method is inherently restricted to a narrow boundary layer near the surface, the results cannot be propagated to large distances in the convex side from the point of radiation.

In this paper, we examine a beam at large distances radiated

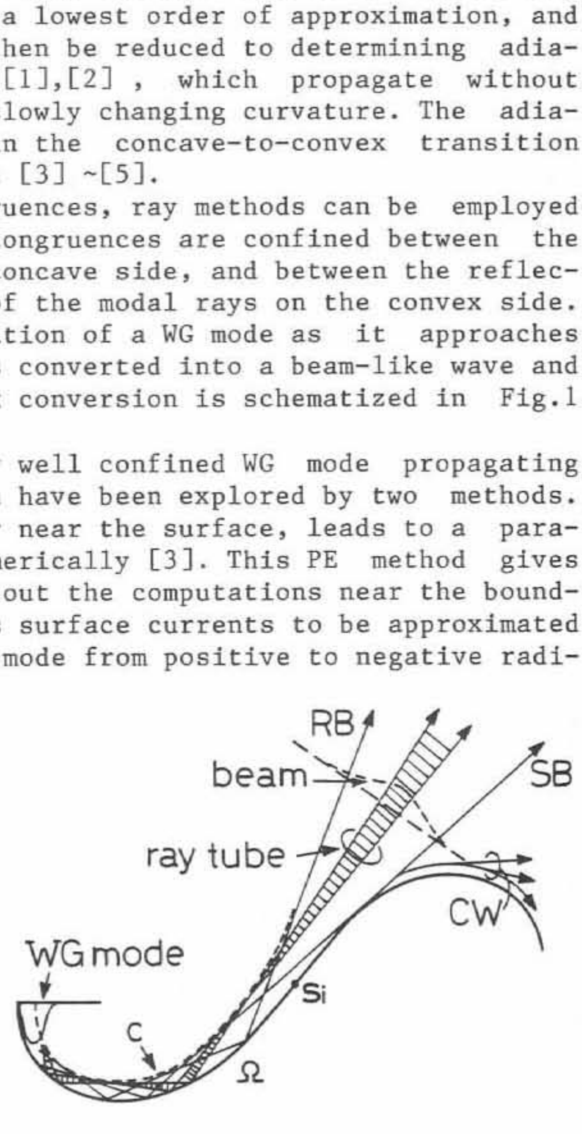

Fig.1. Propagation mechanisms for concave-to-convex boundary(Ω). Modal ray tracing in ray tubes (shaded) schematizes evolution of WG mode through inflection point (s_i) and excitation of creeping wave (CW) on convex side. RB: reflection boundary; SB: shadow boundary; C: modal caustic.

by the adiabatic WG mode which is incident on an inflection point of the concave-to-convex boundary from the concave side. A theory is developed which takes into account for the beam field at large distances from the radiation points. Predictions from the modal ray tracing will be compared with numerical solutions obtained from the new theory for a model surface, and will be used to interpret the numerical results.

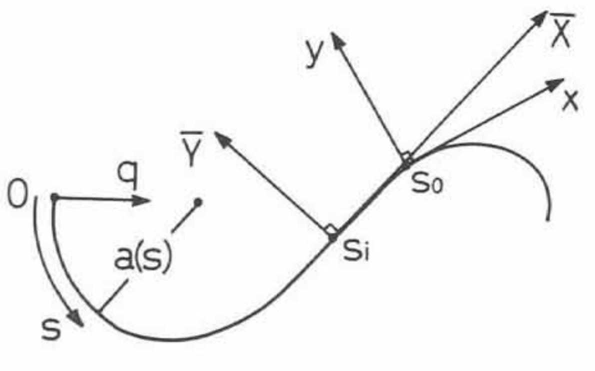

Fig.2. $(s,q),(\overline{X},\overline{Y})$, and (x,y) coordinate systems on concave-to-convex boundary. s_i (inflection point) and s_o (radiation point) are the origin of cartesian coordinate systems $(\overline{X},\overline{Y})$ and (x,y), respectively. $a(s)$: radius of curvature at s.

2. FORMULATION AND SOLUTION

Let s be the arc length of the curve Ω and let q be the coordinate taken along the normal to the boundary at the point s (Fig.2). The radius of curvature a(s) is assumed to change smoothly from positive values, which describe the concave boundary to negative values for the convex boundary, with the inflection point at $s=s_i$ where $a(s_i)=\infty$. We are looking for a high-frequency asymptotic solution to Helmholtz equation: $(\Delta+k^2)u=0$ which vanishes on Ω. Where Δ is the Laplacian in the (s,q) coordinate system [1], [5]. Assuming for u the parabolic ansatz

$$u = U(s, q, k)\exp(iks) \tag{1}$$

one may reduce (1) to the following equation for U :

$$\frac{\partial^2}{\partial q^2} U + 2ik\frac{\partial}{\partial s} U + 2qk^2\frac{1}{a(s)} U = 0 \tag{2}$$

2.1 Adiabatic Whispering Gallery(WG) Mode Solution

In the concave side $a(s)>0$ of the concave-to-convex boundary, well away from the inflection point or for the concave smooth boundaries with variable radius of curvature, the parabolic equation for U in Eq.(2) has solutions [1],[2]:

$$U_m \sim D_m\left[\frac{2}{ka(s)}\right]^{1/6} \frac{Ai[-\sigma_m+q/H(s)]}{Ai'[-\sigma_m]}\cdot\exp(-i\sigma_m\gamma(s)) \tag{3}$$

where

$$H(s)=\left(\frac{a(s)}{2}\right)^{1/3}\cdot k^{-2/3},\qquad \gamma(s)=\left(\frac{k}{2}\right)^{1/3}\cdot\int_0^s a^{-2/3}(\tau)d\tau \tag{3a}$$

and

$$Ai[-\sigma_m] = 0,\quad m=1,\ 2,\ 3,--- \tag{3b}$$

Here, Ai is the Airy function and Dm is the excitation coefficient of the m-th mode. Thus, the parabolic equation in (2) reduces to adiabatic WG modes. The adiabatic mode solutions in Fig.(3) fail when conditions change from concave to convex. The parabolic equation is implemented numerically where it cannot be reduce to the simpler asymptotic forms.

2.2 Parabolic Equation Algorithm

The parabolic equation in (2) remains valid at the transition region where the radius of curvature changes smoothly from $a(s)>0$ to $a(s)<0$

through the inflection point s_i. Near the inflection point where $a(s) \to \infty$, the curvature $K(s)$ can be approximated linearly as $K(s) = 1/a(s) = -a_o(s-s_i)$, where a_o is a positive constant. Introducing the scaled coordinates.

$$X=(s-s_i)a_o^{2/5}k^{1/5}, \quad Y=qa_o^{1/5}k^{3/5} \tag{4}$$

changes the parabolic equation into the frequency-independent form

$$\frac{\partial^2}{\partial Y^2}U + 2i\frac{\partial}{\partial X}U - 2XYU = 0 \quad (5)$$

which is numerically expedient. The inflection point is located in X=0. To track the adiabatic WG mode from the well trapped concave side to the convex side, we have integrated the parabolic equation in (5) numerically, using a difference equation marching scheme with the adiabatic WG mode as input at some initial range $X_o \ll 0$ in the well concave region. Thus, we have obtained the beam-like wave and the creeping wave on the convex side which are excited by the adiabatic WG mode. However, the beam-like wave cannot be propagated to large distances by the PE algorithm since the parabolic equation is restricted to the narrow region near the surface.

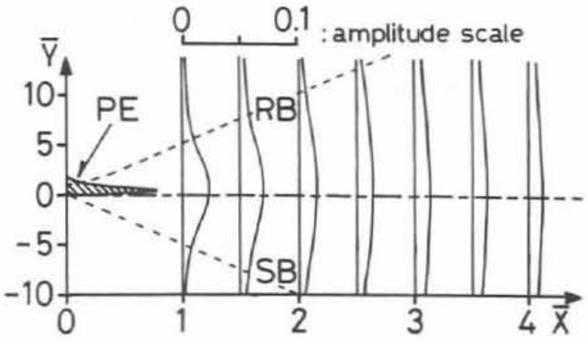

Fig.3. Amplitude plot of dominant mode radiation from the exit of concave boundary with constant curvature ($K(s)$= 20.944). The $\bar{X}$ coordinate coincides with the tangent to the boundary at the exit.

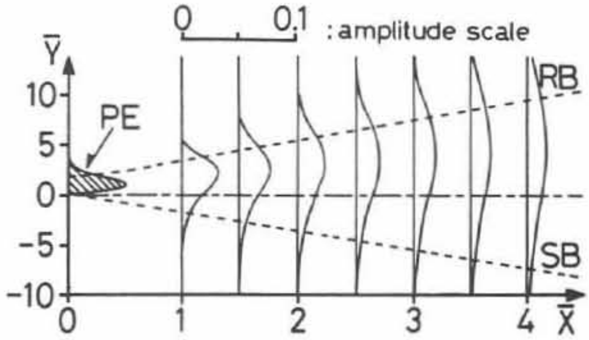

Fig.4. Amplitude plot of adiabatic WG mode radiation from the exit of concave boundary with variable curvature. ($K(s)=-87.730(s-0.239)$, $s \leq 0.239$).

2.3 Fresnel-Kirchhoff Diffraction Formula

We now come to our principal objective : the beam field at large distances excited by the adiabatic WG mode which is incident on the inflection point. We assume that at relatively large distances, the beam field is well approximated by the Fresnel-Kirchhoff diffraction formula [8]. Then, referring to the geometry shown in Fig.2, the beam field Gm(x,y) at the observation point P in the (x,y) coordinate system may be obtained from

$$G_m(x,y) = \sqrt{\frac{k}{2\pi R}}\, e^{ikR - i\pi/4} \int_0^s G_m(0,y')\, e^{ik\Phi(y';x,y)}\, dy' \tag{6}$$

where

$$\Phi(y',x,y) = -\frac{x\,x'}{R} + \frac{(y')^2}{2R} - \frac{(y\,y')^2}{2R^3} + \cdots \tag{6 a}$$

$$R = \sqrt{x^2 + y^2} \tag{6 b}$$

Here, $Gm(0',y')$ is the beam field distribution along the aperture $y=y' > 0$ at $x=0$ (or $s=s_o$) near the inflection point $s=s_i$. We integrate Eq.(6) numerically by using the numerical solutions for $Gm(0,y')$ obtained from the PE algorithm.

3. NUMERICAL RESULTS

We implement numerically the new method developed in §2, in order to calculate the beam fields at various horizontal distances. The PE algorithm in Fq.(5) is initiated well in the concave side by the known adiabatic WG mode input in Eq.(3), and evolves from there by a stepwise marching scheme. Passing through the inflection point, one confirms the radiation of a beam-like wave, exhibiting successively greater detachment from the concave surface. At the point $s=s_0$ where the beam-like wave is just detached from the surface, the Fresnel-Kirchhoff formula in Eq.(6), is applied to radiate the beam further away from the point of the detachment.

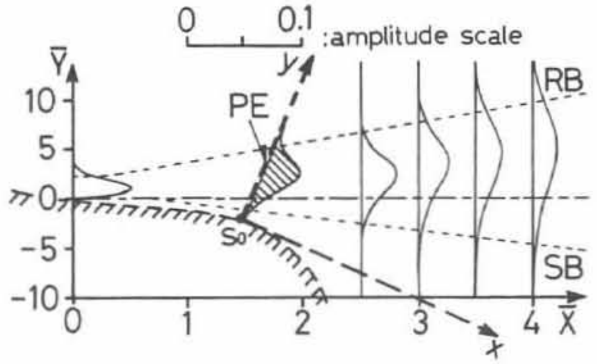

Fig.5. Amplitude plot of beam-like wave radiation from the point $s=s_0$ of concave-to-convex boundary. The curvature is defined as $K(s)=-87.730(s-0.239)$.

The results for the various boundaries are plotted on the $(\bar{X},\bar{Y})$ coordinate system (see Fig.2) in Figs.3-5. Fig.3 is the magnitude plots of beam fields radiated from the exit of the concave boundary with the constant curvature. The beam is radiated in the wide region between the reflection boundary and the shadow boundary. Fig.4 corresponds to the beam radiation from the variable radius of concave boundary. The narrow beam is radiated into the free space. One may note that the beam axis does not coincide with the $\bar{X}$-coordinate or the tangent to the boundary at the exit, but is shifted slightly to the upper side. Fig.5 is the beam radiation from the concave-to-convex boundary. The beam width becomes very narrow, and the beam axis is again shifted to the upper side.

4. CONCLUSION

This analytical-numerical study has furnished a basic understanding of the propagation phenomena over a concave-to-convex boundary excited by the whispering gallery (WG) mode. The theory charts the progress of an adiabatic WG mode, initially trapped, through the inflection point to the convex side. The adiabatic WG mode solutions on the concave side and the beam-like wave on the convex side are connected through the transition region near the inflection point by the parabolic equation algorithm. The beam has been radiated to large distances by the Fresnel-Kirchhoff formula.

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