RAY MODE COUPLING ANALYSIS OF PLANE WAVE SCATTERING BY A TROUGH

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1. Introduction

Recently, attention has been made for measuring radar cross section from open-ended cavities and many investigations have been reported [1 - 5]. When an electromagnetic wave impinges on such open structure, its scattering pattern often exhibits singular characteristics. This is due to the re-radiation contribution from the internal guiding or resonance structure.

In this paper, electromagnetic plane wave scattering by a trough will be discussed. The conventional approach to analyze the scattering field from such structures may be numerical schemes such as the moment method, which are effective for small scatterers, but not for large objects compared with the wavelength. The method used here is the high frequency asymptotic ray technique, which serves an attractive alternative scheme. It is customarily useful to use the modal description for the interior guiding structure region, and ray description for the exterior region. Accordingly, in order to calculate the scattered field efficiently, each description should be retained in the suitable region. Ray-mode conversion between the above two alternative descriptions has to be considered at the opening. Such conversion can be established by rigorous Poisson summation formula. Thus, one can construct the solution with keeping the advantages of both descriptions. In order to take account of the effect of multiple internal reflections, modal couplings at the opening are also considered. In the following discussion, time harmonic factor $e^{-i\omega t}$ is assumed.

2. Formulation of the Problem

As illustrated in Fig.1, H-polarized electromagnetic plane wave:

$$u^i(=H_y^i) = exp(-ikx\cos\theta_0 - ikz\sin\theta_0) \tag{1}$$

impinges on the aperture of a perfectly conducting trough of width a and depth b. $k(=\omega/c)$ denotes free space wavenumber. For simplicity, we shall now formulate the scattering far field. When this incident wave hits aperture's edges, the edge diffracted wave will be generated. According to the GTD formulation, primary edge diffracted waves may be given as [6]

$$\begin{aligned} u_p &= u_p^+ + u_p^-, & (2)\\ u_p^+ &= C(k\rho)D_{+1}(\theta,\theta_0;3\pi/2)exp\{-ika(\cos\theta+\cos\theta_0)/2\}, & (3)\\ u_p^- &= C(k\rho)D_{+1}(\theta+\pi/2,\theta_0+\pi/2;3\pi/2)exp\{ika(\cos\theta+\cos\theta_0)/2\}, & (4)\end{aligned}$$

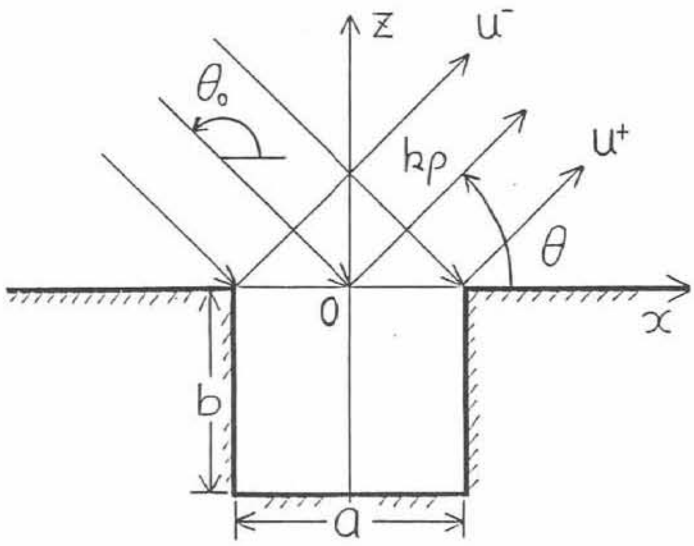

Figure 1: A perfectly conducting trough of width a and depth b, illuminated by H-polarized electromagnetic plane wave of incidence angle θ_0

where $C(x)$ is an asymptotic far field for two dimensional free space Green's function:

$$C(x) = (8\pi x)^{-1/2} e^{ix+i\pi/4}, \tag{5}$$

and $D_\tau(\phi, \phi_0; \phi_w)$ is Keller's edge diffraction coefficient:

$$D_\tau(\phi, \phi_0; \phi_w) = \frac{2\pi}{\phi_w} \sin \frac{\pi^2}{\phi_w} \left\{ \frac{1}{\cos\frac{\pi^2}{\phi_w} - \cos\frac{\phi-\phi_0}{\phi_w}\pi} + \tau \frac{1}{\cos\frac{\pi^2}{\phi_w} - \cos\frac{\phi+\phi_0}{\phi_w}\pi} \right\} \tag{6}$$

for a perfectly conducting wedge of wedge angle $(2\pi - \phi_w)$. One notes that the above primary diffracted waves u_p^+ and u_p^- diverge at the direction of reflection shadow boundary $\theta = \pi - \theta_0$. However the combined contribution $u_p (= u_p^+ + u_p^-)$ becomes finite due to the cancellation of each diverging property.

Thus excited primary edge diffracted waves and the incident plane wave also illuminate inside the indented trough region, and reradiate after experiencing some internal reflections at the wall. These contributions may be treated by tracing infinite number of multiply bouncing rays as in Ref.[1], but this is not numerically efficient. Here, we shall consider these field as re-radiation from the propagating parallel-plane waveguide modes excited by ray-mode coupling at the open end. Such coupling can be obtained easily by utilizing Poisson summation formula [7]. In order to treat modal couplings at the open end, and the multiple reflections of modes at the closed end systematically, a matrix formulation is used here. The re-radiation field u_r can be expressed in a compact form as [3]

$$u_r = C(k\rho)[\mathbf{S}] \sum_{n=1}^{\infty} [\mathbf{B}]^{n-1} [\mathbf{R}^{(1)}], \tag{7}$$

where matrices $[\mathbf{S}], [\mathbf{B}]$, and $[\mathbf{R}^{(1)}]$ denote the modal radiation row vector, modal coupling matrix, and modal excitation column vector at the open end, respectively. Index n designates the number of the reflection of the waveguide modes at the bottom. Each component of these matrices is given as

$$s_{1,p} = \frac{1}{2}\{D_{+1}(\theta, 3\pi/2 - \theta_p; 3\pi/2)(-1)^i e^{-ika\cos\theta/2}$$

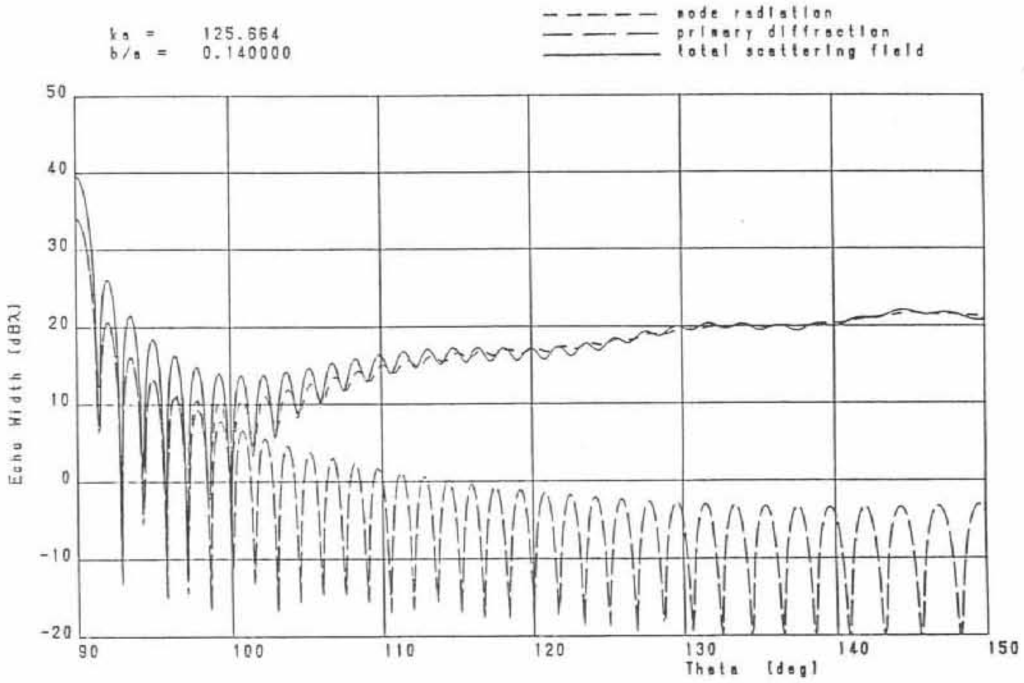

Figure 2: Echo width of a shallow trough. H-polarized plane wave incidence case. $ka = 20\pi, b/a = 0.14$.

$$+D_{+1}(\theta + \pi/2, \theta_i; 3\pi/2)(-1)^i e^{ika\cos\theta/2}\}, \tag{8}$$

$$b_{p,q} = \frac{i\epsilon_p}{8a\zeta_p} D_{+1}(\theta_p, \theta_q; 3\pi/2)\{1 + (-1)^{p+q}\} e^{2i\zeta_p b}, \tag{9}$$

$$r_{q,1} = \frac{i\epsilon_q}{4a\zeta_q}\{D_{+1}(\theta_q, \theta_0 + \pi/2; 3\pi/2) e^{ika\cos\theta_0/2} + D_{+1}(3\pi/2 - \theta_q, \theta_0; 3\pi/2)(-1)^q e^{-ika\cos\theta_0/2}\} e^{2i\zeta_q b}, \tag{10}$$

$$\theta_m = \sin^{-1}\left(\frac{m\pi}{ka}\right), \quad \zeta_m = \sqrt{k^2 - (m\pi/a)^2}, \quad \epsilon_m = \{{}^{2}_{1}\ {}^{m\neq 0}_{m=0}\}, m = p, q. \tag{11}$$

The matrix size is determined by the number of the propagating waveguide modes. One may also rewrite Eq.(7) in a collective closed form as

$$u_r = C(k\rho)[\mathbf{S}]\frac{[\mathbf{I}]}{[\mathbf{I}] - [\mathbf{B}]}[\mathbf{R}^{(1)}] = C(k\rho)[\mathbf{S}][[\mathbf{I}] - [\mathbf{B}]]^{-1}[\mathbf{R}^{(1)}] \tag{12}$$

where [I] denotes an unit matrix. Then the total scattering field u_t is given by a sum of u_p and u_r.

3. Numerical Examples and Discussions

Using the above formulation, numerical calculations have been done extensively. It has been found that the modal re-radiation field u_r plays an important role, and the primary diffraction field u_p doesn't contribute too much to the total field except for the direction of near shadow boundary. Modal re-radiation decreases roughly 10dB after each reflection at the bottom.

Fig.2 shows the echo width of a trough. This is the case for a pretty shallow (2.8λ) and wide (20λ) opening. This result may be compared with the one calculated by the Boundary Integral Method (BIM) [4]. Agreement is found to be excellent even for such pretty shallow case.

4. Concluding Remarks

In this paper, high frequency asymptotic ray technique has been applied to analyze an electromagnetic plane wave scattering by a trough. In the formulation, the modal description has been retained inside the waveguide region. Coupling between the waveguide modes and coupling between modes and rays at the aperture are treated systematically by using matrix form. So far, the multiple reflection effects of the waveguide modes have been taken into account, but the higher order interaction terms such as multiple edge diffractions are not included yet. When the aperture of the trough gets narrower, then the above higher order interaction terms become more important. These aspects are not discussed here, but in a separate paper.

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