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An Asymptotic Series Solution for the Flanged-Waveguide Radiation

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Abstract The problem of radiation from a flanged parallel plate waveguide is re-examined. The technique of the Fourier transform is used to represent the radiation fields in the spectral domain. The simultaneous equations for the radiation field coefficients are formulated and solved to give an asymptotic series solution. The asymptotic series solution is compared with other results to clarify some ambiguities associated with the numerical inaccuracies among different numerical approaches.

1 Introduction

The subject of radiation from a flanged parallel plane waveguide has been long received a great deal of attention due to its important applications to the antenna engineering. Representative theoretical investigations into the flanged waveguide radiation problem are in [1]. Recently, the technique of the moment method [2] has been used to clarify some discrepancies in the numerical results existing among the different techniques as pointed out in [1]. In this paper, we re-examine the problem of the flanged waveguide radiation in the spectral domain which is based on the Fourier transform technique. The objective of this paper is not only to formulate the solution to the flanged-waveguide-radiation problem in an asymptotic series representation but also to compare the series solution to the existing other numerical results. In what follows, we present the expression of the radiation field from the waveguide and compare it with others. A brief summary on the thoeretical development is given in Conclusion.

2 Derivation of Radiation Field: TM^y Case

Consider a parallel-plate waveguide of width $2a$ which has an infinite flange as shown in Fig. 1. Assume that a TM^y incident field E_y^i impinges on the aperture from inside the parallel-plate waveguide. Here, $exp(-jwt)$ time-harmonic variation is assumed throughout.

Then in region (I) of wavenumber k, incident and reflected electric fields are respectively written as

$$\begin{aligned} E_y^i(x,z) &= sina_p(x+a)exp(j\xi_p z) \\ E_y^r(x,z) &= \sum_{m=1}^{\infty} c_m sina_m(x+a)exp(-j\xi_m z) \end{aligned}$$

where $\xi_m = \sqrt{k^2 - a_m^2}, a_m = m\pi/(2a)$

Note that for odd p, $m = 1, 3, 5, ...$, and for even p, $m = 2, 4, 6, ...$

In regin (II) of wavenumber k_0, the transmitted electric field is

$$\begin{aligned} E_y^t(x,z) &= 1/(2\pi)\int_{-\infty}^{\infty} \tilde{E}_y^t(\zeta)e^{-j\zeta x + jk_1 z}\, d\zeta \\ \tilde{E}_y^t(\zeta) &= \int_{-\infty}^{\infty} E_y^t(x,0)e^{j\zeta x}\, dx \\ k_1 &= \sqrt{k_0^2 - \zeta^2} \end{aligned}$$

Note that $\tilde{E}_y^t(\zeta)$ and$E_y^t(x,0)$ are the Fourier-transformed pair.

Since $H_x(x,z) = -1/(j\omega\mu)\partial E_y(x,z)/\partial z$, the corresponding x components of the incident, reflected, and transmitted H-fields are

$$\begin{aligned} H_x^i(x,z) &= -\frac{\xi_p}{\omega\mu} sin a_p(x+a) exp(j\xi_p z) \\ H_x^r(x,z) &= \sum_{m=1}^{\infty} \frac{\xi_m c_m}{\omega\mu} sin a_m(x+a) exp(-j\xi_m z) \\ H_x^t(x,z) &= \frac{-1}{j\omega\mu}\frac{\partial}{\partial z}\int_{-\infty}^{\infty} \frac{1}{2\pi}\tilde{E}_y^t(\zeta)e^{-j\zeta x + jk_1 z}\, d\zeta \end{aligned}$$

To determine unknown coefficient c_m, it is necessary to match the boundary conditions of tangential E- and H-field continuities. First, the tangential E-field continuity along the irregular boundary (-$\infty < x < \infty, z = 0$) yields

$$\begin{aligned} E_y^t(x,0) &= E_y^i(x,0) + E_y^r(x,0) && |x| < a \\ &= 0 && |x| > a \end{aligned}$$

Taking the Fourier transform on the both sides of above equation, we get

$$\tilde{E}_y^t(\zeta) = \int_{-\infty}^{\infty} E_y^t(x,0)e^{j\zeta x}\, dx \tag{2.1}$$

$$= \int_{-a}^{a} [sin a_p(x+a) + \sum_{m=1}^{\infty} c_m sin a_m(x+a)]e^{j\zeta x}\, dx \tag{2.2}$$

Substituting (2.1) into (2.2), and performing integration with respect to x, we obtain

$$\tilde{E}_y^t(\zeta) = K_p(\zeta) + \sum_{m=1}^{\infty} c_m K_m(\zeta) \tag{2.3}$$

$$K_m(\zeta) = \frac{a_m}{(\zeta^2 - a_m^2)}[e^{j\zeta a}(-1)^m - e^{-j\zeta a}] \tag{2.4}$$

Second, the tangential H-field continuity along $-a < x < a, z = 0$, gives

$$\begin{aligned} H_x^t(x,0) &= H_x^i(x,0) + H_x^r(x,0) \\ \frac{1}{2\pi}\int_{-\infty}^{\infty} [K_p(\zeta) + \sum_{m=1}^{\infty} c_m K_m(\zeta)]k_1 e^{-j\zeta x}\, d\zeta &= \xi_p sin a_p(x+a) - \sum_{m=1}^{\infty} c_m \xi_m sin a_m(x+a) \end{aligned}$$

In order to determine the coefficient c_m, we multiply the above equation by $sina_n(x+a)$ and integrate the both sides with respect to x from $-a$ to a, then we obtain

$$\frac{1}{2\pi}[I_{pn} + \sum_{m=1}^{\infty} c_m I_{mn}] \quad = \quad \xi_p a \delta_{np} - \xi_n c_n a \tag{2.5}$$

where

$$I_{mn} = \int_{-\infty}^{\infty} \frac{a_m a_n[(-1)^m e^{j\zeta a} - e^{-j\zeta a}][(-1)^n e^{-j\zeta a} - e^{j\zeta a}]k_1}{(\zeta^2 - a_m^2)(\zeta^2 - a_n^2)} d\zeta$$

I_{mn} may be converted into a fast convergent integral using the the technique of the contour integration such as:

$$I_{mn} = 2\pi a \eta_m \delta_{mn} - (I_1 + I_2)$$

where δ_{mn} is the Kronecker delta, and $\eta_m = \sqrt{k_0^2 - a_m^2}$. The detailed evaluation of I_{mn} along with the expressions of I_1 and I_2 is given in [3]. The first term containing δ_{mn} is a residue contribution at $\zeta = \pm a_m$ whereas I_1, I_2 arise from the integration along the branch-cut which is associated with the branch-point at $\zeta = k_0$. When the operating frequency approaches infinity ($k_0 a \to \infty$), the branch-cut contribution becomes negligible,thus $I_{mn} \to 2\pi a \eta_m \delta_{mn}$.

Substituting I_{mn} into (2-5), we obtain the simultaneous equations for c_m. Solving the simultaneous equations, we may represent the c_m in the following matrix form:

$$C = (U - R)^{-1} Q = Q + RQ + R^2 Q + ...$$

where C is the column matrix of elements c_m, U is the identity matrix, R is the full matrix of elements r_{nm}, and Q is the column matrix of elements of q_n. The expressions of r_{nm}, q_n are given as:

$$\begin{aligned} r_{nm} &= \frac{(I_1 + I_2)|_{mn}}{2\pi(\xi_n + \eta_n)a} \\ q_n &= \frac{(\xi_p - \eta_p)\delta_{np}}{(\xi_n + \eta_n)} + \frac{(I_1 + I_2)|_{pn}}{2\pi a(\xi_n + \eta_n)} \end{aligned}$$

The examination of r_{nm} reveals that $r_{nm} \sim O[1/\sqrt{k_0 a}]$, for $k_0 a > 1$, and c_m may be given as

$$c_m = q_m(1 + O[1/\sqrt{k_0 a}])$$

In case the flanged waveguide is filled with the air ($i.e., k = k_0$), then $\xi_p = \eta_p$, thus

$$q_n \quad = \quad \frac{(I_1 + I_2)|_{pn}}{2\pi a(\xi_n + \eta_n)} \sim O(1/\sqrt{k_0 a})$$

This means that the reflection coefficient, $|c_m|^2$ is of the order $(1/(k_0 a))$. Fig. 2 shows the input impedance Z versus $2a/\lambda$ when p=1, where $Z = [(1 + c_1)/(1 - c_1)]^*$. Note that our computation well agrees with the results in [2], thus reconfirming the numerical accuracy of [1].

3 Concluding Remarks

Using the Fourier transform approach, we obtain the asymptotic series solution for the radiation problem of the flanged waveguide. The series solution is compared with several existing approximate results in order to clarify some ambiguities associated with numerical inaccuracies in the computation of the reflection coefficients.

References

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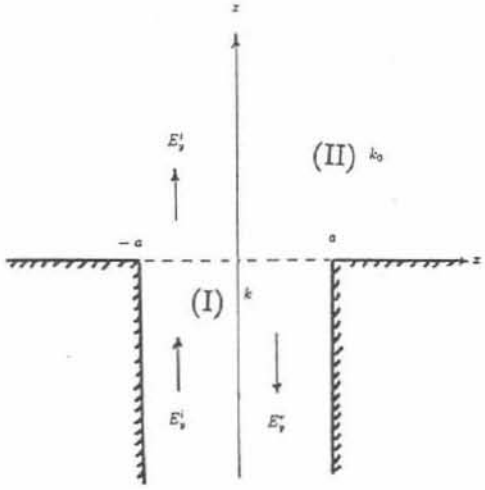

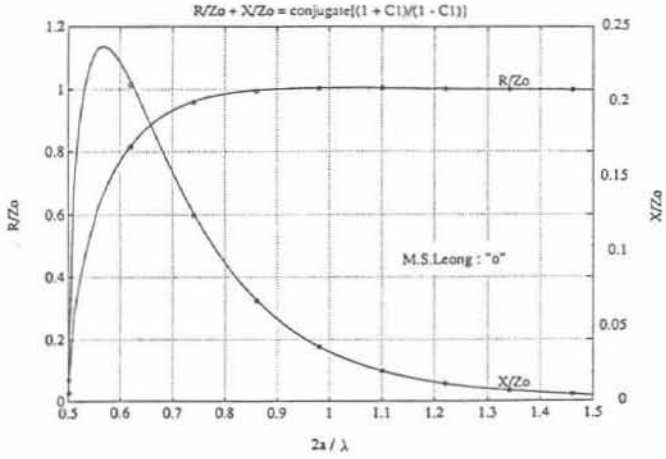

Fig. 1: Geometry of Radiation problem Fig. 2: Behavior of Input Impedance versus $2a/\lambda$.