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The Exact Formulation of a Flanged Rectangular Waveguide Antenna

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1. Introduction

A flanged rectangular waveguide is a typical radiating element of communication and radar systems and its radiation property has been studied as single and coupled radiators by a variety of methods [1]–[5]. It is well known that the cross-polarized component and higher-order modes on the waveguide aperture are indispensable for obtaining an accurate result and many authors have considered them. The inclusion of the edge property in the field is also effective in obtaining a highly accurate and fast convergent solution, but the solution derived by including the proper edge condition is rather sparse [3], particularly, the exact formulation for the case of two different mediums has not yet been reported.

In this paper, we exactly formulate the radiation of an electromagnetic wave from a flanged rectangular waveguide by applying the method of the Kobayashi Potential (KP), and we take into account the proper edge condition in the formulation. The process of analysis is as follows. The axial and transverse components of vector potentials are used to represent the fields in the waveguide and half-space, respectively. Enforcement of the required boundary conditions yields dual integral equations and they are solved by applying the discontinuous properties of the Weber-Schafheitlin (WS) integrals and the projection method. Finally, the problem is reduced to matrix equations whose matrix elements consist of double infinite integrals and series.

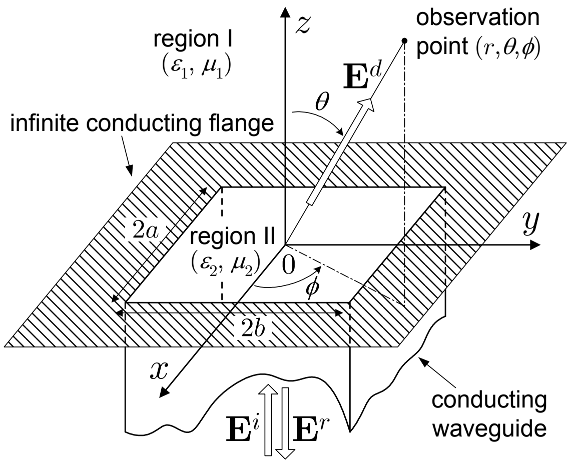

Figure 1: Radiation of an electromagnetic wave from a flanged rectangular waveguide.

2. Statement of the problem

Figure 1 shows the geometry of the problem. The dimension of the aperture is $2a \times 2b$ and its center is chosen as the origin O of (x, y, z). We assume that the infinite flange and waveguide are perfect conductors and the half-space (region I) and inside of waveguide (region II) are filled with isotropic and homogeneous lossless mediums with parameters $(\epsilon_1,\ \mu_1)$ and $(\epsilon_2,\ \mu_2)$, respectively. The problem is to determine the field $\mathbf{E}^d$ radiated from the waveguide into region I and to evaluate the reflected wave $\mathbf{E}^r$ in region II, when the waveguide is excited by TE- and/or TM-modes propagating in the positive z direction ($\mathbf{E}^i$ means the incident wave). In this analysis, the harmonic time dependence $\exp(j\omega t)$ is assumed.

2.1 Field in the waveguide

The fields in the waveguide are represented by a linear combination of the TE- and TM-modal functions and axial components of vector potentials F_z and A_z that satisfy the boundary conditions in

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the waveguide are:

$$\text{TE mode:} \quad \begin{pmatrix} F_z^i \\ F_z^r \end{pmatrix} = a\epsilon_2 \sum_{\substack{m=0\ n=0 \\ (m,n)\neq(0,0)}}^{\infty}\sum^{\infty} \begin{pmatrix} A_{mn}^{(E)} \exp(-jh_{mn}z_a) \\ B_{mn}^{(E)} \exp(jh_{mn}z_a) \end{pmatrix} \cos\frac{m\pi}{2}(\xi+1)\cos\frac{n\pi}{2}(\eta+1), \tag{1a}$$

$$\text{TM mode:} \quad \begin{pmatrix} A_z^i \\ A_z^r \end{pmatrix} = \frac{\kappa_2^2}{\omega} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{pmatrix} A_{mn}^{(M)} \exp(-jh_{mn}z_a) \\ B_{mn}^{(M)} \exp(jh_{mn}z_a) \end{pmatrix} \sin\frac{m\pi}{2}(\xi+1)\sin\frac{n\pi}{2}(\eta+1), \tag{1b}$$

$$h_{mn} = \sqrt{\kappa_2^2 - (m\pi/2)^2 - p^2(n\pi/2)^2}, \quad p = a/b\,(=1/q), \quad \kappa_2 = k_2 a, \quad k_2 = \omega\sqrt{\epsilon_2\mu_2}, \tag{1c}$$

where $\xi = x/a$, $\eta = y/b$, $z_a = z/a$ are the normalized variables.

2.2 Expression of the radiation field

For the radiated waves in the half-space, we use the x and y components of the electric vector potential $\mathbf{F}$ and they are given by Fourier spectral representations. The expressions include some unknown functions which are determined by the following boundary conditions.

$$E_x^d = 0, \quad E_y^d = 0, \quad (x,y) \in D^c, \quad z = +0, \tag{2a}$$

$$E_x^d = E_x^i + E_x^r, \quad E_y^d = E_y^i + E_y^r, \quad (x,y) \in D, \quad z = 0, \tag{2b}$$

$$H_x^d = H_x^i + H_x^r, \quad H_y^d = H_y^i + H_y^r, \quad (x,y) \in D, \quad z = 0. \tag{2c}$$

Here, $D = \{(x,y) \mid |x| < a,\ |y| < b\} \subset \mathbf{R}^2$ represents the domain of the aperture, and D^c is the region on the flange. Enforcement of the above conditions yields three kinds of integral equations and the equations from (2b) and (2c), which include the unknown coefficients $B_{mn}^{(E)}$, $B_{mn}^{(M)}$, can be combined by applying the orthogonality of the waveguide modes ($B_{mn}^{(E)}$, $B_{mn}^{(M)}$ are expressed as surface integrals of the products of the modal functions and aperture electric field that includes the unknown functions). Thus, we have dual integral equations related with the unknown functions (see [5]).

The equation derived from (2a) is satisfied by using the discontinuous property of the WS integrals:

$$\int_0^\infty \frac{J_{2n+\sigma}(t)}{t^\sigma}\cos xt\,dt = \begin{cases} 0 & (|x|>1) \\ \dfrac{(-1)^n\Gamma(\sigma)\Gamma(2n+1)}{2^{1-\sigma}\Gamma(2n+2\sigma)}(1-x^2)^{\sigma-\frac{1}{2}}C_{2n}^{(\sigma)}(x) & (|x|<1) \end{cases}, \tag{3a}$$

$$\int_0^\infty \frac{J_{2n+1+\sigma}(t)}{t^\sigma}\sin xt\,dt = \begin{cases} 0 & (|x|>1) \\ \dfrac{(-1)^n\Gamma(\sigma)\Gamma(2n+2)}{2^{1-\sigma}\Gamma(2n+2\sigma+1)}(1-x^2)^{\sigma-\frac{1}{2}}C_{2n+1}^{(\sigma)}(x) & (|x|<1) \end{cases}, \tag{3b}$$

where $C_\ell^\sigma(x)$ is the Gegenbauer polynomial. Thus, the final forms of F_x^d and F_y^d are given by

$$\begin{aligned} F_x^d = a\epsilon_1 \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\int_0^\infty\int_0^\infty \frac{1}{\zeta(\alpha,\beta)} \Big\{ &\Lambda_{2m}^\sigma(\alpha)\cos\alpha\xi \left[A_{mn}^{(x)}\Lambda_{2n}^\tau(\beta)\cos\beta\eta + B_{mn}^{(x)}\Lambda_{2n+1}^\tau(\beta)\sin\beta\eta\right] \\ &+\Lambda_{2m+1}^\sigma(\alpha)\sin\alpha\xi\left[C_{mn}^{(x)}\Lambda_{2n}^\tau(\beta)\cos\beta\eta + D_{mn}^{(x)}\Lambda_{2n+1}^\tau(\beta)\sin\beta\eta\right]\Big\}\exp\left[-\zeta(\alpha,\beta)\,z_a\right]d\alpha d\beta, \end{aligned} \tag{4a}$$

$$\begin{aligned} F_y^d = a\epsilon_1 \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\int_0^\infty\int_0^\infty \frac{1}{\zeta(\alpha,\beta)} \Big\{ &\Lambda_{2m}^\tau(\alpha)\cos\alpha\xi \left[A_{mn}^{(y)}\Lambda_{2n}^\sigma(\beta)\cos\beta\eta + B_{mn}^{(y)}\Lambda_{2n+1}^\sigma(\beta)\sin\beta\eta\right] \\ &+\Lambda_{2m+1}^\tau(\alpha)\sin\alpha\xi\left[C_{mn}^{(y)}\Lambda_{2n}^\sigma(\beta)\cos\beta\eta + D_{mn}^{(y)}\Lambda_{2n+1}^\sigma(\beta)\sin\beta\eta\right]\Big\}\exp\left[-\zeta(\alpha,\beta)\,z_a\right]d\alpha d\beta, \end{aligned} \tag{4b}$$

$$\Lambda_\ell^\nu(x) = J_{\ell+\nu}(x)/x^\nu, \quad \zeta(\alpha,\beta) = \sqrt{\alpha^2 + p^2\beta^2 - \kappa_1^2}, \quad \kappa_1 = k_1 a, \quad k_1 = \omega\sqrt{\epsilon_1\mu_1}. \tag{4c}$$

This is the Kobayashi potential for the present problem and σ and τ are selected so as to incorporate the edge property in the electric field [6]. When $\epsilon_1 = \epsilon_2$, $\mu_1 = \mu_2$, $\sigma = 7/6$ and $\tau = 1/6$ are selected. $A_{mn}^{(x)} \sim D_{mn}^{(x)}$ and $A_{mn}^{(y)} \sim D_{mn}^{(y)}$ are unknown and they are determined from the other integral equation in which the integrals of $B_{mn}^{(E)}$ and $B_{mn}^{(M)}$ are analytically performed by using the following formulas:

$$\int_{-1}^{1}\left[\int_0^\infty \frac{J_{2n+\sigma}(t)}{t^\sigma}\cos xt\,dt\right]\cos ax\,dx = \pi\frac{J_{2n+\sigma}(a)}{a^\sigma}, \tag{5a}$$

$$\int_{-1}^{1}\left[\int_0^\infty \frac{J_{2n+1+\sigma}(t)}{t^\sigma}\sin xt\,dt\right]\sin ax\,dx = \pi\frac{J_{2n+1+\sigma}(a)}{a^\sigma}. \tag{5b}$$

2.3 Matrix equation

The resultant equation is projected into a functional space of Jacobi's polynomials (or Gegenbauer polynomials) by using the orthogonality of the polynomials:

$$\cos p\xi = 2^{\nu}\Gamma(\nu)\sum_{n=0}^{\infty}(-1)^n(2n+\nu)\frac{J_{2n+\nu}(p)}{p^{\nu}}C_{2n}^{(\nu)}(\xi), \tag{6a}$$

$$\sin p\xi = 2^{\nu}\Gamma(\nu)\sum_{n=0}^{\infty}(-1)^n(2n+\nu+1)\frac{J_{2n+\nu+1}(p)}{p^{\nu}}C_{2n+1}^{(\nu)}(\xi), \tag{6b}$$

$$\int_{-1}^{1}(1-x^2)^{\nu-\frac{1}{2}}C_m^{(\nu)}(x)C_n^{(\nu)}(x)dx = \frac{\pi 2^{1-2\nu}\Gamma(2\nu+n)}{n!(n+\nu)\Gamma^2(\nu)}\delta_{mn}, \quad (\Re\nu > -\tfrac{1}{2}), \tag{6c}$$

where $\delta_{\ell\ell'}$ is the Kronecker delta. Thus, the matrix equation for the expansion coefficients $A_{mn}^{(x)} \sim D_{mn}^{(x)}$ and $A_{mn}^{(y)} \sim D_{mn}^{(y)}$ is finally obtained as follows.

$$\begin{aligned}
&\begin{bmatrix} K_{A(m,n,s,t)}^{(u,v)} + R_\mu S_{A(m,n,s,t)}^{(u,v)} & (-1)^{u+v}p\{G_{A(m,n,s,t)}^{(u,v)} + R_\mu T_{A(m,n,s,t)}^{(u,v)}\} \\ (-1)^{u+v}q\{G_{B(m,n,s,t)}^{(\bar{u},\bar{v})} + R_\mu T_{B(m,n,s,t)}^{(\bar{u},\bar{v})}\} & K_{B(m,n,s,t)}^{(\bar{u},\bar{v})} + R_\mu S_{B(m,n,s,t)}^{(\bar{u},\bar{v})} \end{bmatrix} \begin{bmatrix} X_{mn}^{(u,v)} \\ Y_{mn}^{(\bar{u},\bar{v})} \end{bmatrix} \\
&= 2jR_\mu(-1)^{u+v}\begin{bmatrix} P_{st}^{(u,v)} \\ Q_{st}^{(\bar{u},\bar{v})} \end{bmatrix}, \qquad \begin{cases} (u,v) = (0,0),\ (0,1),\ (1,0),\ (1,1) \\ s = 0,1,2,\cdots, \quad t = 0,1,2,\cdots \end{cases},
\end{aligned} \tag{7a}$$

$$X_{mn}^{(0,0)} = A_{mn}^{(x)}, \quad X_{mn}^{(0,1)} = B_{mn}^{(x)}, \quad X_{mn}^{(1,0)} = C_{mn}^{(x)}, \quad X_{mn}^{(1,1)} = D_{mn}^{(x)}, \tag{7b}$$

$$Y_{mn}^{(0,0)} = A_{mn}^{(y)}, \quad Y_{mn}^{(0,1)} = B_{mn}^{(y)}, \quad Y_{mn}^{(1,0)} = C_{mn}^{(y)}, \quad Y_{mn}^{(1,1)} = D_{mn}^{(y)}, \tag{7c}$$

where $\bar{u} = 1-u$, $\bar{v} = 1-v$, and $R_\mu = \mu_1/\mu_2$. The other symbols are defined by

$$\begin{aligned}
P_{st}^{(u,v)} &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^{m+n}\left(\tfrac{2m+1+u}{2}\pi\right)h_{2m+1+u,2n+v}A_{2m+1+u,2n+v}^{(E)}\Lambda_{2s+u}^{\tau'}\left(\tfrac{2m+1+u}{2}\pi\right)\Lambda_{2t+v}^{\sigma'}\left(\tfrac{2n+v}{2}\pi\right) \\
&+ p\kappa_2^2\sum_{m=0}^{\infty}\sum_{n=1-v}^{\infty}(-1)^{m+n}\left(\tfrac{2n+v}{2}\pi\right)A_{2m+1+u,2n+v}^{(M)}\Lambda_{2s+u}^{\tau'}\left(\tfrac{2m+1+u}{2}\pi\right)\Lambda_{2t+v}^{\sigma'}\left(\tfrac{2n+v}{2}\pi\right),
\end{aligned} \tag{8a}$$

$$\begin{aligned}
Q_{st}^{(u,v)} &= q\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^{m+n}\left(\tfrac{2n+1+v}{2}\pi\right)h_{2m+u,2n+1+v}A_{2m+u,2n+1+v}^{(E)}\Lambda_{2s+u}^{\sigma'}\left(\tfrac{2m+u}{2}\pi\right)\Lambda_{2t+v}^{\tau'}\left(\tfrac{2n+1+v}{2}\pi\right) \\
&- q^2\kappa_2^2\sum_{m=1-u}^{\infty}\sum_{n=0}^{\infty}(-1)^{m+n}\left(\tfrac{2m+u}{2}\pi\right)A_{2m+u,2n+1+v}^{(M)}\Lambda_{2s+u}^{\sigma'}\left(\tfrac{2m+u}{2}\pi\right)\Lambda_{2t+v}^{\tau'}\left(\tfrac{2n+1+v}{2}\pi\right).
\end{aligned} \tag{8b}$$

The matrix elements consist of double infinite integrals and double infinite series, which are given by

$$K_{A(m,n,s,t)}^{(u,v)} = \int_0^\infty\int_0^\infty \frac{\kappa_1^2-\alpha^2}{\zeta(\alpha,\beta)}\Lambda_{2m+u}^{\sigma}(\alpha)\Lambda_{2s+u}^{\tau'}(\alpha)\Lambda_{2n+v}^{\tau}(\beta)\Lambda_{2t+v}^{\sigma'}(\beta)d\alpha d\beta, \tag{9a}$$

$$K_{B(m,n,s,t)}^{(u,v)} = \int_0^\infty\int_0^\infty \frac{q^2\kappa_1^2-\beta^2}{\zeta(\alpha,\beta)}\Lambda_{2m+u}^{\tau}(\alpha)\Lambda_{2s+u}^{\sigma'}(\alpha)\Lambda_{2n+v}^{\sigma}(\beta)\Lambda_{2t+v}^{\tau'}(\beta)d\alpha d\beta, \tag{9b}$$

$$G_{A(m,n,s,t)}^{(u,v)} = \int_0^\infty\int_0^\infty \frac{\alpha\beta}{\zeta(\alpha,\beta)}\Lambda_{2m+1-u}^{\tau}(\alpha)\Lambda_{2s+u}^{\tau'}(\alpha)\Lambda_{2n+1-v}^{\sigma}(\beta)\Lambda_{2t+v}^{\sigma'}(\beta)d\alpha d\beta, \tag{9c}$$

$$G_{B(m,n,s,t)}^{(u,v)} = \int_0^\infty\int_0^\infty \frac{\alpha\beta}{\zeta(\alpha,\beta)}\Lambda_{2m+1-u}^{\sigma}(\alpha)\Lambda_{2s+u}^{\sigma'}(\alpha)\Lambda_{2n+1-v}^{\tau}(\beta)\Lambda_{2t+v}^{\tau'}(\beta)d\alpha d\beta, \tag{9d}$$

$$\begin{aligned}
S_{A(m,n,s,t)}^{(u,v)} &= \pi^2\sum_{m'=0}^{\infty}\sum_{n'=0}^{\infty}\frac{\kappa_2^2-\left(\frac{2m'+1+u}{2}\pi\right)^2}{(1+\delta_{0,2n'+v})\gamma_{2m'+1+u,2n'+v}} \\
&\times \Lambda_{2m+u}^{\sigma}\left(\tfrac{2m'+1+u}{2}\pi\right)\Lambda_{2s+u}^{\tau'}\left(\tfrac{2m'+1+u}{2}\pi\right)\Lambda_{2n+v}^{\tau}\left(\tfrac{2n'+v}{2}\pi\right)\Lambda_{2t+v}^{\sigma'}\left(\tfrac{2n'+v}{2}\pi\right),
\end{aligned} \tag{10a}$$

$$S_{B(m,n,s,t)}^{(u,v)} = \pi^2\sum_{m'=0}^{\infty}\sum_{n'=0}^{\infty}\frac{q^2\kappa_2^2-\left(\frac{2n'+1+v}{2}\pi\right)^2}{(1+\delta_{0,2m'+u})\gamma_{2m'+u,2n'+1+v}}$$

$$\times\,\Lambda^{\tau}_{2m+u}\left(\tfrac{2m'+u}{2}\pi\right)\Lambda^{\sigma'}_{2s+u}\left(\tfrac{2m'+u}{2}\pi\right)\Lambda^{\sigma}_{2n+v}\left(\tfrac{2n'+1+v}{2}\pi\right)\Lambda^{\tau'}_{2t+v}\left(\tfrac{2n'+1+v}{2}\pi\right), \tag{10b}$$

$$T^{(u,v)}_{A(m,n,s,t)}=\pi^2\sum_{m'=0}^{\infty}\sum_{n'=0}^{\infty}\frac{1}{\gamma_{2m'+1+u,2n'+v}}$$
$$\times\,\Lambda^{\tau}_{2m+1-u}\left(\tfrac{2m'+1+u}{2}\pi\right)\Lambda^{\tau'}_{2s+u}\left(\tfrac{2m'+1+u}{2}\pi\right)\Lambda^{\sigma}_{2n+1-v}\left(\tfrac{2n'+v}{2}\pi\right)\Lambda^{\sigma'}_{2t+v}\left(\tfrac{2n'+v}{2}\pi\right), \tag{10c}$$

$$T^{(u,v)}_{B(m,n,s,t)}=\pi^2\sum_{m'=0}^{\infty}\sum_{n'=0}^{\infty}\frac{1}{\gamma_{2m'+u,2n'+1+v}}$$
$$\times\,\Lambda^{\sigma}_{2m+1-u}\left(\tfrac{2m'+u}{2}\pi\right)\Lambda^{\sigma'}_{2s+u}\left(\tfrac{2m'+u}{2}\pi\right)\Lambda^{\tau}_{2n+1-v}\left(\tfrac{2n'+1+v}{2}\pi\right)\Lambda^{\tau'}_{2t+v}\left(\tfrac{2n'+1+v}{2}\pi\right), \tag{10d}$$

where $\gamma_{mn}=\sqrt{(m\pi/2)^2+p^2\,(n\pi/2)^2-\kappa_2^2}=jh_{mn}$. Parameters σ' and τ' are determined by considering the edge property of the magnetic field [6]. When $\epsilon_1=\epsilon_2$, $\mu_1=\mu_2$, $\sigma'=-1/6$ and $\tau'=5/6$ are selected.

2.4 Expression of the far field

The far-field expression can be obtained from Eq. (4) by applying the stationary phase method of integration, and the result is given by

$$F^d_x=\frac{\pi q a^2\epsilon_1}{2}\frac{\exp(-jk_1 r)}{r}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\Big\{\Lambda^{\sigma}_{2m}(\kappa_a)\left[A^{(x)}_{mn}\Lambda^{\tau}_{2n}(\kappa_b)+jB^{(x)}_{mn}\Lambda^{\tau}_{2n+1}(\kappa_b)\right]$$
$$+\,\Lambda^{\sigma}_{2m+1}(\kappa_a)\left[jC^{(x)}_{mn}\Lambda^{\tau}_{2n}(\kappa_b)-D^{(x)}_{mn}\Lambda^{\tau}_{2n+1}(\kappa_b)\right]\Big\}, \tag{11a}$$

$$F^d_y=\frac{\pi q a^2\epsilon_1}{2}\frac{\exp(-jk_1 r)}{r}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\Big\{\Lambda^{\tau}_{2m}(\kappa_a)\left[A^{(y)}_{mn}\Lambda^{\sigma}_{2n}(\kappa_b)+jB^{(y)}_{mn}\Lambda^{\sigma}_{2n+1}(\kappa_b)\right]$$
$$+\,\Lambda^{\tau}_{2m+1}(\kappa_a)\left[jC^{(y)}_{mn}\Lambda^{\sigma}_{2n}(\kappa_b)-D^{(y)}_{mn}\Lambda^{\sigma}_{2n+1}(\kappa_b)\right]\Big\}, \tag{11b}$$

$$\kappa_a=\kappa_1\sin\theta\cos\phi,\quad \kappa_b=q\kappa_1\sin\theta\sin\phi. \tag{11c}$$

The far electric field is computed by the following relations.

$$E_\theta\simeq j(k_1/\epsilon_1)(F_x\sin\phi-F_y\cos\phi),\qquad E_\phi\simeq j(k_1/\epsilon_1)\cos\theta(F_x\cos\phi+F_y\sin\phi). \tag{12}$$

3. Conclusion

We derived the exact solution of radiation field from a flanged rectangular waveguide by using the KP that satisfies the proper edge condition. The problem was reduced to the matrix equation for the expansion coefficients of the diffracted field, and the matrix elements were given by double infinite integrals and double infinite series. By using the derived solution, we can obtain highly accurate numerical results of physical quantities. Since our results can be extended to the problem of a rectangular waveguide array, the rigolous treatment of mutual coupling between the elements will be made in the near future.

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