# Coupling Through a Slit in a Flanged Parallel-Plate Waveguide with a Conducting Strip 

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## 1. Introduction

The radiation characteristics of a flanged parallel-plate waveguide (FPPW) with objects in the vicinity of the aperture have been investigated by some authors [1,2], because one may obtain the physical insights of the radiation properties of more practical structures such as waveguide-fed slot antenna[3], short backfire antenna[4], etc. In the most of the studies, however, it seems that the role of a nearby (conducting) scatterer on the radiation properties of the structure and the maximum coupling (or impedance matching) mechanism have not been discussed throughly.

In this study, the problem of coupling through a slit in a FPPW to a conducting strip near the aperture is considered for the purpose of gaining an insight into the impedance matching (or maximum coupling) phenomena through the investigation of the equivalent admittance properties of the coupling slit and complex powers in the geometry under consideration. Furthermore, the effects of the guide height and the slit width on the radiation properties have been examined along with discussions about the maximum coupling phenomena through a narrow slit due to a nearby conducting strip.

## 2. Theory

Fig. 1 shows a cross sectional view of a flanged parallel-plate waveguide (FPPW) opening into the half ( $\mathrm{z}>0$ ) free space $\left(\mu_{0}, \epsilon_{0}\right.$ ) where is located a conducting strip parallel to the ground plane ( $\mathrm{z}=0$ ). The y-component TEM magnetic field incident on the aperture region can be given by $H_{y}^{\text {inc }}=V_{\text {inc }} /(\eta h) \exp [-j k z]$ in which $V_{\text {inc }}$ is the potential difference across the plates, $k\left(=k_{0} \sqrt{\epsilon_{r}}=2 \pi / \lambda, k_{0}=\omega \sqrt{\mu_{0} \epsilon_{0}}=2 \pi / \lambda_{0}\right)$ is the wavenumber, and $\eta\left(=\eta_{0} / \sqrt{\epsilon_{r}}, \eta_{0}=\sqrt{\mu_{0} / \epsilon_{0}}\right)$ is the intrinsic impedance of the dielectric inside the FPPW.

Following Butler et al. [2], the present problems can be formulated in terms of coupled integro-differential equations for the induced electric current density $J_{x}(x)$ on the conducting strip and the magnetic current density $M_{y}(x)$, equivalent to the slit electric field $E_{x}^{A}(x) \equiv E_{x}(x, 0)$, over the shorted slit as

$$
\begin{align*}
& \left.\left(H_{y}^{J_{0}}+H_{y}^{M_{0}}\right)\right|_{z=0_{+}}=\left.\left(H_{y}^{M}+H_{y}^{s c}\right)\right|_{z=0_{-}},\left|x-X_{a}\right|<a / 2 \quad \text { (across the slit) }  \tag{1}\\
& \quad E_{x}^{J_{0}}(\varrho)+E_{x}^{M_{0}}(\varrho)=0, \varrho \in C \quad \text { (on the surface of strip). } \tag{2}
\end{align*}
$$

Here, the subscript $\mathrm{x}(\mathrm{y})$ indicates an $\mathrm{x}(\mathrm{y})$ component, $H_{y}^{J_{0}}\left(H_{y}^{M_{0}}\right)$ is the magnetic field in the free space due to $J_{x}\left(-M_{y}\right), H_{y}^{M}$ is the magnetic field inside the FPPW due to $M_{y}$ over the shorted slit, and $H_{y}^{s c}$ represents the short circuit magnetic field given by $2 V_{\mathrm{inc}} /(\eta h) \cos k z$. The detailed expressions for the terms in (1) and (2) are given in [2]. In order to solve the equations (1) and (2) numerically using moment method, the unknown distributions $J_{x}\left(x^{\prime}\right)$ and $E_{x}^{A}\left(x^{\prime}\right)$ are expanded in terms of piecewise sinusoidal functions and pulse functions, respectively, and Galerkin's scheme is employed to reduce (1) and (2) to a linear equation system.

From knowledge of the distributions, one may obtain all the quantities of interests such as the voltage reflection coefficient $\Gamma_{V}$ from the slit, the equivalent slit admittance $Y_{s}\left(=G_{s}+j B_{s}\right)$, the reflected power $P_{r}$ from the slit, the coupled power $P_{s}$ through the slit into the exterior free space region, and radiation pattern. All the power quantities are normalized to the incident TEM mode power $P_{i}$, for convenience, by setting $P_{i}=1$.

Equivalent circuit representation [5] for the TEM mode in the guide is depicted in Fig. 2 where
$Y_{C}(=1 / \eta h)$ is the characteristic admittance and $y_{s}\left(=Y_{s} / Y_{C}\right)$ is the normalized equivalent slit admittance obtained from the voltage reflection coefficient $\Gamma_{V}$ as

$$
\begin{equation*}
y_{s}=g_{s}+j b_{s}=\left(1-\Gamma_{V}\right) /\left(1+\Gamma_{V}\right) \tag{3}
\end{equation*}
$$

The equivalent slit susceptance $b_{s}$ is associated with the nonpropagating reactive fields in the vicinity of the slit and so it could be expressed as the sum of $b_{s i}$ and $b_{s e}$, as shown in Fig. 2, in which the reactive powers inside and outside the guide regions are taken into account, while the coupled (or radiated) power $P_{s}\left(=P_{i}-P_{r}\right)$ through the slit into the free space reflected in the conductance $g_{s}$.

In order to explain the role of the conducting strip in achieving impedance matching (or maximum coupling) in the geometry of Fig. 1, radiation and equivalent admittance properties of a slit in a FPPW in the absence of the nearby conducting strip should be examined. The reactive power $Q_{\mathrm{in}}\left(Q_{\mathrm{ex}}\right)$ in the region $V_{\text {in }}\left(V_{\text {ex }}\right)$, see Fig. 3, inside (outside) the PPW can be computed by integrating complex Poynting vector $\left(\underline{E} \times \underline{H^{*}}\right)$ over the surface $S_{\mathrm{i}}\left(S_{\mathrm{e}}\right)$ enclosing $V_{\mathrm{in}}\left(V_{\mathrm{ex}}\right)$ as

$$
\begin{equation*}
Q_{\mathrm{in}(\mathrm{ex})}=\frac{1}{2} \operatorname{Im}\left\{\oint_{\mathrm{s}_{\mathrm{i}(e)}} \underline{\mathrm{E}} \times \underline{\mathrm{H}^{*}} \cdot \underline{\mathrm{ds}}\right\} \tag{4}
\end{equation*}
$$

Then the reactive powers in the lossless medium $V_{\text {in }}$ and $V_{\text {ex }}$ could be expressed by

$$
\begin{equation*}
Q_{\mathrm{in}}=\frac{1}{2} \operatorname{Im}\left\{\sum_{m=1}^{N} \sum_{n=1}^{N} V_{m}\left(V_{n} Y_{m n}^{h}\right)^{*}\right\} \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
Q_{\mathrm{ex}}=\Gamma_{V}-\Gamma_{V}^{*}-Q_{\mathrm{in}} \tag{6}
\end{equation*}
$$

in which

$$
\begin{equation*}
Y_{m n}^{h}=j \Delta^{2} \frac{2 k}{\eta h} \sum_{q=1}^{\infty} \frac{\cos \left[\left(x_{m}+0.5 h\right) q \pi / h\right] \cos \left[\left(x_{n}+0.5 h\right) q \pi / h\right]}{\sqrt{(q \pi / h)^{2}-k^{2}}}\left[\frac{\sin (q \pi \Delta / 2 h)}{(q \pi \Delta / 2 h)}\right]^{2} \tag{7}
\end{equation*}
$$

with $\quad \Gamma_{V}=\sum_{n=1}^{N} V_{n} \Delta / V_{\mathrm{inc}}, x_{n}=X_{a}-a / 2+(n-0.5) \Delta, \quad \Delta=a / N$, and total segment number $N$. Accordingly total reactive power in the vicinity of the slit is $Q_{\mathrm{slit}}=Q_{\mathrm{in}}+Q_{\mathrm{ex}}=2 \operatorname{Im}\left\{\Gamma_{\mathrm{V}}\right\}$. Now the equivalent slit susceptance $b_{s i}$ and $b_{s e}$ could be given as

$$
\begin{equation*}
b_{s i(s e)}=\frac{Q_{\mathrm{in}(\mathrm{ex})}}{Q_{\mathrm{slit}}} b_{s} \tag{8}
\end{equation*}
$$

If the slit is narrow relative to the guide height $h$ as well as the guide wavelength $\lambda$ and the electric field distribution is assumed to be uniform over the slit, then the uniform electric field $E_{s}$ can be obtained as

$$
\begin{equation*}
E_{s}=\frac{2 V_{\mathrm{inc}} / a}{A+j\left(B_{i}+B_{e}\right)}=\frac{2 V_{\mathrm{inc}} / a}{A+j B} \tag{9}
\end{equation*}
$$

in which
$A=1+\frac{k_{o} \eta h}{2 \eta_{o}}, B_{i}=2 k \sum_{q=1}^{\infty} \frac{\cos ^{2}\left[q \pi\left(X_{a}+h / 2\right) / h\right]}{\sqrt{(q \pi / h)^{2}-k^{2}}}\left[\frac{\sin (q \pi a / 2 h)}{(q \pi a / 2 h)}\right]^{2}, B_{e}=\frac{k_{0} \eta h}{\eta_{0} \pi}\left(\log \frac{2}{k_{0} a 1.781}+1.5\right)$.
Now the equivalent admittance $y_{s}$ of a narrow slit is

$$
\begin{equation*}
y_{s}=g_{s}+j b_{s}=\left(1-\Gamma_{V}\right) /\left(1+\Gamma_{V}\right)=(A-1)+j\left(B_{i}+B_{e}\right) \tag{10}
\end{equation*}
$$

where $\Gamma_{V}=E_{s} a / V_{\mathrm{inc}}-1$. From (10), one finds that the impedance matching ( $y_{s}=1+j 0$ ) for the case of narrow slit requires the conditions $A=2$ and $B=0$ (or $B_{e}=-B_{i}$ ), although both of them could not be satisfied without the nearby scatterer (conducting strip). Hence, in order to achieve the impedance matching, the arrangement of a scatterer in the vicinity of the narrow slit should bring a new complex term $A_{s}+j B_{s}$ to be appended to the denominator $A+j B$ in (9) as

$$
\begin{equation*}
E_{s}=\frac{2 V_{\mathrm{inc}} / a}{\left(A+A_{s}\right)+j\left(B+B_{s}\right)}=\frac{2 V_{\mathrm{inc}} / a}{A^{\prime}+j B^{\prime}} \tag{11}
\end{equation*}
$$

The equivalent slit admittance, owing to a nearby scatterer, would be changed to $y_{s}=\left(A^{\prime}-1\right)+j B^{\prime}$ where $b_{s i}$ remained to be $B_{i}$ while $b_{s e}$ is modified to $B_{e}+B_{s}$. For the case that $A^{\prime}=2$ and $B=0$ in (11), the slit admittance is matched to the characteristic admittance of the feed line (i.e., $y_{s}=1+j 0$ ) and the maximum coupling ( $P_{s} \simeq P_{i}$ ) would occur. In order for the slit to be resonated, $B$ in (11) would vanish and then $B_{s}$ should be negative (inductive) because both $B_{i}$ and $B_{e}$ are positive (capacitive), which means that the nearby scatterer should append the required inductive susceptance $B_{s}$ to the geometry having capacitive susceptance component $B\left(=B_{i}+B_{e}\right)$ and small conductance $A-1$ for the case of no nearby scatterer.

## 3. Results and Discussions

The equations (1) and (2) have been solved numerically using moment method by expanding $J_{x}$ and $E_{x}^{A}$ in terms of piecewise sinusoidal functions and pulse functions, respectively, and employing Galerkin's scheme. In Fig. 3, the normalized slit susceptances $b_{s}\left(=b_{s i}+b_{s e}\right)$ are plotted for different aperture-to-guide height ratios; $a / h=0.1,0.4$, and 1 . From the results for $b_{s i}$ and $b_{s e}$ in Fig. 3, one finds that the reactive power $Q_{\mathrm{in}}$ in the waveguide region is much smaller than $Q_{\mathrm{ex}}$ in the free space region as far as the ratio $a / h$ is near unity. However, as the ratio $a / h$ is decreased, the portion of $b_{s i}$ as well as the susceptance $b_{s}$ is increased. It is interesting to note that, for the case of thin waveguide (i.e., $h / \lambda_{0} \ll 1$ ), the susceptance $b_{s e}$ is much greater than $b_{s i}$, irrespective of the ratio $a / h$, as shown in Fig. 3. Another interesting point is that, within the limit of $k a \rightarrow 0, b_{s}$ logarithmically approached to infinity so as to give the voltage reflection coefficient $\Gamma_{v}=-1$ (as it should, for a short ended circuit) while $g_{s}$ remained constant [see (10)], regardless of the slit width $a$, though the divergent behaviors of $b_{s}$ depend on the guide height $h$, similarly as those of a narrow transverse slit in a PPW $[5,7]$.

As the slit width approaches to zero, the unnormalized conductance $G_{s}\left(=g_{s} Y_{C}\right)$ of a thin slit converges to, irrespective of $a / h, 1 / 120 \lambda_{0}$ which has also been observed in the problems of the equivalent series admittance of a narrow transverse slit and the equivalent load admittance of a wide transverse slit in a PPW, in the prior works $[5,7]$.

The effect of slit offset $X_{a}$ on the slit susceptances $b_{s i}$ and $b_{s e}$ for the case of $\epsilon_{r}=1, a / h=0.1$, and $h / \lambda_{0}=0.4$ is shown in Fig. 4 where $b_{s i}$ is increased along with the slit offset $X_{a}$ while $b_{s e}$ is remained as it is.

The equivalent slit admittances for the case that $\epsilon_{r}=1$ and $a / h=1$ are plotted in Fig. 5 where our results agree fairly well with the data in [8] which have been obtained under the assumption that the field distribution in the slit is of the form of the incident TEM wave.

From the above results for the slit admittance, one finds that, in most cases, the slit is impedance mismatched with the feed waveguide especially when the guide height is low or the slit is narrow in a thick waveguide, since the slit has small conductance $g_{s}$ and its susceptance $b_{s}$ does not vanish for the impedance matching $\left(g_{s}=1\right.$ and $\left.b_{s}=0\right)$. Hence it is needed to place some types of scatterers in the vicinity of the slit so that it may be impedance matched to the feed waveguide, as discussed in the foregoing section.

However, it should be noted that the FPPW with large guide height itself behaves as an efficient radiator without conducting strip [for reference, $P_{s} \geq \%$ when $k_{0} h \geq 7, \epsilon_{r}=1$, and $a / h=1$ ], though the impedance matching performance could be improved due to the conducting strip. As an example, maximum coupling(i.e., $P_{s} \simeq P_{i}$ or $y_{s} \simeq 1+\mathrm{j} 0$ ) has been observed for the case that $\epsilon_{r}=1, h=0.29 \lambda_{0}$, $a / h=1, L=0.29 \lambda_{0}, X_{0}=0$, and $Z_{0}=0.227 \lambda_{0}$, where the antenna gain(4.91dBi) is improved from 3.35 dBi without strip ( $P_{s}=91.6 \%$ and $y_{s}=0.77+\mathrm{j} 0.48$ ). In this case, the ratio of -10 dB impedance bandwidth $\Delta f$, over which $P_{s} / P_{i}$ is greater than $90 \%$, to the maximum coupling frequency $f_{0}$ is $\Delta f / f_{0} \simeq 63 \%$ and the antenna gain is greater than 3.65 dBi over the bandwidth. Hence the maximum coupling structure of this type is useful for wideband application.

On the contrary, the maximum coupling structures which consist of a narrow slit, regardless of $a / h$, and a nearby conducting strip have been found to be useful for the high gain antenna application at the expense of the frequency bandwidth. Impedance matching has been achieved, for example, when $h=0.29 \lambda_{0}, a / h=0.1, X_{0}=0, Z_{0}=0.093 \lambda_{0}$, and $L=0.519 \lambda_{0}$, in which case the structure has the antenna gain greater than 6.63 dBi over the -10 dB impedance bandwidth $\Delta f / f_{0} \simeq 11.4 \%$.

Another method for obtaining impedance matching of a narrow slit by use of a nearby conducting strip is to adjust the lateral displacement $X_{0}$ of the strip center for given strip length $L$ and distance $Z_{0}$ from a slit. Maximum coupling phenomena similar to the cavity-type coupling phenomena [9] occurred in the coupling problem through a narrow transverse slit in a PPW with a conducting strip have also been observed in the present geometry of Fig. 1, as expected.

## 4. Conclusion

The problem of electromagnetic coupling through a slit in a flanged parallel-plate waveguide to a nearby conducting strip has been considered with main interests centering on the impedance matching mechanism or maximum coupling phenomenon. The equivalent slit susceptances associated with reactive powers near the coupling slit inside and outside the guide have been obtained. In addition, the effects of the conducting strip on the maximum coupling have been discussed from the viewpoints of equivalent admittance properties and complex powers.

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## References

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Fig. 1. Geometry.


Fig. 3. Slit susceptance.


Fig. 2. Equivalent circuit.


Fig. 4. Slit susceptance.


Fig. 5. Slit admittance.

