CHARACTERISTICS OF A GAUSSIAN BEAM REFLECTED AND TRANSMITTED FROM AN ANISOTROPIC DIELECTRIC SLAB

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1. Introduction

Anisotropic media are used as polarizers, rotators and retardation plates in optical devices. The transmission and reflection from an anisotropic media has been already studied for the incident plane wave[1][2]. However, the interaction of Gaussian beams with an anisotropic media must be studied because these devices are operated under many laser systems in modern optical systems.

One of our authors preliminarily presented a method to analyze the reflection and transmission of a Gaussian beam incident upon a uniaxial anisotropic slab with arbitrary optical axis[3][4]. Landry et. al investigated the Gaussian beam transmission and reflection from a general anisotropic multilayer structure[5]. However, the characteristics of a Gaussian beam at an anisotropic dielectric slab should be studied in more detail.

In this paper, the transmission and reflection of a Gaussian beam from an anisotropic slab with arbitrary optical axis are presented. First, the wave equation is solved using the spectral domain method and Gaussian beams are represented by Fourier transform. Second, applying the boundary conditions, the TE and TM reflection and transmission coefficients are obtained in 2 × 2 matrix format. The final expressions of the beams are given in the forms of integrals and those are evaluated numerically using the FFT algorithm. The power field variations of transmitted and reflected beams are discussed for the thickness of slab.

2. Analysis

2.1 Expression of an incident beam

As shown in Fig.1(a), we consider the reflection and transmission of a Gaussian beam launched into an anisotropic slab. The boundary plane is the xy plane (at z=0, d), media 1 and 3 are isotropic media with the refraction indices n_i and n_t, and media 2 (thickness d) is anisotropic and its ordinary and extraordinary refraction indices are n_o and n_e, respectively. Let ***s*** and ***p*** denote the unit vectors for the electric field polarization; **k** denotes wave vectors of the beams; + and – denote the forward and backward waves in the slab. The wave source is *h* away from the boundary and the basic mode Gaussian beam is incident at angle θ. The time variation $\exp(j\omega t)$ has been suppressed.

We consider that the optical axis ***c*** of the anisotropic medium has an arbitrary direction. As shown in Fig. 1(b), it is assumed that ***c*** makes angle δ with the z-axis and the projection of ***c*** on the xy-plane makes angle ϕ with the x-axis.

$$\mathbf{c} = \cos\phi\sin\delta\,\mathbf{x} + \sin\phi\sin\delta\,\mathbf{y} + \cos\delta\,\mathbf{z} \tag{1}$$

From the Maxwell equations, the wave equation of incident field $\mathbf{E}_i$ in the incident coordinate system (x_i, y_i, z_i) is given by

$$\frac{\partial^2 E^i}{\partial x_i^2} + \frac{\partial^2 E^i}{\partial y_i^2} + \frac{\partial^2 E^i}{\partial z_i^2} + k_i^2 E^i = 0 \tag{2}$$

where $k_i = n_i\omega / c_0$ is the magnitude of the wave factor in the incident region and c_0 is the speed of light in vacuum. To solve the Eq.(2), we use the following Fourier transform:

$$E^i(x_i, y_i, z_i) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_i(\alpha, \beta, z_i)e^{-j(\alpha x_i + \beta y_i)}d\alpha d\beta \tag{3}$$

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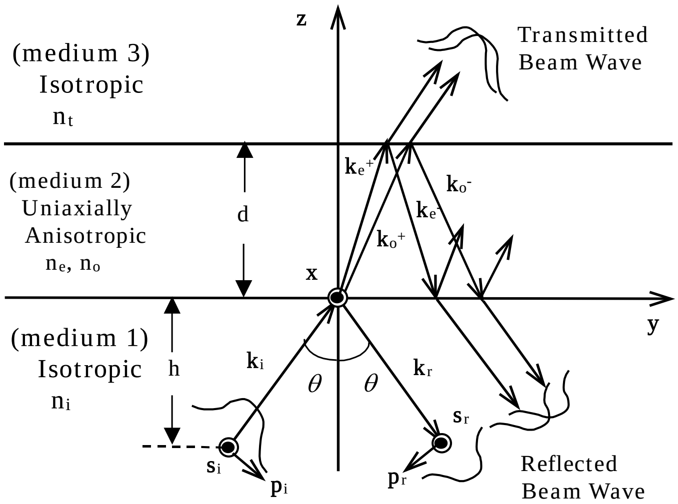


Fig. 1(a) Uniaxially anisotropic slab and beams.

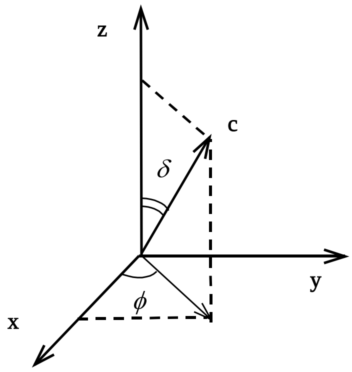


Fig. 1(b) Direction of the optical axis **c**.

By substituting Eq.(3) into Eq.(2), we obtain

$$\frac{d^2 E_i}{dz_i^2} + (k_i^2 - \alpha^2 - \beta^2) E_i = 0 \tag{4}$$

Eqs.(2) and (4) is solved using Fourier transforms and transformation of coordinates between (x_i, y_i, z_i) and (x, y, z). The incident beam is, therefore, given in the general form by Eq.(5) where the polarization direction of the field are considered and it is expressed as the sum of the $\mathbf{s}_i$ (TE wave) and $\mathbf{p}_i$ (TM wave) whose amplitudes are A_s and A_p, respectively.

$$\mathbf{E}^i(x,y,z) = \frac{w_1 w_2}{4\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (A_s \mathbf{s}_i + A_p \mathbf{p}_i) X(\alpha,\beta) \exp\{-j(\mathbf{k}_i \cdot \mathbf{r} + \gamma h / \cos\theta)\} d\alpha d\beta \tag{5a}$$

$$X(\alpha,\beta) = \exp\{-({w_1}^2\alpha^2 + {w_2}^2\beta^2)/4\} \tag{5b}$$

where w_1 and w_2 are the spot size of the x_i and y_i axis, respectively. Here, $\mathbf{k}_i$ is the wave vector of the incident beam, $\mathbf{s}_i$ is a unit vector perpendicular to both **y** and $\mathbf{k}_i$, on the other hand, $\mathbf{p}_i$ is a unit vector perpendicular to both $\mathbf{k}_i$ and $\mathbf{s}_i$.

$$\mathbf{s}_i = \frac{\mathbf{y} \times \mathbf{k}_i}{|\mathbf{y} \times \mathbf{k}_i|}, \quad \mathbf{p}_i = \frac{\mathbf{k}_i \times \mathbf{s}_i}{|\mathbf{k}_i \times \mathbf{s}_i|} \tag{6}$$

2.2 Expression of the reflected and transmitted beams

Consider the phase matching and the radiation conditions at the boundaries (z=0 and d). The reflected and transmitted waves as well as the ordinary and extraordinary in the anisotropic medium are assumed to be

[reflected wave]

$$\mathbf{E}^r(x,y,z) = \frac{w_1 w_2}{4\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (B_s \mathbf{s}_r + B_p \mathbf{p}_r) X(\alpha,\beta) \exp\{-j(\mathbf{k}_r \cdot \mathbf{r} + \gamma h / \cos\theta)\} d\alpha d\beta \tag{7}$$

[ordinary and extraordinary waves]

$$\begin{aligned}\mathbf{E}^m(x,y,z) = \frac{w_1 w_2}{4\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &(C_o \mathbf{o}_+ e^{-j\mathbf{k}_o^+ \cdot \mathbf{r}} + C_e \mathbf{e}_+ e^{-j\mathbf{k}_e^+ \cdot \mathbf{r}} + D_o \mathbf{o}_- e^{-j\mathbf{k}_o^- \cdot \mathbf{r}} + D_e \mathbf{e}_- e^{-j\mathbf{k}_e^- \cdot \mathbf{r}}) \\ &\times X(\alpha,\beta) \exp\{-j\gamma h / \cos\theta)\} d\alpha d\beta\end{aligned} \tag{8}$$

[transmitted wave]

$$\mathbf{E}^t(x,y,z) = \frac{w_1 w_2}{4\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (E_s \mathbf{s}_t + E_p \mathbf{p}_t) X(\alpha,\beta) \exp\{-j(\mathbf{k}_t \cdot \mathbf{r} + \gamma h / \cos\theta)\} d\alpha d\beta \tag{9}$$

where $\mathbf{s}_r$, $\mathbf{p}_r$, $\mathbf{s}_t$, and $\mathbf{p}_t$ are unit vectors of reflected and transmitted waves in the isotropic medium, on the other hand, $\mathbf{o}_\pm$ and $\mathbf{e}_\pm$ are unit vectors for the polarization directions of the ordinary and extraordinary waves in the anisotropic medium:

$$\mathbf{s}_r = \frac{-\mathbf{y}\times\mathbf{k}_r}{|-\mathbf{y}\times\mathbf{k}_r|},\ \ \mathbf{p}_r = \frac{\mathbf{k}_r\times\mathbf{s}_r}{|\mathbf{k}_r\times\mathbf{s}_r|},\ \ \mathbf{s}_t = \frac{-\mathbf{y}\times\mathbf{k}_t}{|-\mathbf{y}\times\mathbf{k}_t|},\ \ \mathbf{p}_t = \frac{\mathbf{k}_t\times\mathbf{s}_t}{|\mathbf{k}_t\times\mathbf{s}_t|} \tag{10}$$

$$\mathbf{o}_\pm = \frac{\mathbf{k}_o{}^\pm\times\mathbf{c}}{|\mathbf{k}_o{}^\pm\times\mathbf{c}|},\ \ \mathbf{e}_\pm = \frac{(\mathbf{k}_e{}^\pm\times\mathbf{c})\times\mathbf{P}_e{}^\pm}{|(\mathbf{k}_e{}^\pm\times\mathbf{c})\times\mathbf{P}_e{}^\pm|},\ \ \mathbf{P}_e^\pm = (n_e^2 - n_o^2)(\mathbf{k}_e^\pm\cdot\mathbf{c})\mathbf{c} + n_o^2\mathbf{k}_e^\pm \tag{11}$$

Also, the magnetic field is given by

$$\mathbf{H} = \frac{1}{\omega\mu_0}\mathbf{k}\times\mathbf{E} \tag{12}$$

In Eqs.(7)-(9), *B, C, D* and *E* (functions of α, β) are the unknown coefficients of the electric field amplitudes and are determined from the continuity conditions in term of x, y components of the electric and magnetic fields at boundary surfaces (z=0 and *d*).

Since the integral form of the beam wave cannot be found analytically, the FFT is used to evaluate the integrals numerically.

3. Numerical results

Figs. 2(a)-(d) show the normalized power field distributions of the transmitted and reflected beams due to the *s*-polarized incident Gaussian beam whose beam spot size *w* is 988λ (the beam center angle of incidence $\theta = 30°$) for the slab thickness *d*. The coordinate axes are normalized by the wavelength λ. The media 1 and 3 are chosen to be free space ($n_i = n_t = 1.0$) and the medium 2 is chosen to be a calcite (negative crystal; $n_0 = 1.6565$ and $n_e = 1.4857$) and optical axis direction is taken to be $\delta = 90°$ and $\phi = 45°$.

Here Eqs.(7) and (9) were respectively transformed into reflected coordinates (x_r, y_r, z_r) and transmitted coordinates (x_t, y_t, z_t) from coordinates (x, y, z). The parameters are adopted for the FFT calculations as the following:

Sampling point number N_α (along x_r or x_t) and N_β (along y_r or y_t)

$$N_\alpha = 32,\ \ N_\beta = 256 \tag{13}$$

Sampling interval Δt_α (along x_r or x_t) and Δt_β (along y_r or y_t)

$$\Delta t_\alpha = \frac{4\pi}{N_\alpha w/\lambda},\ \ \Delta t_\beta = \frac{4\pi}{N_\beta w/\lambda} \tag{14}$$

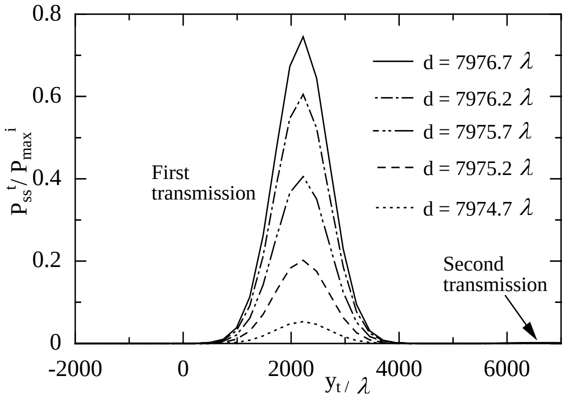
Also, the normalized power field is calculated by

$$P_{ss,sp}^{r,t} = \left| E_{ss,sp}^{r,t} \big/ E_{\max}^{i} \right|^2 \tag{15}$$

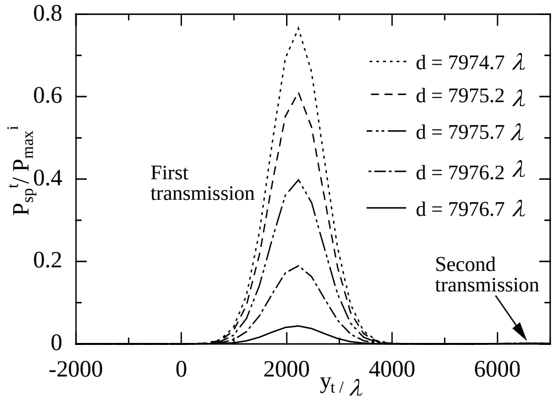
It is found that the reflected wave has two peaks (first and second reflection) as shown in Fig. 2(c). The first reflection wave is a reflected wave at the boundary between media 1 and 2 when a beam wave is incident. The second reflection wave is a reflected wave that is reflected once at the boundary between the media 2 and 3 then emerges out of media 1.

Corresponding to the reflected wave, the first transmission wave is seen as shown in Figs. 2(a)(b), however, the second transmission wave almost equals to zero. In this case, the first transmission wave involves the ordinary and extraordinary waves that depend on the relation between an optical axis and an incident beam polarization. The power field P_{ss}^t varies from minimum for d=7974.7 to maximum for d=7976.7 and it varies periodically for the slab thickness *d*. Conversely, P_{sp}^t varies from maximum for d=7974.7 to minimum for d=7976.7, that is, the variation changes by half a period.

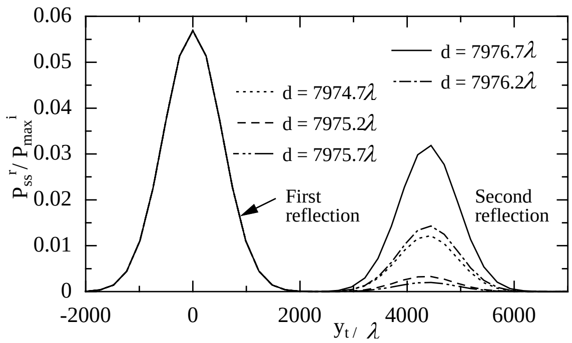
On the other hand, the first reflection wave does not change for the slab thickness *d* because it does not propagate through an anisotropic dielectric slab. The second reflection P_{ss}^r changes by quarter a period compared with P_{ss}^t. Also, P_{sp}^r changes by half a period compared with P_{ss}^t.



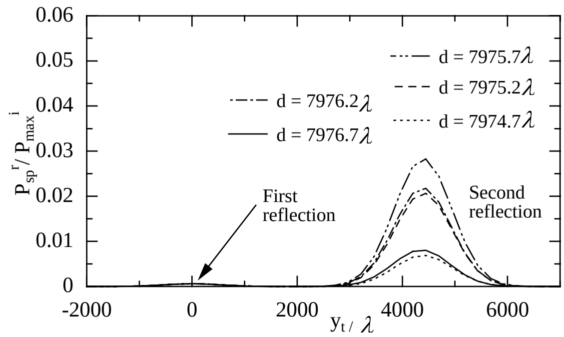
(a) P_{ss}^{t}



(b) P_{sp}^{t}



(c) P_{ss}^{r}



(d) P_{sp}^{r}

Fig. 2 The normalized power field distributions of transmission and reflection beams for the slab thickness d when s polarization is launched into calcite ($n_0 = 1.6565$, $n_e = 1.4857$, $\phi = 45°$, $\delta = 90°$) with incident beam angle $\theta = 30°$ and beam width w=500 λ.

4. Conclusions

The characteristics of reflected and transmitted beams when the three dimensional Gaussian beam is launched into the anisotropic dielectric slab whose optical axis has arbitrary direction is described. Solutions were numerically calculated by using FFT. It was found that s component of the transmission power field changes by half a period compared with p component of that for the slab thickness d when first transmission wave involves the ordinary and extraordinary waves.

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