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Analysis of scattering problem by an imperfection of finite extent in a plane surface

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1.Introduction

In this paper, a new method based on the mode-matching method in the sense of least squares [1], [2] for analyzing the two dimensional scattering problem of plane wave incidence to the infinite plane surface with an arbitrary imperfection of finite extent. The semi-infinite upper and lower regions of that surface are a vacuum and a perfect conductor, respectively.

For this problem, the Rayleigh hypothesis [3] assert that the scattered field may be expanded in terms of outward-going wave functions and represented by the ordinary Fourier transform whose spatial spectrum is not band-limited in the upper region of that surface including the boundary. However, it was shown that the Rayleigh hypothesis could not be applied without a limiting condition by van den Berg and Millar and so on [4], [5]. Namely, the condition about the profile of the imperfection in a surface was derived under which the hypothesis is rigorously valid. For example, when the imperfection is sinusoidal periodic groove structure of finite extent and the length of a period is D and a half depth of the groove is A, the condition is $2\pi A/D < 0.448$. Moreover, if the profile of grooves is rectangular, Rayleigh hypothesis is can not be applied.

In this paper, a new method is presented for analyzing the above-mentioned scattering problem. This method is based on the mode-matching method in the sense of least squares. In this method, the approximate scattered wave is represented by the integral transform with band-limited spectrum of plane waves. This approximate wave function is determined in such way that the mean-square boundary residual is minimized.

2. Formulation of the problem

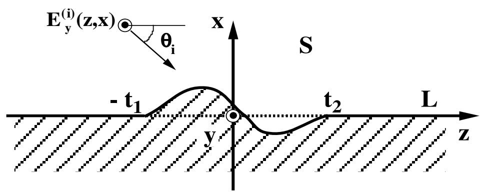

Fig.1 Geometry of the plane surface with an arbitrary imperfection of finite extent.

Figure 1 shows the geometry of the plane surface with an imperfection of finite extent. The upper region of the boundary L is vacuum and the lower region is filled with a perfect conductor. The boundary L is given as a function of the z-axis as follows:

$$x(z) = \begin{cases} \xi(z) , & -t_1 \le z \le t_2 \\ 0 , & -\infty \le z < -t_1 , \ t_2 < z < \infty \end{cases} \qquad (1)$$

The incident wave is a TE plane wave polarized to y direction and given as follows:

$$E_y^{(i)}(z,x) = \exp\left[j \left\{ \beta_i z - \kappa(\beta_i) x \right\} \right] \qquad (2)$$

$$\beta_i = k_0 \cos\theta_i \ , \ \kappa(\beta_i) = k_0 \sin\theta_i \qquad (3)$$

where, $k_0 (= 2\pi/\lambda_0)$ and θ_i are wave number and the angle of incidence measured from z-axis, respectively. The time factor $\exp(-j\omega t)$ is suppressed. When the plane wave of Eqs. (2) and (3) is incident to the boundary L, the total field $\Psi^{(t)}(z,x)$ is given by

$$\Psi^{(t)}(z,x) = f(z,x) + \Psi^{(s)}(z,x) \qquad (4)$$

where, f(z, x) is the sum of the incident plane wave and the reflected wave by the perfect plane surface having no imperfection of finite extent

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and given as follows:

$$f(z,x) = -2j\sin\{\kappa(\beta_i)x\}\exp(j\beta_i z)\ , \quad (5)$$

and then $\Psi^{(s)}(z,x)$ is the scattered wave generated by the imperfection. The total field satisfies the boundary condition as follows:

$$\Psi^{(t)}(z,x(z)) = f(z,x(z)) + \Psi^{(s)}(z,x(z))\ . \quad (6)$$

In this method, the approximate wave function for exact scattered wave $\Psi^{(s)}(z, x)$ is represented by the integral transform with the band-limited spectrum of plane waves as follows:

$$\Psi_W(z,x) = \frac{1}{2\pi}\int_{-w}^{w}\psi_w(h)\,\phi(h,z,x)\,dh\ , \quad (7)$$

where, $\psi_w(h)$ is the spectra defined as

$$\psi_w(h) = \begin{cases}\psi_w(h)\ , & |h| < w\\ 0\ , & |h| < w\end{cases}\ , \quad (8)$$

and $\phi(h,z,x)$ is a plane wave given by

$$\phi(h,z,x) = \exp[j\{\kappa(h)x + hz\}]\ (9)\ , \kappa(h) = \left(k_0^2 - h^2\right)^{\frac{1}{2}}\ (10)$$

3. Algorithm

The approximate wave function is determined in such way that the mean-square boundary residual is minimized. From concept of the method of least square, the following mean-square error related with boundary condition on L is defined.

$$\Omega_w = \int_{-\infty}^{\infty}\left|\Psi_W(z) + f(z)\right|^2 dz \Big/ \int_{-t_1}^{t_2}|f(z)|^2 dz \quad (11)$$

The integration of the approximate wave function of Eq. (7) is discretized as follows:

$$\Psi_W(z,x) \cong \frac{1}{2\pi}\sum_{m=1}^{N} A_m\int_{C_m}\phi(h,z,x)\,dh$$
$$(m = 1,2,\cdots,N) \quad (12)$$

where, C_m is the infinitesimal interval of integration and A_m is the amplitude of the discretized spectrum. Substituting Eq. (12) to Eq. (11), each A_m is determined in order to minimize mean-square error of Eq. (11) in such way that

$$\partial\Omega_w/\partial A_m^* = 0\ , \ (m = 1,2,\cdots,N)\ , \quad (13)$$

where, $*$ denotes complex conjugate quantity. Consequently, each A_m is obtained by the following set of simultaneous linear equations:

$$A_m\int_{C_m}dh + \frac{1}{2\pi}\sum_{m=1}^{N}A_m\int_{C_m}\int_{C_n}K(h',h)\,dh'dh + \int_{C_m}F(h)\,dh$$
$$= 0\ , \quad (m = 1,2\cdots,N;\ n = 1,2,\cdots N)\ . \quad (14)$$

It is considered that when mean-square error of Eq. (11) converges to zero, namely, the boundary value of the approximate wave function converges in the mean sense to that of exact scattered wave, the former converges uniformly to the latter in the arbitrary closed subregion of S which is the upper region of the boundary L of Fig.1 [6].

4. Examples of analyses

(1) Scattering problems by sinusoidal periodic groove structures of finite extent.

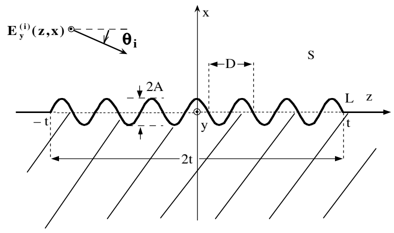

Fig.2 Geometry of surface of perfect conductor with sinusoidal periodic groove structure of finite extent.

In this case, Figure 2 shows the boundary L with a sinusoidal periodic groove structure of finite extent. The profile of the boundary is given by

$$x = \xi(z) = \begin{cases}A\cos Kz\ , & |z| \le t\\ 0\ , & |z| > t\end{cases}\ , \ K = \frac{2\pi}{D}\ , \quad (15)$$

where D is the length of one period and A is a half depth of the groove, respectively.

Figure 3 shows the far-field pattern of the scattered wave when a plane wave is incident at the angle of incidence 90 degree. The length of imperfection part is denoted as 2t=10.5D. Figure 3 shows that the pattern is symmetrical about x-axis. It has good agreement with the physical consideration because the surface shown by figure 3 is symmetrical about x-axis. Figure 4 shows mean-square error of boundary condition on L as a function of bandwidth of approximated wave function when the depth is changed from $0.05\lambda_0$ to $0.15\lambda_0$. This figure shows that the error decreases monotonously with increasing bandwidth for each depth of grooves. The minimum errors for those depths are less than 8%. When the periodical length D is equal to $1.1547\lambda_0$, the region of parameter

A that satisfies the condition $2\pi A/D< 0.448$ under which Rayleigh hypothesis is rigorously valid is $A< 0.0823\lambda_0$. Consequently, it is concluded that scattering problems beyond the condition mentioned above can be analyzed precisely by the present method.

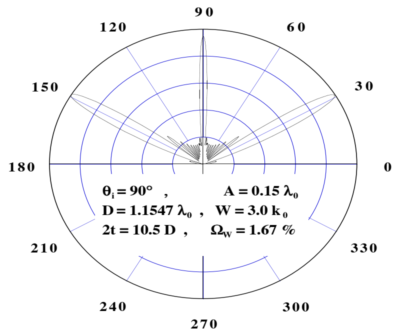

Fig.3 Far-field pattern of the scattered wave when a plane wave is incident at $\theta_i = 90^\circ$.

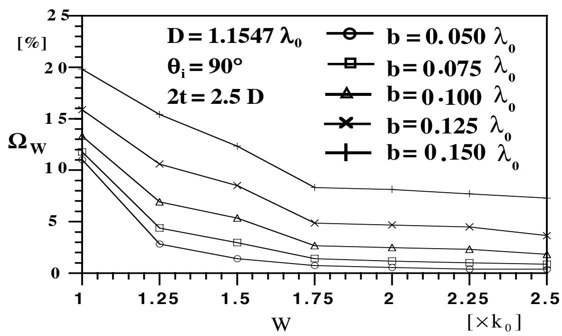

Fig.4 Mean-square error of boundary condition on L, as a function of a bandwidth.

(2) Scattering problems by rectangular periodic groove structure of finite extent.

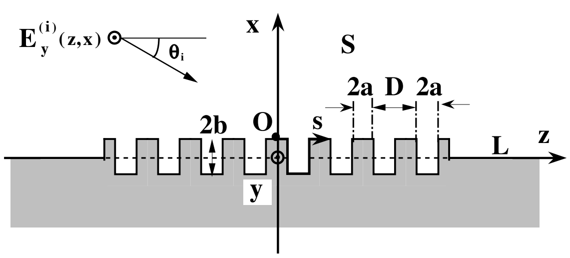

Fig.5 Geometry of surface with rectangular periodic groove structure of finite extent.

In this case, discussion is developed for the plane wave incidence to the surface with an imperfection formed by rectangular periodic groove structure of finite extent as shown by figure 5. In this analysis, z and x coordinates on the boundary L are represented by the parameter s which is the length measured from the origin o along the surface to make those coordinates single valued functions. In figures from 6 to 10, the incident angle of plane wave is 90 degree and the imperfection consists of rectangular groove structure of 2.5 periods. Scattering problems are analyzed in the case that depths of the grooves are from $0.05\lambda_0$ to $0.15\lambda_0$. Band-limited spectra of approximate wave functions have bandwidth in the region from $1k_0$ to 4 k_0. Figure 6 shows the far-field pattern of the scattered wave. This figure shows that the pattern is symmetrical about x-axis. It has good agreement with the physical consideration because the surface shown by figure 5 is symmetrical about x-axis. Figure 7 shows mean-square error of boundary condition on L as a function of bandwidth of approximated wave function. This figure shows that the error decreases monotonously with increasing bandwidth. The minimum errors for those depths are less than 6%. Figure 8 shows the error about energy conservation. This figure shows that the error decreases monotonously with increasing bandwidth. The errors for those depths are less than 1%. The figure 9 shows that boundary values of approximate wave functions converge in mean to the exact boundary value as bandwidth increases. Figure 10 shows that spectra of approximate wave functions approach to a convergent value with increase of bandwidth.

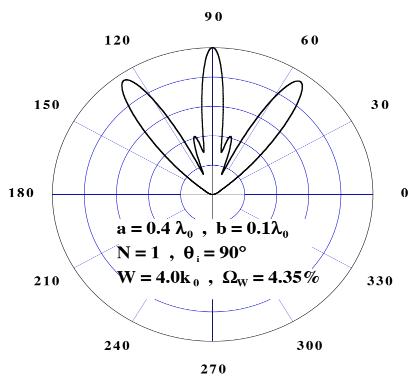

Fig.6 Far-field pattern of the scattered wave when a plane wave is incident on the surface with rectangular periodic groove structure of finite extent at $\theta_i = 90^\circ$.

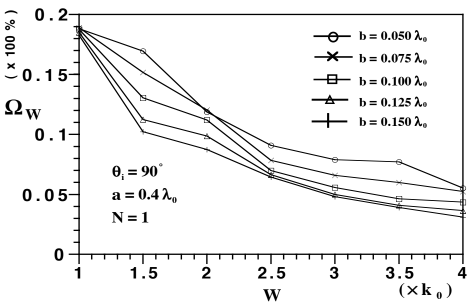

Fig.7 Mean-square error of boundary condition on L, which has the rectangular periodic groove structure of finite extent, as a function of a bandwidth.

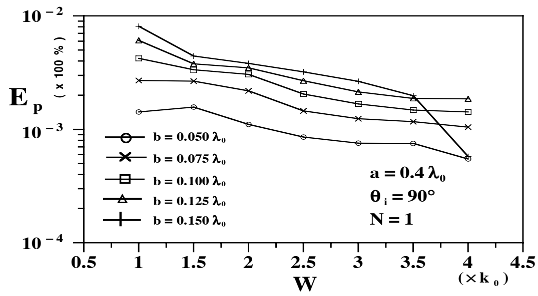

Fig.8 Errors of energy conservation related to the approximate scattered waves as a function of a bandwidth.

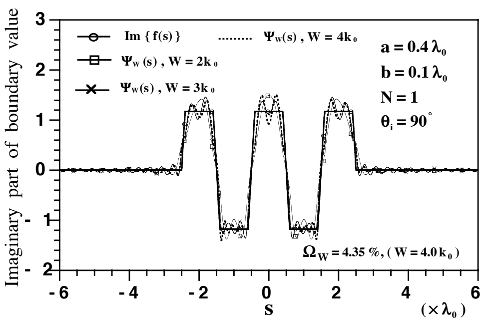

Fig.9 Convergences in mean of the boundary values of approximate scattered waves to the exact boundary value.

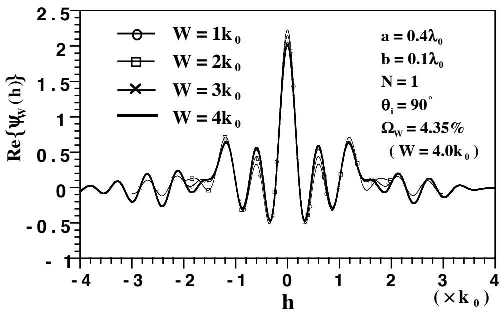

Fig.10 Convergences of band-limited spectra of approximate scattered waves as a function of bandwidth.

5. Conclusions

In this paper, a new method for analyzing the two-dimensional scattering problem of plane wave incidence to the infinite plane surface with an arbitrary imperfection of finite extent. This method is based on the mode-matching method in the sense of least squares. The present method is applied to two cases in which imperfections consist of sinusoidal and rectangular groove structures of finite extent. In the first case, the problem beyond the condition in which the Rayleigh hypothesis is valid can be analyzed by the present method precisely. In the second case where groove profile is rectangular and the Rayleigh hypothesis cannot be applied, scattering problems can be analyzed by the present method precisely. Consequently, it is concluded that the present method is effective for analyzing scattering problems mentioned above.

References

[1] M. Tomita:"Thin-Film Waveguide with a Periodic Groove Structure of Finite Extent",J.Opt.Soc.Am.A, vol.6,no.9,pp.1455-1464(1989).
[2] M. Tomita:"Analysis of Grating Coupler by Mode-Matching Method in the Sense of Least Squares",IEICE Trans.,vol.E73,no.4,pp.554-565(1990).
[3] Lord Rayleigh:" On the dynamical theory of gratings", Proc.Roy.Soc.,vol.79,pp.399-416(1907).
[4] P.M.van den Berg and J.T.Fokkema:"The Rayleigh hypothesis in the theory of reflection by a grating", J.O.S.A., vol.69, no.1, pp.27-31(1979).
[5] R.F. Millar:"The Rayleigh assumption in scattering by a periodic surface II", Proc.Camb.,Phil.Soc.,vol.69, pp.217-225(1971).
[6] Y. Okuno and T. Matsuda:" Mode-matching method with a higher-order smoothing procedure for the numerical solution of diffraction by a grating", J.Opt.Soc.,vol. 73,no.10,pp.1305-1311(1983).