RESONANCE FEATURES OF A CASCADED DIFFRACTION GRATING INTERACTING WITH A PLANE ELECTROMAGNETIC WAVE

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Statement of the problem

The accurate modeling and engineering of the millimeter-wave circuits and devices require accounting of resonance effects existence. The high merit of such effects causes applying of rigorous methods in their investigation, in particular – using of integral equation technique. Cascaded diffraction grating often is a resonance element of such equipment. The resonance interaction of electromagnetic waves with such grating are well-known [1,2]. However, the electromagnetic wave scattering by the cascaded grating with element's arbitrary shape is not widely presented in the literature yet. This paper is devoted to the last case.

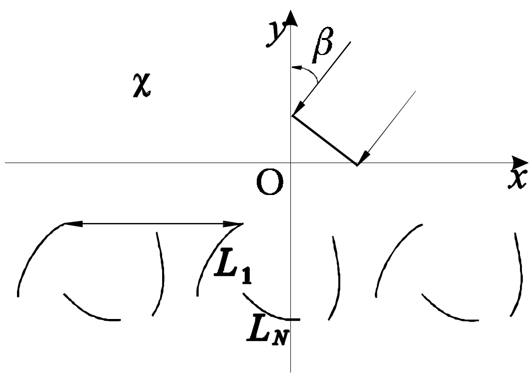


Fig.1. The geometry of the problem.

Consider multielement d-periodic grating in homogeneous isotropic media with the wave number χ. The period of the grating may contains N cylindrical perfectly conducting screens with generatrices parallel to Oz axis. Cross-section of the screens by xOy plane form the open smooth Lyapunov-type contours L_k, $k=\overline{1,N}$ (Fig. 1). The grating is irradiated by an unit-amplitude plane electromagnetic wave with exp[$-i\omega t$] time dependence inciding at β angle to Oy axis. Such problem is reducible to Helmholtz equation solution which satisfies the following conditions: Dirihlet or Neumann on the arcs L_k, $k=\overline{1,N}$; Meixner-type near the screen' ribs (L_k arc end-points); absence of the wave propagating from infinity (except the exciting one).

Green function of the problem

The diffracted field is sought for as a single-layer potential (TM-polarization):

$$E(z)=\exp\left[-i\chi\Im\{ze^{-i\beta}\}\right]+\frac{\pi i}{2}\sum_{u=-\infty}^{+\infty}\sum_{k=1}^{N}\int_{L_k} j_k(t_k)H_0^{(1)}(\chi\,|z-t_k+ud\,|)ds_k\ ,\ t_k\in L_k\ , \tag{1a}$$

or as a double-layer potential (TE-polarization):

$$H(z)=\exp\left[-i\chi\Im\{ze^{-i\beta}\}\right]+\frac{\pi i}{2}\sum_{u=-\infty}^{+\infty}\sum_{k=1}^{N}\int_{L_k} m_k(t_k)\frac{\partial}{\partial n_k}H_0^{(1)}(\chi\,|z-t_k+ud\,|)ds_k\ ,\ t_k\in L_k\ . \tag{1b}$$

Here $H_0^{(1)}(z)$ is the Hankel function of the first kind; $\partial/\partial n_k$ is normal derivative at point $t_k\equiv t_k(s_k)=x_k(s_k)+iy_k(s_k)$ with an arc abscissa s_k of L_k contour; $j_k(t_k)$, $m_k(t_k)$ are the unknown functions which are proportional to the density of longitudinal or transversal currents induced at the scatterers; u is the period number; symbol $\Im$ denotes the imaginary part of a complex variable. Due to the considered two dimensions of the problem (all values do not depend on the longitudinal coordinate) here and below the variable $z=x+iy$ denotes the view point's complex coordinate.

By carrying the sum by u under the integral in (1a,b) and future using of the Floquet periodic condition we obtain:

$$E(z)=\exp\left[i\chi\left(\Re\{z\}\sin\beta-\Im\{z\}\cos\beta\right)\right]+2\pi\sum_{k=1}^{N}\int_{L_k} j_k(t_k)G(t_k,z)ds_k,t_k\in L_k\ ; \tag{2a}$$

$$H(z)=\exp[i\chi(\Re\{z\}\sin\beta-\Im\{z\}\cos\beta)]+2\pi\sum_{k=1}^{N}\int_{L_k} m_k(t_k)\frac{\partial}{\partial n_k}G(t_k,z)ds_k, t_k\in L_k . \qquad (2b)$$

Here $G(t,z)$ is periodic Green function with the following representation:

$$G(t_k,z)=\frac{i}{4}H_0^{(1)}(\chi r_k)+\frac{i}{4}\sum_{\substack{u=-\infty\\u\neq 0}}^{+\infty}H_0^{(1)}(\chi\,|z-t_k-ud\,|)\exp[i\chi du\sin\beta],\ r_k=|z-t_k|. \qquad (3)$$

To calculate the Green function in the so-called grazing points and to avoid a convolution of the weakly convergent series in other cases [3] we used the known integral presentation of the Hankel function, the technique of the integration path transformation and the special Lagrange-type interpolation polynomial of several variables [4], what allow us to calculate the following Green function's asymptotics at infinity

$$G^{\pm\infty}(t_k,z)=\frac{i}{2\chi d}\sum_{m=-n}^{n_+}\left(\frac{\exp[i\chi(\pm\Im\{z\}\cos^*(\beta,m)+\Re\{z\}\sin^*(\beta,m))]}{\cos^*(\beta,m)}\times\right.$$
$$\left.\times\exp[i\chi(\mp\Im\{t_k\}\cos^*(\beta,m)-\Re\{t_k\}\sin^*(\beta,m))]\right). \qquad (4)$$

Here $\cos^*(\beta,m)=\sqrt{\cos^2\beta-\frac{4\pi m}{\chi d}\sin\beta-\left(\frac{2\pi}{d}m\right)^2}$, $\Im\{\cos^*(\beta,m)\}\geq 0$; $\sin^*(\beta,m)=\frac{2\pi}{\chi d}m+\sin\beta$.

For the one-mode case ($\chi d(1\pm\sin\beta)<2\pi$) the Green function estimation (4) is the following:

$$G^{\pm\infty}(t_k,z)=\frac{i\exp[i\chi(\pm\Im\{z\}\cos\beta+\Re\{z\}\sin\beta)]}{2\chi d\cos\beta}\exp[i\chi(\mp\Im\{t_k\}\cos\beta-\Re\{t_k\}\sin\beta)].$$

Thus the scattered field at the infinity looks like:

$$E^s(z)\Big|_{\Im\{z\}\to\pm\infty}=\frac{\pi i}{\chi d\cos\beta}\sum_{k=1}^{N}\int_{L_k} j_k(t_k)\exp[i\chi(\pm\Im\{z-t_k\}\cos\beta+\Re\{z-t_k\}\sin\beta)]ds_k$$

for the case of *TM*-polarization and

$$H^s(z)\Big|_{\Im\{z\}\to\pm\infty}=\frac{\pi i}{\chi d\cos\beta}\sum_{k=1}^{N}\int_{L_k} m_k(t_k)\frac{\partial}{\partial n_k}(\exp[i\chi(\pm\Im\{z-t_k\}\cos\beta+\Re\{z-t_k\}\sin\beta)])ds_k$$

for the case of *TE*-polarization.

The grating's reflection R coefficient is given by the next formula [5]:

$$R=\frac{\{E\vee H\}^s(z)\Big|_{\Im\{z\}\to+\infty}}{\exp[i\chi(\Im\{z\}\cos\beta+\Re\{z\}\sin\beta)]}; \qquad (5)$$

Here symbol $\vee$ means «or»

Solution of the problem

To receive the solution of the problem we construct the system of integral equations from (2a,b) by satisfying the boundary conditions on the contours L_k, $k=\overline{1,N}$. Accounting the condition on the screen's rib allow us to write the unknown function as $j(\tau)=j_0(\tau)(1-\tau^2)^{-1/2}$ for the *TM*-case and $m(\tau)=m_0(\tau)(1-\tau^2)^{1/2}$ for the *TE*-case. Note, that the Green function, as an infinite sum of Hankel functions, has a logarithmic singularity. To solve the obtained system of integral equations we used the specially constructed quadrature formulae [6].

Numerical results and their discussion.

As an example, consider the case when one grating's period contains a couple of screens. Contours L_k are parabolic arcs with the end-points $a+il(k$-1) and vertexes $-ib+il(k$-1) (Fig.2).

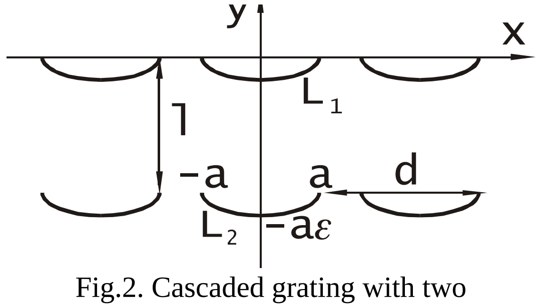


Fig.2. Cascaded grating with two curvilinear screens on a period

Parametric arc's equations in the complex plane $z=x+iy$ look like

$$t_k(\tau)=a(\tau+i\varepsilon(1-\tau^2))+il(k-1),\ \tau=[-1;1], \varepsilon=b/a, k=1,2 ,$$

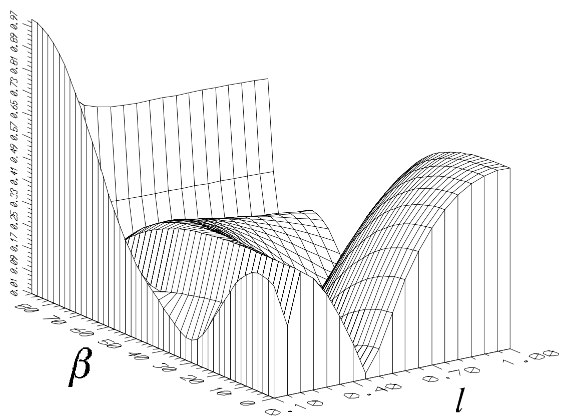
where l is distance between screens. The wave number χ is postulated to be real.

For the algorithm testing the reflection coefficient R was calculated for two-layered cascaded plane grating ($\varepsilon_1=\varepsilon_2=0$) with parameters: $\chi d=\pi$, $2a=0.5d$, $l=0.5d$ at it radiating by a plane *TE*-polarized electromagnetic wave. The obtained results for some incidence angle (in degrees) are presented in the following Table.

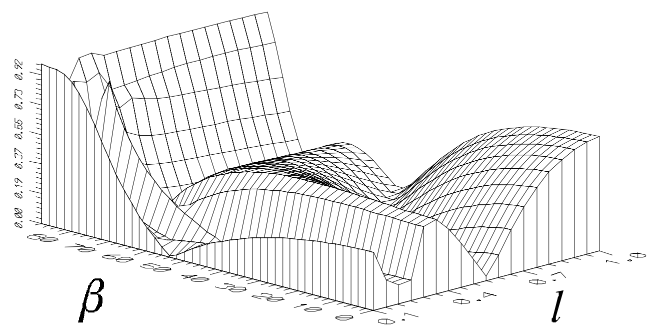
Incidence angle β,°	Reflection coefficient R	Incidence angle β,°	Reflection coefficient R
0	.327338 .327343	60	.211052 .211054
20	.243105 .243110	80	.153818 .153818
40	.004243 .004248	85	.086354 .086354

Data over the lines are obtained by our approach. The number of nodes in the used quadrature formula in the case of one screen was $n=6$. Results under the lines were calculated by Dr. Akira Matsushima from Kumamoto University, Japan [7]. We can see a good coincidence – the difference does not exceed $5.\times10^{-6}$ for all angles. The number of nodes of quadrature $n=6$ guarantees the convergence at the fifth digit for the considered wave length and geometrical parameters. So, we may state the high efficiency of the presented scheme as far as it is a universal one.

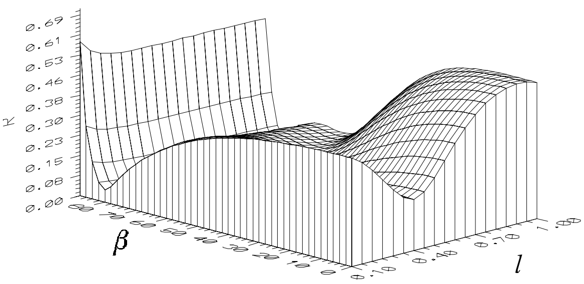
We have investigated the influence of a distance l between the gratings layers on the behaviour of the reflection coefficient (5) depending on the incidence angle β at different curvature of the upper (ε_1) and lower (ε_2) scatterers for the case $\chi d=\pi$, in the case of *TE*-polarization (Fig. 3a-d). On the all mentioned above figures the incidence angle β runs from 0° to 88° with 2° step; the distance between layers l runs from $0.1d$ to $1.0d$ with $0.05d$ step; the dimension of the grating element $2a=0.5d$, the elements curvature is $\varepsilon=-\varepsilon_1=\varepsilon_2=0.25$ (Fig. 3a), $\varepsilon=-\varepsilon_1=\varepsilon_2=0.5$ (Fig. 3b), $\varepsilon=\varepsilon_1=\varepsilon_2=0.25$ (Fig. 3c), $\varepsilon=\varepsilon_1=\varepsilon_2=0.5$ (Fig. 3d).



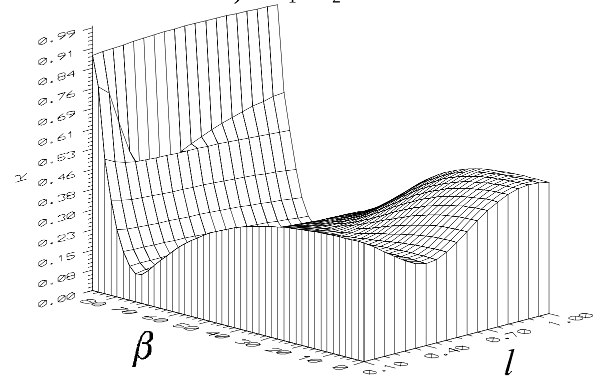
a) $\varepsilon=\varepsilon_1=-\varepsilon_2=0.25$



b) $\varepsilon=\varepsilon_1=-\varepsilon_2=0.5$



c) $\varepsilon=\varepsilon_1=\varepsilon_2=0.25$



d) $\varepsilon=\varepsilon_1=\varepsilon_2=0.5$

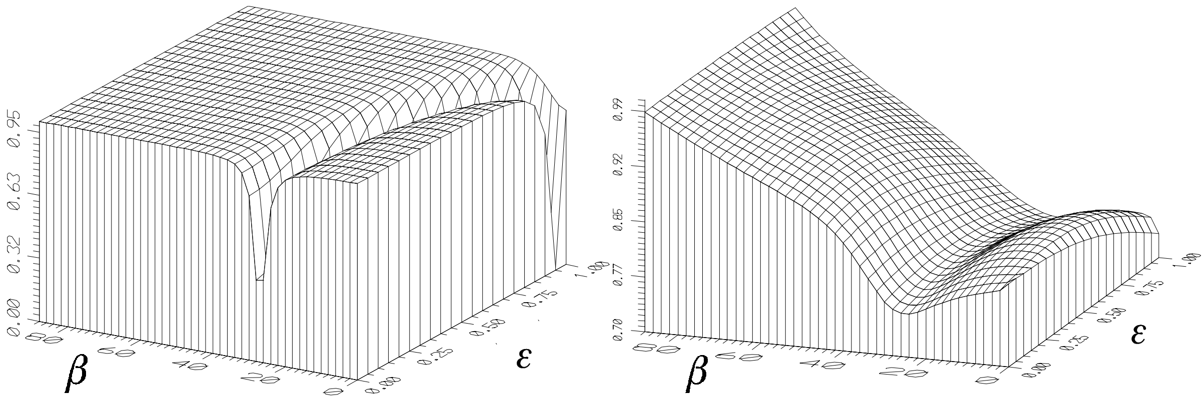
Fig. 3. Reflection coefficient of a two-layered cascaded curvilinear grating.

The converging tests have been performed at the resonance points. It was found that at numbers of quadrature nodes $n=10$ and $n=20$ the results differ at the fifth digit. That is $n=10$ guarantees four reliable digits. The results at the grazing points χd_g have been replaced by the ones calculated at $\chi d_g\pm10^{-4}$.

In the case of the *TM*-polarized plane electromagnetic wave radiating the investigated structure we can observe another behavior of the reflected coefficient. Fig. 4 presents the "incidence angle – screens' curvature" dependencies of the reflection coefficient of the same grating. The incidence angle β runs from 0° to 88° with 2° step; the elements curvature $\varepsilon=\varepsilon_1=\varepsilon_2$ runs from 0.0 to 1.0 with 0.05 step; the dimension of the grating element $2a=0.5d$, the distance between layers $l=d$. At the grating's period equal to $\chi d=\pi$, what is equivalent to the case of $d=0.5l$, the grating is in fact non-transparent (Fig. 4a).

The presented results prove the fact, that the considered two-layered curvilinear grating has strongly pronounced resonance features.

To conclude the paper we observe that the presented above periodic Green function presentation as well as the technique of integral's path modification allow to develop a rigorous approach in solution of the problem of *TM*- and *TE*-polarized plane wave diffraction on arbitrary-shaped cascaded grating. It allows to construct an efficient (from computer-resource point view) algorithm and to provide an accurate numerical analysis of the grating's scattering features in the resonance range for arbitrary incidence angle and geometrical parameters of the structure.



the grating's period $\chi d=\pi$ (a) the grating's period $\chi d=2\pi$ (b)

Fig. 4. The reflecton coefficient of the two-layered cascaded grating.
The dimension of the grating element $2a=0.5d$, the distance between layers $l=d$, *TM*-polarization.

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