Transient scattering from a smooth dielectric cylindrical object with a concave-convex boundary

T. OHMORI, H. IKUNO and M. NISHIMOTO
Department of Electrical Engineering and Computer Science,
Kumamoto University, Kurokami 2-39-1, Kumamoto 860, Japan

1. Introduction : An extended ray theory (ERT), that is an extension of the ordinary ray theory and the GTD to the complex coordinate space, interprets some new aspects about scattering from two-dimensional objects[1],[2]. When a scatterer is composed of dielectrics, the scattering phenomenon is more complicated than that of a perfectly conducting object, because of surface scattering and volume one. The volume scattering from a circular cylinder and a sphere have been interpreted as contributions for geometrical optics (GO) rays and diffracted rays transverse in the interior of a scatterer[2]-[4]. It is possible to analyze the scattering mechanism on an arbitrary shaped dielectric object by using the ERT, so that it may be revealed in terms of reflection, refraction and diffraction events.

We investigate transient scattering from a smooth dielectric cylinder with concave-convex boundary in a homogeneous medium, e.g., light scattering by an air bubble in water. To do so, we classify scattering process into six elementary processes and scattering centers can be obtained by charts for these elementary processes in the ERT[5]. After obtained scattering centers, we reconstruct transfer functions by calculating the amplitude and phase of rays determined by the intuitive role of the ordinary ray theory and the GTD, here, we have done for GO rays. To check the validity of the ERT solutions are compared with those given by reference solutions provided by the Yasuura method[6].

2. Extended ray theory

We consider a plane wave scattering from a dielectric cylinder whose contour is given by $\rho=\rho(\phi)$ and relative refractive-index is $N<1$[7] as shown in Fig.1. In the ERT, scattering process can be represented by reflection and refraction events and diffraction events which are denoted by symbols "**G**" and " **D**", respectively, and classified into six elementary processes,

(G1) **G**, (G2) $\mathbf{G}^{m+2}$, (D1) **D**, (D2) $\mathbf{D}\mathbf{G}^{m}\mathbf{D}$,

(D3) $\mathbf{G}^{m+1}\mathbf{D}$, (D4) $\mathbf{D}\mathbf{G}^{m+1}$ (m=0,1,2,..)

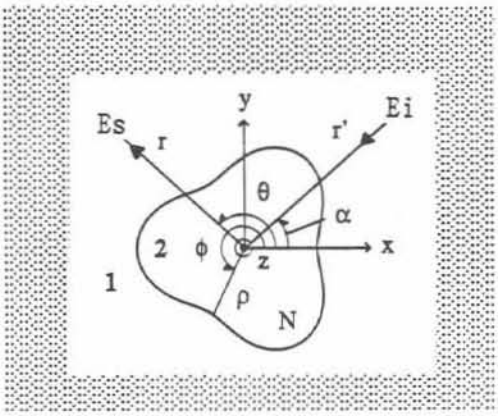


Fig.1 Scattering model and its coordinate system.

(G1) stands for specular reflection events, (G2) m-time internal reflection events, and (D1), (D2), (D3) and (D4) diffraction events. Here we only consider GO contributions. With the help of ray tracing technique, we make charts for (G1) and (G2) which show the relation between observation angles and incidence points of GO rays. Using these charts, we can calculate real scattering centers and complex ones. Then, we reconstruct transfer function of specularly reflected rays (SR) and m-times internally reflected rays (IRm),

$$H(ka) = \Sigma H_{SR}(ka) + \sum_{m} \Sigma H_{IRm}(ka) \quad (1)$$

$$H_{SR}(ka) = R_{11}(\phi_{SR})\, J_{SR}^{-1/2} \exp(-jk\Phi_{SR}) \quad (2)$$

$$H_{IRm}(ka) = T_{12}(\phi_0)\left[\prod_{q=1}^{m} R_{22}(\phi_q)\right] T_{21}(\phi_{m+1}) \left[\prod_{q=0}^{m+1} J_q^{-1/2}\right] \exp(-jk\Phi_{IRm}) \quad (3)$$

where R_{11}, R_{22} and T_{12}, T_{21} denote reflection coefficients and refraction ones, respectively. Φ_{SR} and Φ_{IRm} are ray path lengths, and $J_{SR}^{-1/2}$ and $J_q^{-1/2}$ $_{(q=0,1,..)}$ are given by the conservation of energy. Therefore, transient response waveform from a dielectric object can be expressed by Fourier synthesis in terms of the frequency domain transfer function and the input pulse spectrum[1].

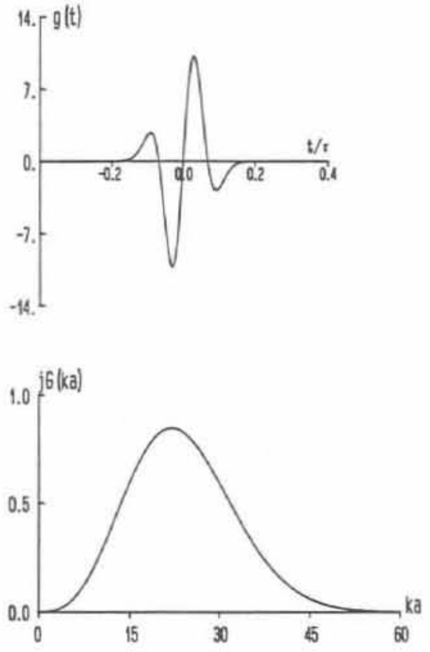


Fig.2 TDG pulse and its spectrum.

3. Examples and discussion

We consider scattering from a periodically deformed cylinder with concave-convex boundary contour described by

$$\rho(\phi) = a\{1 - \delta\cos(3\phi)\}, \quad 0.1 < \delta < 1 \quad (4)$$

where "a" denotes the average radius. Transfer functions for δ=0.2, N=0.75, an incidence angle α=0 and an observation angle $\theta=\pi/6$ have been calculated at 1100 data points at increments Δka=0.05 over $0.05 \leq ka \leq 55.0$ for E- and H-polarized field, because of the spectrum of TDG pulse $(n=18/\tau)$[1] as shown in Fig.2.

To get scattering centers for specular reflection and m-time internal reflection, we make charts for elementary processes (G1) and (G2) as shown in Fig.3. Using these chart, we can obtain real scattering centers and complex ones of rays; complex ray contributions are significant in shadowed side of caustics ($d\theta/d\phi_i$=0) for a deformed cylinder (δ=0.2). The scattering centers for a circular cylinder and a deformed cylinder obtained here are shown in Table 1 and their ray paths in Fig.4. Then, transient response waveforms are reconstructed by these GO rays in the ERT.

It is numerically shown that a TDG pulse response waveform for a deformed cylinder (δ=0.2), whose data is calculated by Yasuura method[6], is more complicated than that for a circular cylinder as shown in Fig.5 (a),(b); we have similar results for H-polarized field. We can trace all of response waveforms on time-axis by using the result in Table 1 and their response waveforms can be reconstructed by using Eqs.(1)-(3) as shown in Fig.5 (c),(d). In comparison with reference solutions and ERT solutions, the scattering mechanism for these examples can be explained by real ray contributions and complex ones in terms of specular reflection events and internal reflection events and for H-polarization case, too. The results show that we have weak diffraction events on the problem considered here.

4. Conclusion

The ERT is a useful technique for analyzing scattering mechanism on an arbitrary shaped dielectric object. We will investigate diffracted rays such as creeping rays and reflected creeping rays[1] in the near future.

References: [1] H.Ikuno and L.B.Felsen, IEEE Trans. AP., **36**, p.1272, 1988. [2] H.Ikuno et al., Proc.1989 URSI Int.Symp. EM Theory B, p.178, Stockholm. [3] Y.M.Chen, J. Math. Phys., **5**, p.820, 1964. [4] H.M.Nussenzveig, J. Math. Phys., **10**, p.82, 1969. [5] H.Ikuno et al., IEE Japan, **EMT-91-126**, p.1, 1991. [6] H.Ikuno and K.Yasuura, Radio Sci., **13**, p.937, 1978. [7] N.Fiedler-Ferrari et al., Phys. Rev. A, **43**, p.1005, 1991.

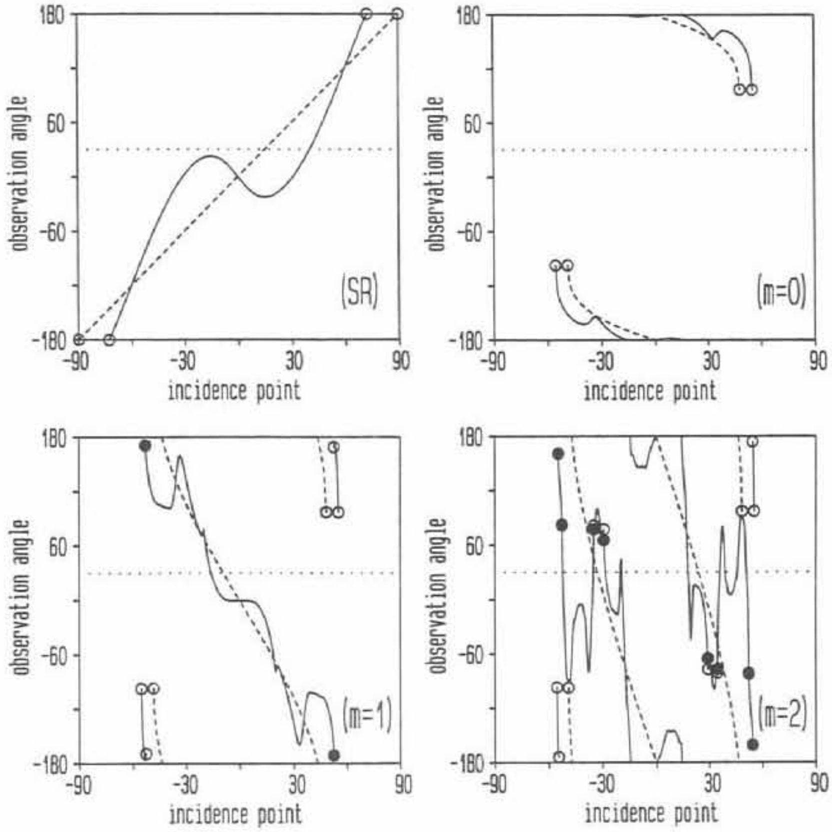


Fig.3. The charts for the relation between observation angles and incidence points for GO rays. Dashed and solid lines denote a circular cylinder case and a deformed cylinder case ($\delta=0.2$), respectively. Circle dots indicate creeping rays or diffracted rays which hit on boundary tangentially and propagate along boundary in the interior side of a scatterer. Solid dots indicate GO rays that are tangential on boundary in the interior side. $N=0.75$, $\alpha=0$.

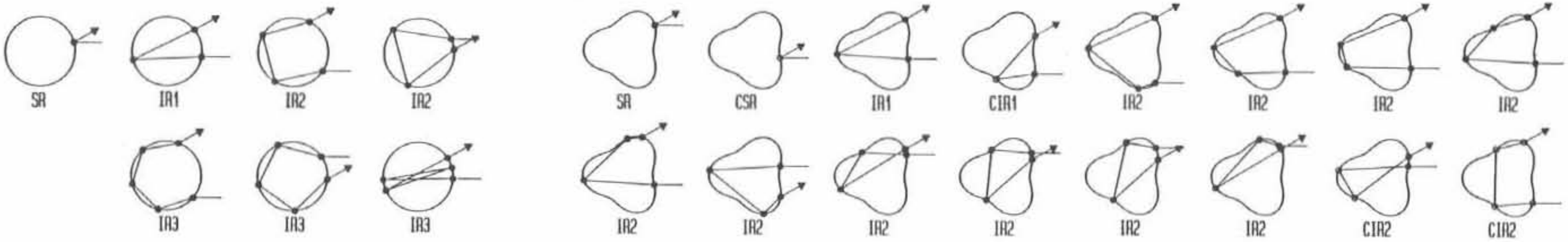
Table 1. Incidence points (ϕ i) and time(t/τ) of contributed rays ($\theta=\pi/6$) obtained by the charts in Fig.3.

(a) a circular cylinder($N=0.75, \alpha=0, \theta=\pi/6$).

ray	ϕ i(degree)	t/ τ
SR	15.00000	0.00000
IR1	-8.95379	1.44545
IR2	-32.55887	1.69021
IR2	24.14194	1.93938
IR3	-39.86275	1.75619
IR3	35.07851	2.07513
IR3	-3.45952	2.95802

(b) a deformed cylinder($\delta=0.2, N=0.75, \alpha=0, \theta=\pi/6$).

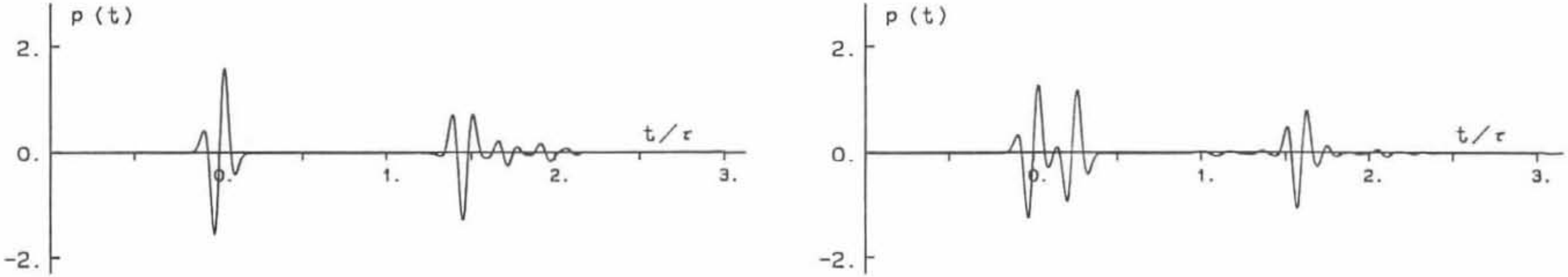
ray	ϕ i (degree)	t/ τ	Im[Φ]
SR	40.15360	0.00000	0.00000
CSR	-14.62891 + 8.20126j	0.24424	-0.01141
IR1	-15.75984	1.57493	0.00000
CIR1	-38.09994 - 8.25955j	1.08946	-0.12732
IR2	-51.92502	1.72670	0.00000
IR2	-34.94103	1.65892	0.00000
IR2	-28.25640	1.67477	0.00000
IR2	-20.01271	1.60819	0.00000
IR2	-19.21908	1.62131	0.00000
IR2	18.17073	2.08832	0.00000
IR2	36.12394	1.74158	0.00000
IR2	39.05301	1.66865	0.00000
IR2	45.92415	1.67507	0.00000
IR2	50.84665	1.79344	0.00000
CIR2	21.42095 + 2.46263j	1.84207	-0.03093
CIR2	-39.95132 - 2.05315j	1.36288	-0.03184



(a) a circular cylinder.

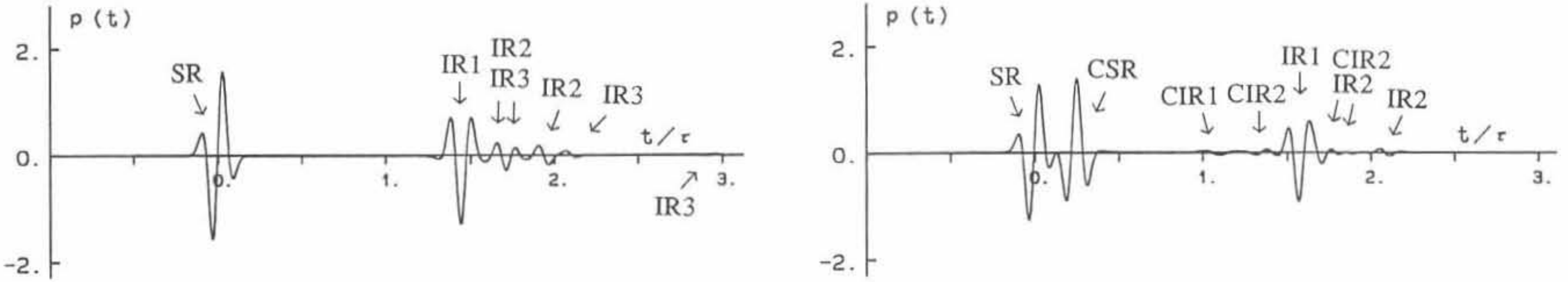
(b) a deformed cylinder (δ=0.2)

Fig.4. Ray paths of rays as shown in Table 1.



(a) Exact solution for a circular cylinder.

(b) Reference solution by Yasuura method for a deformed cylinder (δ=0.2).



(c) Reconstructed solution by ERT for a circular cylinder.

(d) Reconstructed solution by ERT for a deformed cylinder (δ=0.2).

Fig.5. TDG pulse (n=18/τ) response waveforms for E-polarized field (N=0.75, α=0, θ=π/6).