GREEN FUNCTION AND RADIATION OVER TWO-DIMENSIONAL RANDOM SURFACE

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ABSTRACT

A stochastic Green function representing the radiation field from a point source above a two-dimensional homogeneous random surface is constructed by integrating the random wave field for a plane wave incidence. Statistical properties and radiation patterns of the coherent and incoherent fields are evaluated using the asymptotic expression of the Green function.

INTRODUCTION

It is shown in the preceeding works[1-3] that the wave field scattered by a random surface can be better treated as a stochastic functional of the homogeneous random surface without any difficulties such as encountered in the perturbation and renormalization theories. In a previous report[4] a two-dimensional (2D) Green function for a point source above a 1D random surface was construceted from the random wave field for a plane wave incidence, and from its stochastic asymptotic expression various characteristics of the coherent and incoherent field were derived concerning the radiation and propagation over a random surface. The present paper aims to develop a similar treatment for the 3D Green function representing the radiation field from a point source placed above a 2D random surface so as to obtain the statistical characteristics of the radiation over the random surface.

PLANE WAVE SCATTERING BY TWO-DIMENSIONAL RANDOM SURFACE

Let the point r in 3D space R_3 be represented either by the spherical or cylindrical coordinates: $\mathbf{r} = (r,\theta,\phi) = (\rho,z)$ and $\rho = (\rho, \phi)$, where $\rho = |\rho| = r \sin\theta$, $z = r\cos\theta$, ρ being the radial vector in R_2. Let the 2D random surface be given by a homogeneous Gaussian random field:

$$z = f(\rho) = \int_{-\infty}^{\infty} e^{i\lambda\cdot\rho} F(\lambda) dB(\lambda), \quad \rho \in R_2 \tag{1}$$

where $dB(\lambda)$ denotes the 2D complex Gaussian random measure: $\langle dB(\lambda)^* dB(\lambda')\rangle = \delta_{\lambda\lambda'} d\lambda$, $\langle\ \rangle$ denoting average and * complex conjugate. $|F(\rho)|^2$ gives the spectral density and its integration over $\lambda \in R_2$ gives the variance σ^2 of the random surface. The random wave field ψ satisfies the wave equation and either Dirichlet or Neumann condition on the random surface, which can be approximated by the following equivalent boundary condition at z = 0:

$$[\ \psi + f\frac{\partial\psi}{\partial z}\ ]_{z=0} = 0 \qquad \text{(Dirichlet)} \qquad (2)$$

$$[-(\nabla f\cdot\nabla\psi) + \frac{\partial\psi}{\partial z} + f\,\frac{\partial^2\psi}{\partial z^2}\ ]_{z=0} = 0 \qquad \text{(Neumann)} \qquad (3)$$

For a plane wave incident at angle θ_0 the stochastic anlogue of the Floquet solution gives the random wave field of the form

$$\psi(\rho,z|\lambda_0) = e^{i\lambda_0\rho}\,[e^{-iS(\lambda_0)z} \mp e^{iS(\lambda_0)z} \mp U(\rho,z|\lambda_0)] \qquad (4)$$

(- in $\mp$ is for Dirichlet and + for Neumann) where we have put

$$\lambda = |\lambda| = k\sin\theta_0,\quad S(\lambda_0) = (k^2 - \lambda_0^2)^{1/2} = k\cos\theta_0 \qquad (5)$$

where $k_0 = (k,\theta_0,0) = (\lambda_0,\ S(\lambda_0))$ gives the wave vector of the specularly reflected wave. The first and the second term of (4) give the primary field for $\sigma^2 = 0$. The third term U giving the scattered field due to randomness is a homogeneous random field in ρ and can be expanded in terms of Wiener-Hermite orthogonal functionals:

$$U(\rho,z|\boldsymbol{\lambda}) = e^{iS(\lambda)z}A_0(\lambda) + \int_{R_2} e^{i\lambda_1\cdot\rho + iS(\lambda+\lambda_1)z}\,A_1(\lambda_1|\lambda)dB(\lambda_1) + \cdots \qquad (6)$$

where we have shown only first two terms. The Wiener kernels A_0 and A_1 can be obtained approximately by solving the boundary condition (2) or (3).[2)]

GREEN FUNCTION OVER RANDOM SURFACE

Let the source point above the random surface be at $r_0 = (\rho_0,\ z_0)$. Then the Green function satisfies the equation

$$(\nabla^2 + k^2)G(\rho,z|\boldsymbol{\rho}_0,z_0) = -\,\delta(\rho - \rho_0)\,\delta(z - z_0), \qquad z,z_0 > 0 \qquad (7)$$

the boundary condition (2) or (3) and the radiation condition. The Green function can be constructed from (4) as follows:

$$G(\rho,z|\rho_0,z_0) = G_0(\rho,z|\rho_0,z_0) \mp G_s(\rho,z|\rho_0,z_0), \qquad z,z_0 > 0 \qquad (8)$$

$$G_0(\rho,z|\rho_0,z_0) = \frac{i}{8\pi^2}\int_{R_2} e^{i\lambda\cdot(\rho-\rho_0)}[e^{iS(\lambda)|z - z_0|} \mp e^{iS(\lambda)(z+z_0)}]\frac{d\lambda}{S(\lambda)} \qquad (9)$$

$$G_s(\rho,z|\rho_0,z_0) = \frac{i}{8\pi^2}\int_{R_2} e^{i\lambda\cdot(\rho-\rho_0)}\ U(\rho,z|\lambda)\frac{e^{iS(\lambda)z_0}}{S(\lambda)}d\lambda, \qquad (10)$$

where G_0 gives the Green function for the flat surface ($\sigma^2 = 0$) readily expressible in terms of spherical Hankel functions. G_s gives the scattered wave due to surface roughness. We put $\rho_0 = 0$ in the following. By (6) the (11) can be further divided into the coherent part g_c and the incoherent part g_{ic}, namely, $G_s = g_c + g_{ic}$:

$$g_c(\rho,z|0,z_0) = \frac{i}{8\pi^2}\int_{R_2} e^{i\lambda\rho+iS(\lambda)(z+z_0)}\,\frac{A_0(\lambda)}{S(\lambda)}\,d\lambda \qquad (11)$$

$$g_{ic}(\rho,z|0,z_0) = \int_{R_2} K(\lambda,z,z_0|\lambda_1)dB(\lambda_1) + \cdots \qquad (12)$$

$$K(\rho,z,z_0|\lambda_1) = \frac{i}{8\pi^2}\int_{R_2} e^{i(\lambda+\lambda_1)\cdot\rho+iS(\lambda+\lambda_1)z+iS(\lambda)z_0}\frac{A_1(\lambda_1|\lambda)}{S(\lambda)}\,d\lambda \qquad (13)$$

It is interesting to note that the coherent field (11) can be rewritten as

$$g_c(\boldsymbol{\rho},z|0,z_0) = \int_{R_3} g_0(\mathbf{r} - \mathbf{r}')Q(\mathbf{r}')d\mathbf{r}', \quad \mathbf{r} = (\boldsymbol{\rho},z) \tag{14}$$

$$Q(\mathbf{r}) = \delta(z + z_0)\frac{1}{(2\pi)^2}\int_{R_2} e^{i\boldsymbol{\lambda\rho}} A_0(\boldsymbol{\lambda})d\boldsymbol{\lambda} \tag{15}$$

where $g_0(\boldsymbol{\rho},z|0,0) = e^{ikr}/4\pi r$ denotes the free space Green function and $Q(\mathbf{r})$ gives an equivalent source distribution for coherent scattering produced on the plane at the depth $z = -z_0$.

ASYMPTOTIC FORM OF GREEN FUNCTION AND POWER FLOW

We obtain the asymptotic form of the Green function at large distance from the source and the surface; $kr = k\sqrt{\rho^2 + z^2} \to \infty$, assuming the source be fixed not very high above the surface. The asymptotic expression for the total coherent part $G_c = G_0 + g_c$ is calculated to be

$$G_c(\boldsymbol{\rho},z|0,z_0) \sim \frac{e^{ikr}}{4\pi r}[e^{-ikz_0\cos\theta} \pm e^{ikz_0\cos\theta}(1 + A_0(k\sin\theta,\phi))], \quad kr \to \infty \tag{16}$$

and the coherent power flow per unit solid angle in the direction (θ,ϕ) is

$$P_c(\theta,\phi;z_0)/P_{00} = |e^{-ikz_0\cos\theta} \mp e^{ikz_0\cos\theta}[1 + A_0(k\sin\theta,\phi)]|^2 \tag{17}$$

where $P_{00} = 1/(4\pi)^2$ gives the power flow per unit solid angle of a monopole.

The asymptotic expression for the incoherent part is calculated to be

$$g_{ic}(\boldsymbol{\rho},z|0,z_0) \sim \frac{e^{ikr}}{4\pi r} k\cos\theta \int_{R_2} e^{iS(\boldsymbol{\lambda}_s-\boldsymbol{\lambda})z_0}\frac{A_1(\boldsymbol{\lambda}|\boldsymbol{\lambda}_s-\boldsymbol{\lambda})}{S(\boldsymbol{\lambda}_s-\boldsymbol{\lambda})}dB(\boldsymbol{\lambda}) \tag{18}$$

$$\boldsymbol{\lambda}_s = k\sin\theta\cos\phi\, \mathbf{e}_x + k\sin\theta\sin\phi\, \mathbf{e}_y \tag{19}$$

where $\boldsymbol{\lambda}_s$ denotes the projection of the scattering wave vector $k\mathbf{r}/r$ onto X-Y plane. Then the average incoherent power flow per unit solid angle is

$$P_{ic}(\theta,\phi;z_0)/P_{00} = k^2\cos^2\theta \int_{R_2} e^{-\mathrm{Im}S(\boldsymbol{\lambda}_s-\boldsymbol{\lambda})z_0}\left|\frac{A_1(\boldsymbol{\lambda}|\boldsymbol{\lambda}_s-\boldsymbol{\lambda})}{S(\boldsymbol{\lambda}_s-\boldsymbol{\lambda})}\right|^2 d\boldsymbol{\lambda} \tag{20}$$

$$= \cos\theta \int_0^{2\pi}\int_0^{\pi/2} \frac{S(\theta,\phi|\theta_0,\phi_0)}{\cos\theta_0}\sin\theta_0 d\theta_0 d\phi_0 \tag{21}$$

where the surface wave part is neglected in (21) assuming $kz_0 \gg 1$, and

$$S(\theta,\phi|\theta_0,\phi_0) = k^2\cos\theta|A_1(\boldsymbol{\lambda}_s - \boldsymbol{\lambda}_0|\boldsymbol{\lambda}_0)|^2 \tag{22}$$

denotes the angular distribution of the incoherent scattering[2] for the plane wave with the incident angle (θ_0,ϕ_0). The power conservation law

$$1 = (1/4\pi P_{00})\int_0^{2\pi}d\phi\int_0^{\pi/2}[P_c(\theta,\phi;z_0) + P_{ic}(\theta,\phi;z_0)]\sin\theta\, d\theta \tag{23}$$

can be used to check the accuracy of approximate Wiener kernels A_0 and A_1.

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Fig.1 Coordinates and Wave Vectors

Fig.2 Angular Distribution of Total Coherent Power Flow (Dirichlet) solid line: $\sigma^2 = 0$ (height4), $k\ell = 0.5$, broken line: $k\ell = 1.0$, dotted line: $k\ell = 2.0$ (ℓ: correlation length)

Fig.3 Angular Distribution of Incoherent Power Flow (Dirichlet)

Fig.4 Angular Distribution of Total Coherent Power Flow (Neumann)

Fig.5 Angular Distribution of Incoherent Power Flow (Neumann)