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ON NATURAL MODES EXCITED BY INCIDENT PULSE

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1. INTRODUCTION

Current developments in high resolution radar and remote sensing technology have created interest in the investigation of scattering and diffraction of transient waveforms from conducting and dielectric bodies of various shapes, because analysis of transient scattering is very useful for target identification and inverse scattering problems. Based on the Singularity Expansion Method (SEM)[1], which is one of the techniques available for solving transient electromagnetic problems, we can obtain the transient response by summing up an infinite number of natural modes. The eigenfrequencies of these natural modes are correspond to the poles of the scattered field in complex frequency domain, and are called the natural frequencies or resonance frequencies. Since the natural modes are excited by an incident pulse, it is very important to know the relationship between incident pulse and excited natural modes when we analyze the transient scattering.

In this paper, we extract the natural frequencies from pulse responses by applying Prony's method[2][3] and investigate the relationship between incident pulse and excited natural modes. First we calculate the natural frequencies of perfectly conducting cylinder by using Yasuura's method for eigenvalue problems[4]–[6]. Yasuura's method used here is one of the efficient and reliable numerical methods for electromagnetic boundary value problems [7][8]. Second we calculate the pulse responses from the object by using Yasuura's method for ordinary scattering problems and Fourier synthesis technique [8][9], and extract the natural frequencies from them by applying Prony's method. By comparing the extracted natural frequencies with those obtained by Yasuura's method for eigenvalue problems, we investigate the relationship between the incident direction of the excitation pulse and the excited natural modes.

2. CALCULATION OF NATURAL FREQUENCIES

Let us consider a perfectly conducting cylindrical object with mirror symmetric axes as shown in Fig.1. The surface of it is smooth and expressed by $\rho'=\rho'(\theta')$ where prime denotes the point on the surface. According to Yasuura's method, we express the scattered field Ψ by the truncated modal expansion as follows:

$$\Psi(\rho, \theta) = \sum_{n=-N}^{N} c_n(N)\, \varphi_n(\rho, \theta), \qquad \varphi_n(\rho, \theta) = H_n^{(2)}(k\rho)\exp(jn\theta) \tag{1}$$

where $c_n(N)$ is the unknown expansion coefficients, φ_n is the modal function which satisfies the Helmholtz equation, and $H_n^{(2)}$ is the Hankel function of the second kind. The natural frequencies, which are the singularities of the scattered field in a complex frequency domain, can be characterized as the eigenfrequencies of the scattered field. According to the formulation of the eigenvalue problems described in Ref. [4][5], our problem is reduced to the problem of minimizing the following positive definite Hermitian form :

$$\Omega(\gamma_N, N) = \boldsymbol{C}^*(N)\, H(\gamma_N, N)\, \boldsymbol{C}(N) \tag{2}$$

under the constraint

$$C^*(N) \cdot C(N) = 1 \tag{3}$$

where $C(N)$ is the column vector whose elements are unknown expansion coefficients, $H(\gamma_N, N)$ is the $(2N+1) \times (2N+1)$ Hermitian matrix whose elements are inner product of the modal functions on the surface, $\gamma_N = k_N a$ is a frequency normalized by a characteristic length a such as radius of circumscribed circle, and asterisk denotes the Hermite conjugate of. Therefore, our problem is reduced to the problem of searching the complex frequency γ_N which minimizes the positive definite Hermitian form $\Omega(\gamma_N, N)$. It is guaranteed that the approximate natural frequency γ_N uniformly converges to the true natural frequency as the number of truncation N tends to infinity [5]. Furthermore, the extended version called "Yasuura's method with smoothing procedure" [10] is also available for accelerating the convergence of the solution.

In actual numerical calculation, we discretize the Hermitian form and employ the QR decomposition algorithm in order to reduce the amount of numerical computation [6].

3. EXTRACTION OF NATURAL FREQUENCIES BY PRONY'S METHOD

In this section, we briefly explain about the extraction of the natural frequencies from the pulse response by using Prony's method [2][3]. The late time transient response can be represented as a summation of exponentially damped sinusoids, i.e.,

$$I(t) = \sum_{m=1}^{M} A_m \exp(s_m t) \tag{4}$$

where the s_m are the poles in the complex frequency domain and correspond to the natural frequencies to be found. We can find the s_m from a discrete set of sampled transient data $I(n\Delta t)$ $(n=0,1,2, \ldots, 2M-1)$ where Δt is the size of the time-stepping interval. For convenience, we denote $I(n\Delta t)$ by I_n. Then I_n satisfy the linear difference equation of order N which may be written as

$$\sum_{p=0}^{M} \alpha_p I_{p+k} = 0, \qquad p+k = n = 0, 1, 2, \cdots, 2M-1 \tag{5}$$

where the roots of the algebraic equations

$$\sum_{p=0}^{M} \alpha_p Z^p = 0 \tag{6}$$

are $\exp(s_m \Delta t) = Z_m$, $m=1, 2, \ldots, M$. If in Eq.(5) α_M is defined equal to 1, then the α_p may be obtained by solving

$$\sum_{p=0}^{M-1} \alpha_p I_{p+k} = -I_{M+k} . \tag{7}$$

Once the α_p have been found, then the roots Z_m of Eq.(6) can be found and poles are obtained by

$$s_m = (\ln Z_m) / \Delta t . \tag{8}$$

4. NUMERICAL RESULTS AND DISCUSSIONS

As the example, we choose the peanut shaped object which is expressed as follows:

$$\rho'(\theta') = (a^2 \cos^2\theta' + b^2 \sin^2\theta')^{1/2}, \tag{9}$$

where a and b are major and minor axes respectively (see Fig.1). For convenience, we define the parameter of deformation $\delta = b/a$ $(0 \le \delta \le 1)$. When $\delta = 1$ the cross section of the object is a circle of radius a, and when δ is small the cross-section of it has concave-convex portions.

First, we show the natural frequencies of this object obtained by using Yasuura's method for eigenvalue problems as described in Section 2. Here we consider the case of H-polarization. Table 1 shows the obtained some dominant natural frequencies when $\delta = 1$ (circular cylinder) and $\delta = 0.5$ (peanut shaped cylinder). This result shows that the natural frequencies of a circular cylinder are degenerate and they split into two sub-layers when the object is deformed [4]. For circular cylinder we can easily show that the natural frequencies are the roots of $dH_n^{(2)}(\gamma)/d\gamma = 0$ for H-polarization. The index n in Table 1 corresponds to the order of the Hankel function and index μ represents the label of splitting sub-layers.

Next, we extract the natural frequencies from the pulse responses by applying Prony's method. As the incident pulse, we choose the modulated Gaussian pulse. Table 2 shows the extracted natural frequencies for $\theta_i = 0°$, $90°$ and $45°$. When $\theta_i = 45°$ both of the splitting poles are extracted, but when $\theta_i = 0°$ and $90°$ one of them is extracted and the other is not. From this result, we can find that the extracted natural frequencies depend upon the incident angle, and this fact indicates that the incident pulse selectively excites the natural modes. This fact can be explained by employing the field distribution of the natural modes on the surface. Taking into account the symmetry of the object and the quantum condition (resonance condition) of the natural modes, the field distributions of the natural modes on the surface are considered as illustrated in Fig.2 (four modes are shown). By considering the symmetry of the object, we can easily find that the modes of case 1 and 3 are excited for $\theta_i = 0°$, the modes of case 2 and 3 are excited for $\theta_i = 90°$, and all modes are excited for $\theta_i = 45°$. If we assume that the field distributions of case1, 2, 3 and 4 in Fig.2 are correspond to the natural modes labeled $(n,\mu) = (1,0)$, $(1,1)$, $(2,0)$ and $(2,1)$ respectively, then we can consistently explain the result of extraction of Table 2.

5. CONCLUSIONS

We extract the natural frequencies from the pulse responses by applying Prony's method and investigate the relationship between the excitation pulse and the excited natural modes. It is found from this results that the natural modes are selectively excited corresponding to the incident direction of the excitation pulse.

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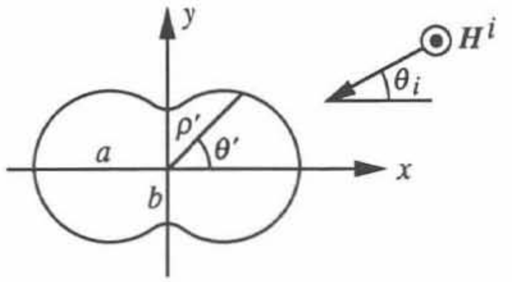

Fig. 1. Cylindrical object expressed by
$\rho'(\theta') = (a^2 \cos^2\theta' + b^2 \sin^2\theta')^{1/2}$

Table 1. Natural frequencies obtained by Yasuura's method for eigenvalue problems.

n	$\delta = 1.0$ (circle)	$\delta = 0.5$ (peanut)
1	$0.501 + j\,0.644$	$0.554 + j\,0.864$ ($\mu = 0$) $0.662 + j\,0.687$ ($\mu = 1$)
2	$1.434 + j\,0.835$	$1.717 + j\,0.904$ ($\mu = 0$) $1.733 + j\,1.066$ ($\mu = 1$)
3	$2.374 + j\,0.968$	$2.873 + j\,1.194$ ($\mu = 0$) $2.815 + j\,1.080$ ($\mu = 1$)

Table 2. Natural frequencies extracted by Prony's method. ($\delta = 0.5$).

n	μ	$\theta_i = 0°$	$\theta_i = 90°$	$\theta_i = 45°$
1	0	$0.572 + j\,0.894$	———	$0.565 + j\,0.820$
	1	———	$0.670 + j\,0.678$	$0.637 + j\,0.699$
2	0	$1.717 + j\,0.904$	$1.704 + j\,0.906$	$1.719 + j\,0.902$
	1	———	———	$1.745 + j\,1.051$
3	0	$2.878 + j\,1.197$	———	$2.877 + j\,1.198$
	1	———	$2.819 + j\,1.092$	$2.823 + j\,1.099$

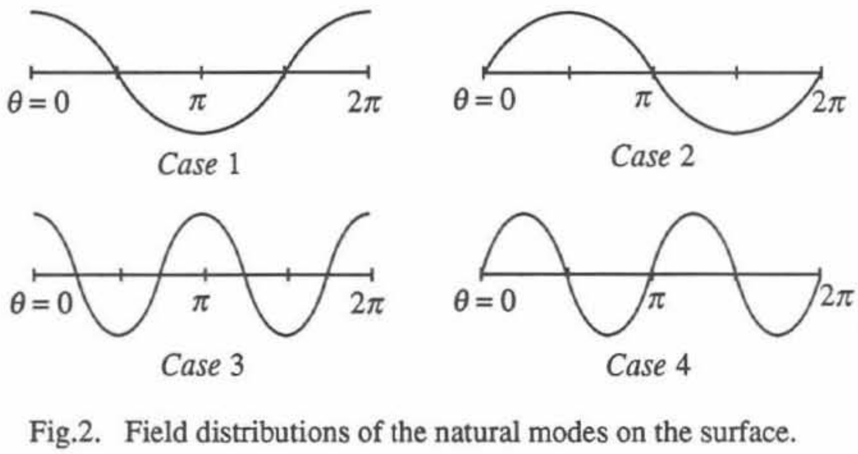

Fig.2. Field distributions of the natural modes on the surface.