BACKSCATTERING FROM A PERFECTLY CONDUCTIVE SLIGHTLY RANDOM SURFACE: CROSS POLARIZATION

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This paper deals with the depolarization in the backscattering from a perfectly conductive slightly random surface by a probabilistic theory developed by the author[1][2]. We present a new formula for the depolarized backscattering cross section, which cannot be obtained by Rice's perturbation method[3]. The perfectly conductive surface is assumed to be a homogeneous and isotropic gaussian random field written by a Wiener integral:

$$z = f(T^{\mathbf{r}}\omega) = \int_{R^2} F(\lambda)\, e^{i\lambda r}\, dB(\lambda) \tag{1}$$

where ω is a probability parameter denoting a sample point in the sample space Ω, T is the measure-preserving transformation in Ω, $\mathbf{r}=(x,y)$ and $\lambda=(\lambda_x,\lambda_y)$ are two-dimensional vectors on the x-y plane $R^2 = (-\infty \le x, y \le \infty)$, and $dB(\lambda)$ is the complex gaussian random measure with $\langle dB(\lambda)\rangle=0$, $\langle dB(\lambda')dB(\lambda)\rangle = \delta(\lambda+\lambda')d\lambda d\lambda'$, the angle brackets denoting the averaging over Ω. $|F(\lambda)|^2$ is the roughness spectrum of the random surface, which, for numerical calculation, will be assumed to be gaussian:

$$|F(\lambda)|^2 = \frac{\sigma^2 \kappa^2}{\pi} \exp(-\kappa^2 \lambda^2) \tag{2}$$

where σ^2 is the RMS height of the surface and κ is the correlation radius. We denote the electric field by $E(z,\mathbf{r},\omega)$, which satisfies $\mathbf{n} \times E = 0$ on the random surface. For a slightly rough surface with gentle sloping [$k\sigma \ll 1$ and $\kappa > \sigma$] ,however, we employ the effective boundary condition on the average surface z = 0 [4]

$$\mathbf{E}_0(0, \mathbf{r},\omega) + f(T^{\mathbf{r}}\omega)\frac{\partial}{\partial z}\mathbf{E}_0(0, \mathbf{r},\omega) + \mathrm{grad} f(T^{\mathbf{r}}\omega)\cdot E_z(0, \mathbf{r},\omega) = 0, \tag{3}$$

where we have put $E = \mathbf{E}_0 + \mathbf{e}_z E_z$, subscripts o and z indicating the projection of a three dimensional vector on the x-y plane and its z component, respectively. We write the wave vector k and the polarization vectors associated with k as

$$k = \mathbf{k}_0 + e_z k_z,\ k_0^2 = k_0^2,\ k_z = \sqrt{k^2 - k_0^2}$$

$$\mathbf{e}_H(\mathbf{k}) = \mathbf{k}_0 \times \mathbf{e}_z / k_0,\ \mathbf{e}_V(k) = k \times \mathbf{e}_H(\mathbf{k})/k \tag{4}$$

where subscript H and v indicate horizontal and vertical polarization respectively. We then write the electric field as

$$E(z,\mathbf{r},\omega) = E^0(z,\mathbf{r}) + E^S(z,\mathbf{r},\omega) \tag{5}$$

where E^0 is the unperturbed electric field over the nonfluctuating surface z = 0 and E^S is the perturbed field due to the surface roughness. For a horizontally polarized plane wave incident we write

$$E^0(z,\mathbf{r}) = e^{iK_0x}\,[\, -\mathbf{e}_H(\mathbf{K}')e^{-iK_zz} + \mathbf{e}_H(\mathbf{K})e^{iK_zz}\,] \tag{6}$$

$$\mathbf{K}' = \mathbf{K}_0 - K_z\mathbf{e}_z,\quad \mathbf{K} = \mathbf{K}_0 + K_z\mathbf{e}_z,\quad \mathbf{K}_0 = K_0\,\mathbf{e}_x = k\sin(\theta_0)\mathbf{e}_x$$

$$K_z = (\,k^2 - K_0^2)^{(1/2)} = k\cos(\theta_0) > 0,\ |\mathbf{K}|=|\mathbf{K}'| = k \tag{7}$$

where the plane of incidence has been taken in the x-z plane, $\mathbf{K}'$ and $\mathbf{K}$ are the wave vectors of incident wave and of the specularly reflected wave respectively, and θ_0 is the angle of incidence shown in Fig. 1. Regarding the scattered field as a stochastic functional of $dB(\boldsymbol{\lambda})$ the complex gaussian random measure, we represent $E^s(z, \mathbf{r}, \omega)$ in terms of Wiener-Hermite expansion:

$$\begin{aligned}
\mathbf{E}^s(z,\mathbf{r},\omega) &= e^{iK_0x}\left\{ \mathbf{e}_H(\mathbf{K})A_0e^{iK_zz} + \mathbf{e}_V(\mathbf{K})B_0e^{iK_zz} \right\} \\
&+ e^{iK_0x}\int_{R^2}\left\{A_1(\boldsymbol{\lambda})\mathbf{e}_H[\boldsymbol{\lambda}] + B_1(\boldsymbol{\lambda})\mathbf{e}_V[\boldsymbol{\lambda}]\right\}e^{i\boldsymbol{\lambda}\mathbf{r} + ik_z(\boldsymbol{\lambda})z}\,h^{(1)}[dB(\boldsymbol{\lambda})] \\
&+ e^{iK_0x}\int_{R^2}\int_{R^2}\left\{A_2(\boldsymbol{\lambda},\boldsymbol{\lambda}')\mathbf{e}_H[\boldsymbol{\lambda}+\boldsymbol{\lambda}'] + B_2(\boldsymbol{\lambda},\boldsymbol{\lambda}')\mathbf{e}_V[\boldsymbol{\lambda}+\boldsymbol{\lambda}']\right\} \\
&\times e^{i(\boldsymbol{\lambda}+\boldsymbol{\lambda}')\mathbf{r} + ik_z(\boldsymbol{\lambda}+\boldsymbol{\lambda}')z}\,h^{(2)}[dB(\boldsymbol{\lambda}),dB(\boldsymbol{\lambda}')] + \cdots
\end{aligned} \tag{8}$$

where we have put $\mathbf{e}_H[\boldsymbol{\lambda}] = \mathbf{e}_H(\mathbf{k}(\boldsymbol{\lambda}))$ and $\mathbf{e}_V[\boldsymbol{\lambda}]=\mathbf{e}_V(\mathbf{k}(\boldsymbol{\lambda}))$ and

$$\mathbf{k}(\boldsymbol{\lambda}) = \mathbf{k}_0(\boldsymbol{\lambda}) + \mathbf{e}_zk_z(\boldsymbol{\lambda}),\qquad \mathbf{k}_0(\boldsymbol{\lambda}) = \mathbf{K}_0 + \boldsymbol{\lambda},$$

$$k_z(\boldsymbol{\lambda}) = \sqrt{k^2 - \mathbf{k}_0^2(\boldsymbol{\lambda})},\ \mathrm{Im}[k_z(\boldsymbol{\lambda})] \geq 0,\ k_z(0) = K_z, \tag{9}$$

The $\boldsymbol{\lambda}$'s are Bragg vectors relating to scattering directions, $h^{(n)}$'s are random functions called the Wiener-Hermite differentials, A_n and B_n are deterministic coefficients of functional expansion to be solved. Once A_n and B_n are obtained, we have the random wave field by (8) and hence we can calculate any statistical properties of the scattering. Particularly, the depolarized backscattering cross section $\sigma_{hv}^B(\theta)$ can be given by

$$\begin{aligned}
\sigma_{hv}^B(\theta) &= 4\pi k^2\cos^2(\theta)\Big\{ |B_1(-2k\sin\theta\,\mathbf{e}_x)|^2 \\
&+ 2!\int_{R^2}|B_2(-2k\sin\theta\,\mathbf{e}_x - \boldsymbol{\lambda}',\ \boldsymbol{\lambda}')|^2\,d\boldsymbol{\lambda}' + \cdots\Big\},
\end{aligned} \tag{10}$$

where subscript hv indicates horizontal-transmission vertical reception. We substitute (1)(5)(6) and (8) into (3) the approximate boundary condition, and use the orthogonality relation and the recurrence formula of $h^{(n)}$, given in Ref.[1]. Then we obtain a set of equations for A_n and B_n, which are approximately solved to have a second order solution involving A_0,B_0,A_1, B_1, A_2,and B_2. Inserting B_1

and B_2 so obtained into (10), we find the depolarized backscattering cross section:

$$\sigma_{hv}^{B}(\theta) = \frac{32\pi k^8 \cos^4(\theta)}{|1+\cos\theta\, Z_{sh}(k\sin\theta)|^2 \; |\cos\theta + Z_{sv}(k\sin\theta)|^2}$$

$$\times \int_0^{2\pi}\int_0^{\infty} |F(q^+)F(q^-)|^2 \cos^2(\alpha)\sin^2(\alpha)$$

$$\times \left| \frac{(1-p^2)^{(1/2)}}{1+(1-p^2)^{(1/2)} Z_{sh}(kp)} - \frac{1}{(1-p^2)^{(1/2)} + Z_{sv}(pk)} \right|^2 p\,dp\,d\alpha, \quad (11)$$

where we have put $q^{\pm} = k[(\sin\theta \pm p\cos\alpha)^2 + p^2\sin^2\alpha]^{(1/2)}$. Z_{sh} and Z_{sv} are corrections due to multiple scattering. They are effective surface impedance for horizontally and vertically polarized wave

$$Z_{sh}(K_0) = k\int_{R^2} \left[k_z(\lambda) + \frac{\lambda_y^2}{k_z(\lambda)} \right] |F(\lambda)|^2 \, d\lambda, \quad (12)$$

$$Z_{sv}(K_0) = \frac{1}{k}\int_{R^2} \left[K_z^2 k_z(\lambda) + \frac{k^2\lambda_x^2}{k_z(\lambda)} \right] |F(\lambda)|^2 \, d\lambda, \quad (13)$$

If we put $Z_{sh}=0$ and $Z_{sv}=0$, Eq. (11) becomes identical with the result by the second order perturbation; but it diverges unphysically. Because of the corrections due to multiple scattering, however, our solution (11) always yields a finite backscattering cross section. For the gaussian roughness spectrum (2) the two-dimensional integrals in (11)-(13) are easily reduced to one-dimensional ones, which are evaluated numerically. Figure 2 illustrates $\sigma_{hv}^{B}(\theta)$ for several values of $k\chi$. We also deal with a case where a vertically polarized plane wave is incident. It is then shown that $\sigma_{hv}^{B}(\theta) = \sigma_{vh}^{B}(\theta)$ holds.

References

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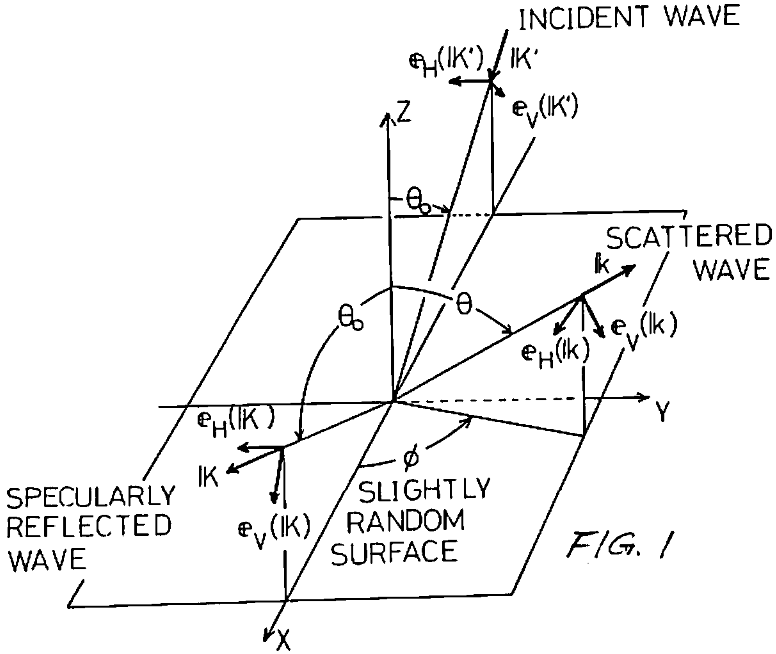

FIG. 1

FIG. 1 Scattering of electromagnetic plane wave from a perfectly conductive slightly random surface.

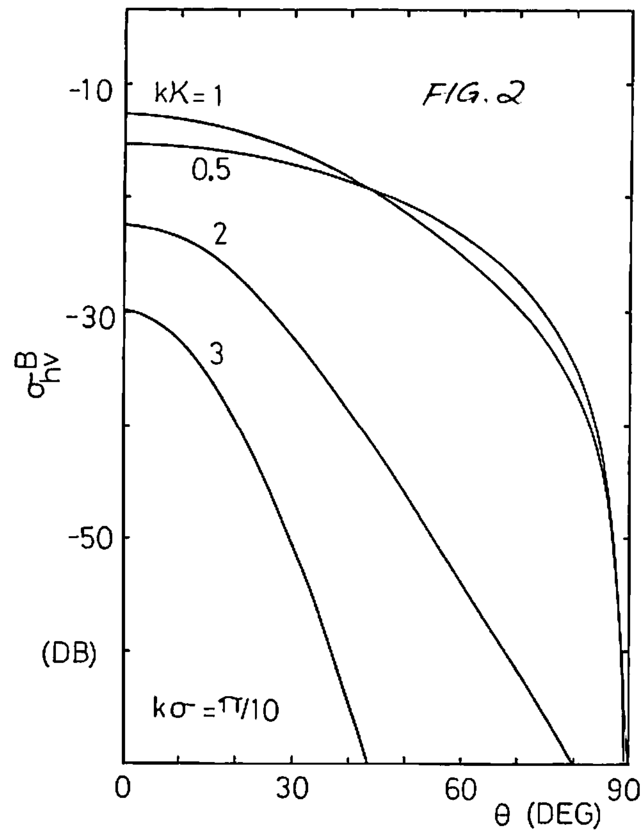

FIG. 2

FIG. 2 Depolarized backscattering cross section for the gaussian roughness spectrum (2). σ and K indicates RMS surface height and the correlation distance of the random surface, respectively.