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Numerical Analysis of Multiple Scattering from 3-D Arbitrarily Shaped Dielectric Objects

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1. Introduction

Analysis of an electromagnetic wave scattering by a system of three-dimensional (3-D) dielectric objects has been important for various science and engineering disciplines, and the multiple scattering from simply shaped objects such as spheres and spheroids have been analyzed extensively using quasi-analytical techniques.[1]-[3] In addition to such works, in order to obtain more precise scattering properties of variously shaped objects, multiple scattering from nonspherical and nonspheroidal objects have also been analyzed using some numerical techniques.[4]-[6]

One of the reliable methods to analyze a 3-D scattering from variously shaped objects is the Yasuura method. Using this method, single scattering from variously shaped objects have been analyzed and valuable numerical data such as radar cross sections and pulse responses have been provided so far.[7]-[9] Recently, this method has been applied to the multiple scattering from perfectly conducting objects and the scattering from Chevichev particle with concave surface has been analyzed.[10],[11]

In this paper, the Yasuura method to analyze the multiple scattering by 3-D dielectric objects with various shapes is shown. To formulate the problem, the scattered field outside the objects is expressed by a superposition of individual fields scattered by each objects and the individual field is expanded by Hankel function based spherical vector wave functions. Also, inside each object, the transmitted field is expanded by Bessel function based spherical vector wave functions. By matching the boundary condition in a least square sense using an appropriate discterizing rule, the unknown coefficients of the expansions are computed and numerical solutions that converge to true ones can be obtained. Using the computed coefficients, various scattering properties in the near and the far region can be computed. Here, radar cross sections of two dielectric objects are computed as a function of a distance between them and the multiple scattering effects are examined. Throughout this paper, the time factor $e^{j\omega t}$ is assumed and suppressed.

2. Formulation of the problem

Here we consider a system of arbitrarily shaped 3-D dielectric objects illuminated by a plane wave with a wave number k. The number of the objects and the centers of each objects are denoted by N_s and $O_i (i = 1, 2, \cdots, N_s)$, respectively, and the permittivity and the permeability of the ith object are given ε_i and μ_i, respectively as shown in Figure 1. The direction of incidence and the angle between the incident electric field vector and the plane of incidence are denoted by $(\theta_{inc}, \phi_{inc})$ and α, respectively. The scattered electric field $\mathbf{E}^s$ at a point P outside the objects is given by a superposition of individual scattered fields from each of the objects such as $\mathbf{E}^s(\mathbf{r}) = \sum_{i=1}^{N_s} \mathbf{E}_i^s(\mathbf{r}_i)$, where $\mathbf{r}$ and $\mathbf{r}_i$ are position vectors of the point P in the global coordinate system of the origin O and ith coordinate system of the origin O_i, respectively and $\mathbf{E}_i^s$ denotes the scattered electric field from the ith object. Referring to the Yasuura method [8], [9], the scattered electric field from

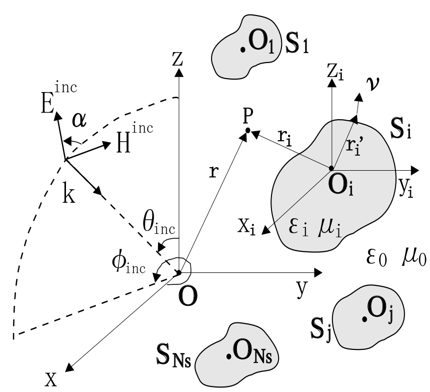

Fig.1 Geometry of the problem.

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the ith object is approximated as

$$\mathbf{E}^s_{i,N}(\mathbf{r}_i) = \sum_{n=1}^{N} \sum_{m=-n}^{n} \{a^i_{mn}(N)\mathbf{m}^{(4)}_{mn}(\mathbf{r}_i) + b^i_{mn}(N)\mathbf{n}^{(4)}_{mn}(\mathbf{r}_i)\} \tag{1}$$

where $a^i_{mn}(N)$ and $b^i_{mn}(N)$ are unknown coefficients to be determined, N is a truncation size, and $\mathbf{m}^{(4)}_{mn}$ and $\mathbf{n}^{(4)}_{mn}$ are spherical vector wave functions [12] defined by

$$\mathbf{m}^{(4)}_{mn}(\mathbf{r}_i) = \nabla \times \{\mathbf{r}_i h^{(2)}_n(kr_i) Y_{mn}(\theta_i, \phi_i)\}, \qquad \mathbf{n}^{(4)}_{mn}(\mathbf{r}_i) = \frac{1}{k}\nabla \times \mathbf{m}^{(4)}_{mn}(\mathbf{r}_i). \tag{2}$$

In Eq.(2), (r_i, θ_i, ϕ_i) denotes a spherical coordinate of the point P, and $h^{(2)}_n$ and Y_{mn} are a spherical Hankel function of the second kind and a spherical harmonic, respectively.

The transmitted field inside the jth object is also expanded by the spherical vector wave functions as

$$\mathbf{E}^t_{i,N}(\mathbf{r}_i) = \sum_{n=1}^{N} \sum_{m=-n}^{n} \{c^i_{mn}(N)\mathbf{m}^{(1)}_{mn}(\mathbf{r}_i) + d^i_{mn}(N)\mathbf{n}^{(1)}_{mn}(\mathbf{r}_i)\} \tag{3}$$

where $c^i_{mn}(N)$ and $d^i_{mn}(N)$ are unknown coefficients to be determined, and $\mathbf{m}^{(1)}_{mn}$ and $\mathbf{n}^{(1)}_{mn}$ are spherical vector wave functions defined by substitutingg h^2_n with the spherical Bessel function j_n in Eq.(2)

Using the Faraday's law and the relations $\nabla \times \mathbf{m} = k\mathbf{n}$ and $\nabla \times \mathbf{n} = k\mathbf{m}$, we can obtain approximate expressions of the different magnetic ($\mathbf{H}$) fields from those of the corresponding electric ($\mathbf{E}$) fields

The unknown coefficients $a^i_{mn}(N)$, $b^i_{mn}(N)$, $c^i_{mn}(N)$ and $b^i_{mn}(N)$ in Eq.(1) are determined so as to match the boundary condition on the surface of the objects in a least square sense, that is, so as to minimize a discretized norm defined by

$$\begin{aligned}
&\Omega(N, L_\theta, L_\phi) = \\
&\quad \sum_{j=1}^{N_s}[\sum_{I=1}^{L_\theta}\sum_{J=1}^{L_\phi} |\nu(\mathbf{r}'_{j,IJ}) \times \{\sum_{i=1}^{N_s} \mathbf{E}^s_{i,N}(\mathbf{r}'_{j,IJ}) + \mathbf{E}^{inc}(\mathbf{r}'_{j,IJ}) - \mathbf{E}^t_{j,N}(\mathbf{r}'_{j,IJ})\}|^2 J(\mathbf{r}'_{j,IJ})\omega(\mathbf{r}'_{j,IJ}) \\
&\qquad / \sum_{I=1}^{L_\theta}\sum_{J=1}^{L_\phi} |\nu(\mathbf{r}'_{j,IJ}) \times \mathbf{E}^{inc}(\mathbf{r}'_{j,IJ})|^2 J(\mathbf{r}'_{j,IJ})\omega(\mathbf{r}'_{j,IJ})] \\
&\quad +[\sum_{I=1}^{L_\theta}\sum_{J=1}^{L_\phi} |\nu(\mathbf{r}'_{j,IJ}) \times \{\sum_{i=1}^{N_s} \mathbf{H}^s_{i,N}(\mathbf{r}'_{j,IJ}) + \mathbf{H}^{inc}(\mathbf{r}'_{j,IJ}) - \mathbf{H}^t_{j,N}(\mathbf{r}'_{j,IJ})\}|^2 J(\mathbf{r}'_{j,IJ})\omega(\mathbf{r}'_{j,IJ}) \\
&\qquad / \sum_{I=1}^{L_\theta}\sum_{J=1}^{L_\phi} |\nu(\mathbf{r}'_{j,IJ}) \times \mathbf{H}^{inc}(\mathbf{r}'_{j,IJ})|^2 J(\mathbf{r}'_{j,IJ})\omega(\mathbf{r}'_{j,IJ})]
\end{aligned} \tag{4}$$

where L_θ and L_ϕ denote the total numbers of sampling points concerning θ_i and ϕ_i, respectively, and ν, J and ω are the normal unit vector, the Jacobian and the weight at the sampling point $\mathbf{r}'_{j,IJ}$, respectively, and $\mathbf{E}^{inc}$ and $\mathbf{H}^{inc}$ denote incident electric and magnetic fields, respectively as shown in Fig.1. The prime of $\mathbf{r}'_{j,IJ}$ indicates that the variable pertains to the surface of the objects. In Eq.(4), if $i \neq j$, the spherical vector wave functions $\mathbf{m}^{(4)}_{mn}(\mathbf{r}'_i)$ and $\mathbf{n}^{(4)}_{mn}(\mathbf{r}'_i)$ that expand the scattered field $\mathbf{E}^s_{i,N}$ are translated from the ith coordinate system to the jth one using a vector addition theorem [13],[14]. By calculating the unknown coefficients that minimize the discretized norm, we can obtain numerical solutions that converges true ones as the truncation size N increases.[15]

3. Accuracy checks and numerical examples

In order to check the accuracy of solutions and investigate scattering properties of multiple objects, here we consider two dielectric nonspherical objects whose surface are described by

$$r_i'(\theta_i') = a_i(1 + \delta_i \cos \eta_i \theta_i'), \quad | \delta_i | < 1, \quad i = 1, 2, \cdots, N_s \tag{5}$$

where a_i, δ_i, and μ_i denote the mean radii, deformation parameters, and waviness parameters of the objects, respectively. These objects are located on z-axis and the distance between the centers of the objects is denoted by d

First, in order to determine the total number of sampling points L_θ and L_ϕ, convergence properties of the discretized norms $\Omega(N, L_\theta, L_\phi)$ as a function of L_θ and L_ϕ for a fixed truncation size N are computed. The results are shown in Fig.2 and it is seen that each discretized norm $\Omega(N, L_\theta, L_\phi)$ takes constant values when $L_\theta/(N+1)$ and $L_\phi/(N+1)$ are greater than 2. In Figs.3 and 4, the discretized norms and radar cross sections as a function of the truncation size N are shown when L_θ and L_ϕ are fixed to $2(N+1)$. It is shown that the discretize norm monotonically decrease and the radar cross sections converge as N increases. From these results, the total numbers of sampling points should be $L_\theta = L_\phi = 2(N+1)$.

To investigate the multiple scattering effect, radar cross sections of two nonspherical objects located on z-axis are computed. In Fig. 5, the radar cross sections as a function of d for different permittivity ε_i are shown. Due to the multiple scattering effects, it is seen that the radar cross sections have oscillatory behaviors, and the nearer the two objects located, the more strongly the radar cross sections oscillate. This effect is more striking as the permittivity takes larger values.

4. Conclusion

Multiple scattering by 3-D arbitrarily shaped dielectric objects is formulated by using the Yasuura method. By expanding the scattered fields and the transmitted fields by spherical vecor wave functions and matching the boundary condition with the sampling points $L_\theta = L_\phi = 2(N+1)$, it is shown that numerical solutions converging to true ones can be obtained. To examine the effect of multiple scattering, radar cross sections of two nonspherical objects as a function of distance between them are computed. It is shown that the coupling effect appears on the radar cross section as an oscillatory behavior.

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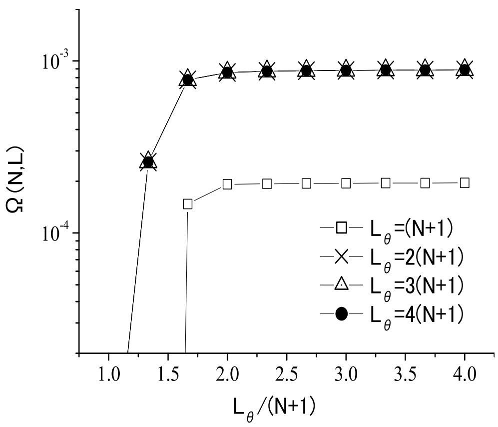

Fig.2 Discretized norms vs. total numbers of sampling points. ($N = 5, N_s = 2, \eta_1 = \eta_2 = 2, \delta_1 = \delta_2 = 0.1, a_1 = a_2 = a, d = 3a, ka = 1, \theta_{inc} = \pi/2, \phi_{inc} = 0, \alpha = 0$)

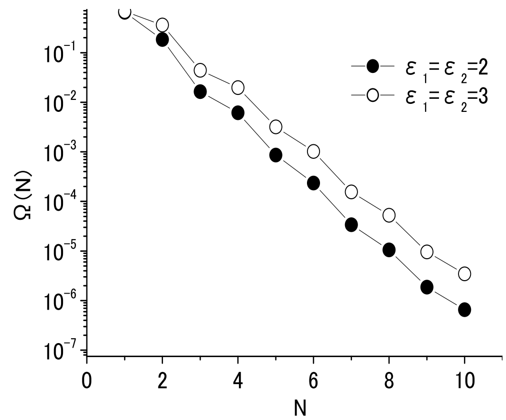

Fig.3 Discretized norms vs. truncation size. ($N_s = 2, \eta_1 = \eta_2 = 2, \delta_1 = \delta_2 = 0.1, a_1 = a_2 = a, d = 3a, ka = 1, \theta_{inc} = \pi/2, \phi_{inc} = 0, \alpha = 0$)

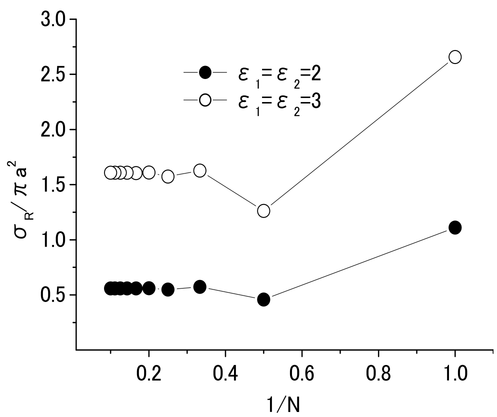

Fig.4 Radar cross section vs. truncation size. ($N_s = 2, \eta_1 = \eta_2 = 2, \delta_1 = \delta_2 = 0.1, a_1 = a_2 = a, d = 3a, ka = 1, \theta_{inc} = \pi/2, \phi_{inc} = 0, \alpha = 0$)

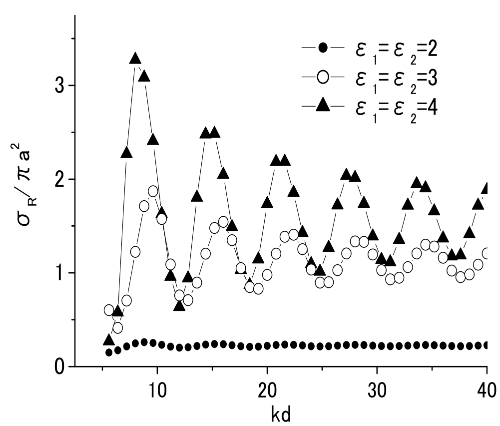

Fig.5 Radar cross sections vs. distance between the centers of two objects d. ($N_s = 2, a_1 = a_2 = a, \eta_1 = \eta_2 = 2, \delta_1 = \delta_2 = 0.1, ka = 2, \theta_{inc} = \pi/2, \phi_{inc} = 0, \alpha = 0$)