Study on the design of maximum directivity of a receiving array with mutual coupling

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1. Introduction

Reciprocity theorem holds between a transmitting antenna pattern and a receiving antenna pattern[1], if its feeding circuit is passive one. In a Yagi-Uda array and a reactance loaded array, which is often called ESPAR antenna[2], it can be verified analytically that the transmitting antenna pattern is identical to the receiving one[3]. However, it may not be guaranteed in the case of an adaptive array, outputs of the elements of which are weighted and summed by digital beamformers.

In this paper, in order to ensure the reciprocity of the directional pattern between the transmitting array and the receiving one, we investigate whether the directional pattern of the receiving array with maximum directivity coincide to that of the transmitting array which is excited to obtain maximum directivity under the existence of mutual coupling[4]. To achieve this purpose, first we derive the expression of the directivity of the receiving array with mutual coupling. Next, we investigate the matched load impedance for each element of the array under the existence of mutual coupling and derived the simultaneous equation for determining the matched loads. Nevertheless we use the correct formula for the receiving array directivity and the matched load for each element, there is discrepancy between the transmitting array pattern and the receiving one. Considering this problem, we show that the output voltages across the matched load impedance should not be summed up, but that they should be summed after transformed by the matching circuit. Finally, we present the result that the receiving array pattern with maximum directivity coincide to that of the transmitting array.

2. Directivity of a receiving array antenna

To obtain a maximum directivity of a receiving array, it is necessary to derive an equation of the directivity and to know a matched load impedance of each element of the array with mutual coupling between elements. In this section, we derive the expression of the receiving array antenna based on the definition of the directivity of a receiving antenna and obtain the matched load impedances of elements.

2.1 Directivity of a receiving antenna [5]

Figure 1 shows a receiving array and the coordinates for its analysis. In this figure, the array consists of dipole elements and is arranged linearly, however, both a kind of elements and the arrangement of elements can be arbitrary. Suppose the direction of the incident plane wane E_i is $(q_i, j_i,)$. The voltage of each element X can be expressed by the following equation,

$$\mathbf{X}=\mathbf{Z}_L\left(\mathbf{?}_a+\mathbf{Z}_L\right)^{-1}\mathbf{X}' \tag{1}$$

, where Z_a is the generalized impedance matrix of the array antenna, Z_L is the load impedance matrix only with diagonal elements and X' is the vector determined by the incident plane wave. We set the maximum direction of the array antenna to be $q=q_0$, $j=j_0$, then the directivity of the array can be expressed following the definition of the directivity of a receiving antenna as follows.

$$G_r(q_0,j_0)=\frac{\left[\text{Receiving power from the direction of }(q_0,j_0)\right]}{\dfrac{1}{4p}\left[\text{Total receiving power}\right]} =\frac{\left|\sum_{i=1}^{N}x_i(q_0,j_0)w_i^*\right|^2}{\dfrac{1}{4p}\sum_{i=1}^{N}\int_0^{p}\int_0^{2p}\left\{\left|x_i(q,j)w_i^*\right|^2\big/2\,\mathrm{Re}(Z_{Li})\right\}dq\,dj} \tag{2}$$

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Here ,w_i's are the weights for the outputs of the elements and correspond to the excitation coefficients of the transmitting array. When the incident waves are in the horizontal plane, G_r in equation (2) can be expressed by the next equation,

$$Gr(j_0) = \frac{\left| x_1(j_0)w_1^* + x_2(j_0)w_2^* + \cdots + x_N(j_0)w_N^* \right|^2}{P_{L1}^a |w_1|^2 + P_{L2}^a |w_2|^2 + \cdots + P_{LN}^a |w_N|^2} \tag{3}$$

, where P_{Li}^a's are the average power received by the i-th element. If the elements are identical and the mutual coupling does not exist, all P_{Li}^a's are the same and the expression of the directivity by eq.(3) equals to that of the SNR (Signal to the Noise Ratio) of the receiving array. The value of the directivity of eq.(3) depends on the weights w_i. The weights which maximize the directivity can be determined by the method of Lagrange multipliers and are obtained as follows .

$$w_i = \frac{x_i(j_0)}{P_{Li}^a} \tag{4}$$

2.2 Matched loads for elements of an array

In analysis of an adaptive array antenna, the complex conjugate of the self-impedance of an element, Z_{ii} is usually used as the load impedance of the element. However, mutual couplings exist between elements, Z_{ii} is not matched load to the output of the element any more. To realize the design of the receiving array with the maximum directivity, we should find the matched load impedances for the elements with mutual couplings between elements, but they have not been obtained except two elements which corresponds to two elements diversity. The matched load of the output terminal should be equal to the complex conjugate of the output impedance of that element, when the matched load impedances are connected at the other output terminals as shown in Fig. 2. For example, at the terminal 1-1', the following equation can be derived[5].

$$Z_{in1} = \frac{V_1}{I_1} = \frac{\left\| \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & & \\ \vdots & & \ddots & \\ Z_{N1} & \cdots & \cdots & Z_{NN} \end{bmatrix} + \begin{bmatrix} 0 & 0 & & 0 \\ 0 & Z_{L2} & & \\ & & \ddots & \\ 0 & & & Z_{LN} \end{bmatrix} \right\|}{\left\| \begin{bmatrix} Z_{22} & \cdots & \cdots & Z_{2N} \\ \vdots & \ddots & & \\ \vdots & & \ddots & \\ Z_{N2} & \cdots & \cdots & Z_{NN} \end{bmatrix} + \begin{bmatrix} Z_{L2} & 0 & & 0 \\ 0 & \ddots & & \\ & & \ddots & \\ 0 & & & Z_{LN} \end{bmatrix} \right\|} = Z_{L1}^* \tag{5}$$

Equation (5) can also be expressed as Equation (6) which is convenient for the solution by the numerical method.

$$\begin{vmatrix} Z_{11} - Z_{L1}^* & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} - Z_{L2} & & \\ \vdots & & \ddots & \\ Z_{N1} & & & Z_{NN} - Z_{LN} \end{vmatrix} = 0 \tag{6}$$

As for the other terminals, similar equations are derived as the equation (6), which is the simultaneous equation, the unknowns of which are Z_{L1}, Z_{L2}, …, and Z_{LN}. This simultaneous equation is non-linear, but the numerical solution can effectively be obtained in the case of five element array[3].

3. Antenna pattern of the receiving array with maximum directivity

We calculated the receiving antenna pattern with maximum directivity using equation (3) and (4) Moreover, the load impedances are determined by equation (6), instead of Z_{ii}. Figure 3 shows the calculation result with the corresponding pattern of the transmitting array of three elements. Judging from the reciprocal property of the transmitting and receiving array, the patterns of them should be coincide, however, they are different, especially, in a main beam region. We are going to investigate the reason of the difference in the next section.

4. Determination of the output voltages of elements

In section three, we calculate the array pattern by weighting the output voltages across the load impedances Z_{Li}'s and afterward summing them, which seem to be equivalent load impedances, but not real loads. In a practical radio frequency circuit, terminal loads are real and antenna outputs are transformed through the matching circuits from complex quantities to real ones. Figure 4 and 5 are the illustrations, how we determine the output voltages which are weighted and summed into a total output of the array. Figure 6 shows the calculation result of the receiving array pattern which are almost coincide to the transmitting array pattern, which is solved by using the property of eigenvalue of Hermit quadratic form.

5. Conclusion

In order to solve the problem of maximizing the directivity of a receiving array antenna with mutual coupling, we derive the expression of the directivity based on the definition of the directivity of a receiving antenna. The simultaneous equation for determining the matched loads of the array elements with mutual coupling was derived and solved. This equation is very general and can be used for arbitrary elements and configuration of the array. We also showed that the terminal output voltage should be summed into a total output after transformed from complex quantity to real one by matching circuit. Finally, we showed the receiving array pattern with maximum directivity coincided to the transmitting array one. These properties between transmitting arrays and receiving ones can be applied to solve various antenna design problerms for optimizing radio channels.

References

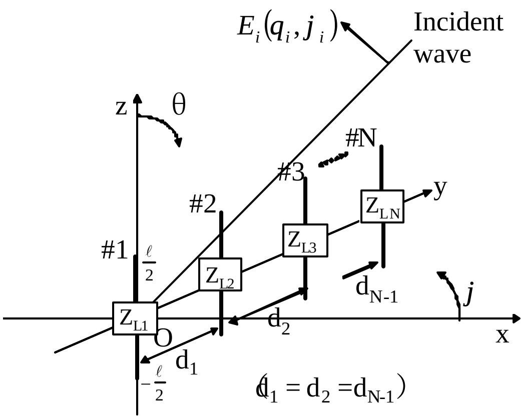
[1] H Uchida, and Y. Mushiake, "VHF-Antenna," Chapter 2, pp.35-50, Corona pub. co., Aug. 1966..

[2] T. Ohira, and K. Iigusa, "Electronically steerable parasitic array radiator antenna," IEICE Trans. on Electron., vol.J87-C, no.1, pp.12.31, Jan. 2004.

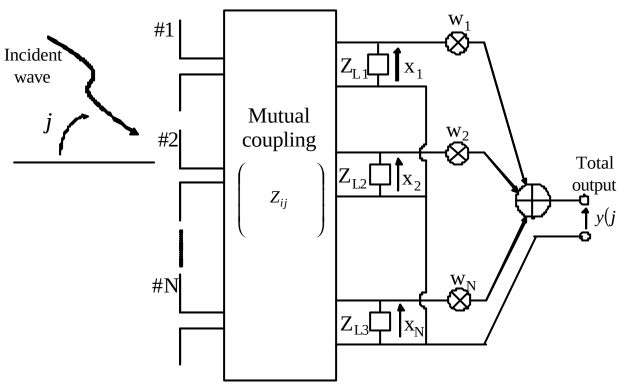
[3] K. Kagoshima, S. Obote, Y. Ichikawa, M. Yamane, and T. Tanaka, "Pattern synthesis of and array antenna with maximum directivity and nulls at specified directions," Fifth Int. Conf. on Antenna Theory and Techniques (ICATT '05), in Kiev, Ukraine, May 24-27, 2005, to be presented.

[4] N. Inagaki, and T. Sekiguchi, "Exact design of an array of linear antennas with optimum directivity," IEICE Trans. on Commun., vol.53-B, no.11, pp.687-692, Nov. 1970.

[5] K. Kagoshima, S. Obote, Y. Ichikawa, and M. Yamane, "Study on the maximum receiving power and voltage in an array antenna with mutual coupling," Technical report of IEICE, A•P2004-191, pp.5-10, Dec. 2004.



(a) Element arrangement and its coordinate



(b) Equivalent circuit

Fig. 1 Receiving dipole array and its equivalent circuit.

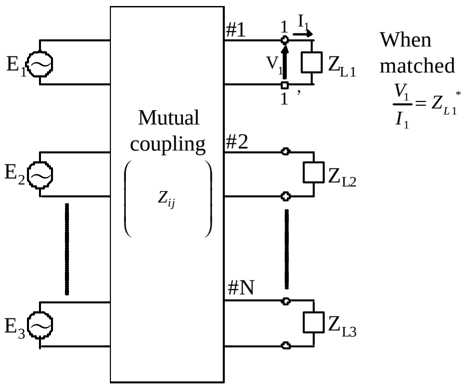


Fig. 2 Equivalent circuit of a receiving array.

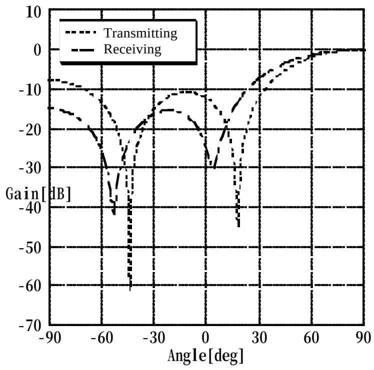
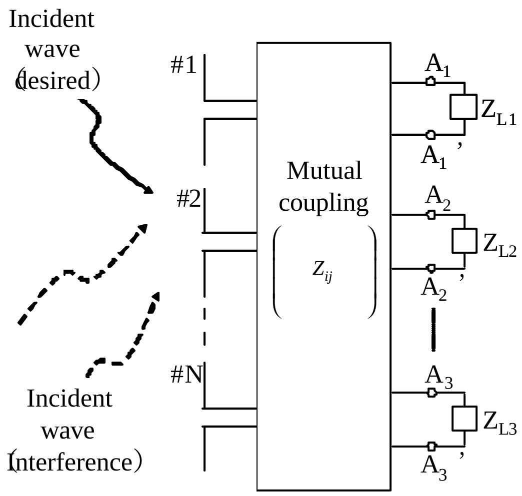
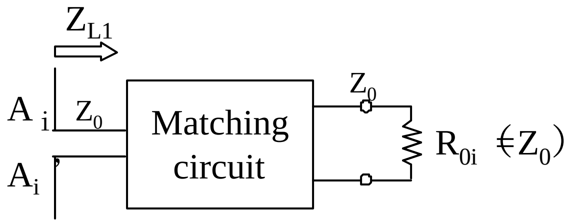


Fig. 3 Comparison of antenna pattern of transmitting mode and receiving mode.

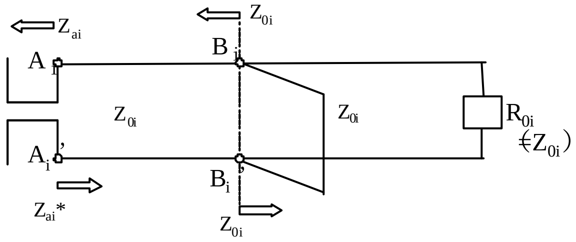


(a) Outer problem

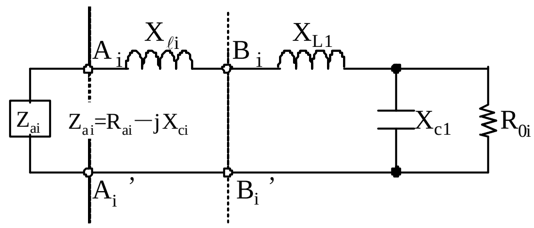


(b) Inner problem

Fig. 4 Equivalent circuit for calculating output voltages of a receiving array.

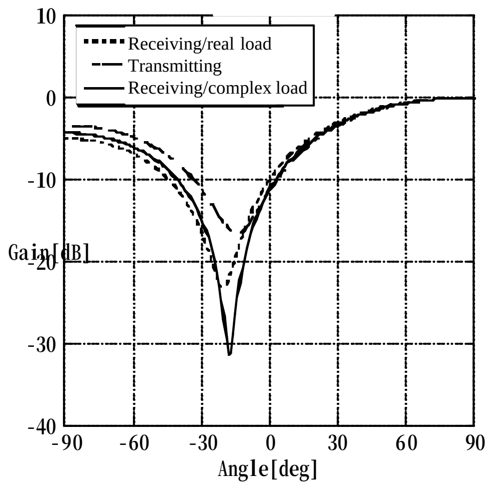


(a) Stub type

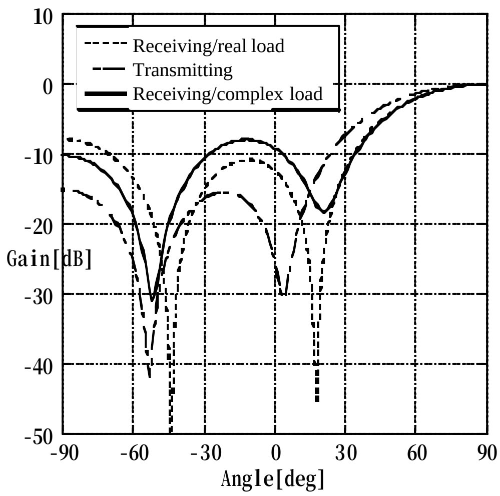


(b) L, C type

Fig. 5 Matching circuit for antenna elements.



(a) 2 element ($d_1=0.25\,l$, $\ell_1=0.5\,l$, $\ell_2=0.45\,l$)



(b) 3 element ($d_1= d_2=0.25\,l$, $\ell_1=\ell_2=\ell_3=0.5\,l$)

Fig. 6 Comparison of a transmitting and a receiving antenna pattern.