On an Expression of Antenna Factor for Transmitting Small Loop Antenna in Liquid

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Abstract – **To evaluate specific absorption rate (SAR) of radio devices operated in MHz band, a small loop antenna can be used to calibrate a probe for estimating field intensity in the tissue-equivalent liquid. In our proposed method, field intensity radiated by a small loop antenna can be expressed in terms of its antenna factor in transmitting mode. In this paper, distance property of the antenna factor is analytically derived to determine a fitting curve.**

Index Terms — **Probe calibration, MHz band, SAR, Antenna factor for transmitting loop, Liquid.**

1. Introduction

With increasing use of radio devices operated in MHz, for example, transmitting/receiving devices for wireless power transmission (WPT), specific absorption rate (SAR) of such devices, which is one of evaluation indexes for radio-wave protection guidelines, should be measured. At the frequency of 6.78 MHz, where WPT system would be used, we proposed a method based on antenna factor in transmitting mode for a reference loop antenna, which is referred as loop antenna factor (LAF) in this paper [1]. It is an extension of the method to find field intensity in terms of LAF in free space in MHz band derived by Greence [2]. Our modification is to change $j\beta$ into $\gamma = \alpha + j\beta$, or attenuation constant in the medium, α, is added to the original formulation, where β is phase constant in the medium or liquid in this paper. The procedure and related formulation for evaluating the field intensity in the liquid radiated by the reference circular loop antenna is described in [1]. We use an extension of Greene's formulation and 2-antenna method, which can be used to determine the far-field gain of the reference loop antenna by facing two identical antennas.

In the process of finding the field intensity, we use the antenna factor in transmitting mode, LAF, of the reference loop antenna. This quantity is an important ones in our formulation, and in practice it is convenient to express LAF as a function of the distance from the antenna. In this paper, we analytically derive an approximation of LAF in terms of the distance, and show some examples for related curve fitting.

2. Derivation of Distance Property of LAF

As shown in Fig. 1, two identical antennas are faced in the lossy homogenous medium or liquid with perfect polarization matched. S parameters between the two antennas can be measured as a function of the distance between them, d. Then, LAF can be expressed as

$$F_t = \left[\frac{1}{2\omega\mu K(d)} \frac{|S_{21}|}{\sqrt{(1-|S_{11}|^2)(1-|S_{22}|^2)}}\right]^{1/2}, \tag{1}$$

where

$$K(d) = \frac{\sqrt{(1+\alpha R_0)^2 + (\beta R_0)^2}\, e^{-\alpha R_0}}{2\pi R_0^3} \times \left\{1 + \frac{15}{8}\left(\frac{a}{R_0}\right)^4 + \frac{315}{64}\left(\frac{a}{R_0}\right)^8\right\}, \tag{2}$$

$$R_0 = \sqrt{d^2 + 2a^2}\,, \tag{3}$$

and a is the radius of the circular loop antenna. The above derivation can be found in [1].

R_0 in the exponential term included in $K(d)$ can be approximated as

$$R_0 = d\left(1 + \frac{2a^2}{d^2}\right)^{1/2} \approx d\left(1 + \frac{a^2}{d^2}\right) = d + \frac{a^2}{d}, \tag{4}$$

provided $r \gg a$. In the other terms, R_0 can be approximated as $R_0 = d$. Then, $K(d)$ can be expanded as

$$\begin{aligned} K(d) &\approx \frac{\sqrt{(1+\alpha d)^2 + (\beta d)^2}\, e^{-\alpha d} e^{-\frac{\alpha a^2}{d}}}{2\pi d^3} \\ &\quad \times \left\{1 + \frac{15}{8}\left(\frac{a}{R_0}\right)^4 + \frac{315}{64}\left(\frac{a}{R_0}\right)^8\right\} \\ &= \frac{\sqrt{\alpha^2+\beta^2}}{2\pi d^2}\sqrt{1 + \frac{2\alpha}{\alpha^2+\beta^2}\frac{1}{d} + \frac{1}{\alpha^2+\beta^2}\frac{1}{d^2}} \\ &\quad \times e^{-\alpha d}\left\{1 + \left(-\frac{\alpha a^2}{d}\right) + \frac{1}{2!}\left(-\frac{\alpha a^2}{d}\right)^2 + \cdots\right\} \\ &\quad \times \left\{1 + \frac{15}{8}\left(\frac{a}{R_0}\right)^4 + \frac{315}{64}\left(\frac{a}{R_0}\right)^8\right\} \\ &= \frac{\sqrt{\alpha^2+\beta^2}}{2\pi}\frac{e^{-\alpha d}}{d^2}\left(1 + \frac{a_1}{d} + \frac{a_2}{d^2} + \cdots\right), \end{aligned} \tag{5}$$

where $a_1, a_2, \cdots$ are constants which include α, β, and a. On the other hand, the term including $|S_{21}|$, $|S_{11}|$ and $|S_{22}|$ in (1) can be expanded as [3]

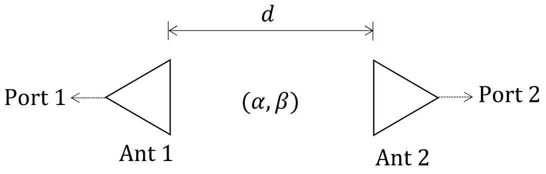

Fig. 1. Configuration of two identical antennas in lossy homogenous medium of (α, β).

$$\frac{|S_{21}|}{\sqrt{(1-|S_{11}|^2)(1-|S_{22}|^2)}} = \frac{G_f e^{-\alpha d}}{2\beta d}\left(1+\frac{b_1'}{d}+\frac{b_2'}{d^2}+\cdots\right) \quad (6)$$

where G_f is the far-field gain of the loop antenna, and $b_1', b_2', \cdots$ are constants. By substituting (5) and (6) into (1), LAF can be expressed as

$$F_t = \left(d\,\frac{b_0 + b_1/d + b_2/d^2 + \cdots}{1 + a_1/d + a_2/d^2 + \cdots}\right)^{1/2}, \quad (7)$$

where $b_0 = G_f/4\pi\omega\mu\beta\sqrt{\alpha^2+\beta^2}$, $b_k = b_0 b_k'$ $(k = 1,2,\cdots)$. Then, the distance property of LAF can be also expressed as

$$F_t \approx \sqrt{d}\,\frac{c_3 + c_4/d + c_5/d^2}{1 + c_1/d + c_2/d^2}, \quad (8)$$

where $c_1 = a_1/2$, $c_2 = a_2/2 - a_1/8$, $c_3 = \sqrt{b_0}$, $c_4 = b_1/2\sqrt{b_0}$, and $c_5 = b_2/2\sqrt{b_0} - b_1^2/8b_0^{3/2}$.

For example, in the close region of the antenna where the distance from the antenna is very small or $d \ll 1$, LAF can be approximated as

$$F_t = \left(\frac{b_1 + b_2/d + b_3/d^2 + \cdots}{1 + a_1/d + a_2/d^2 + \cdots}\right)^{1/2}, \quad (9)$$

Thus, we can see that LAF converges with a constant, $\sqrt{b_1}$ as $1/d$ approaches zero.

3. Some Examples of Curve Fitting

Fig. 2 shows the distance property of measured LAF for our shielded circular loop antennas [4] with the perimeter length of $C = 50$ mm and 200 mm operated in the saline solution with the concentration of 0.04 mol/l [5], at the frequency of 6.78 MHz. In the same graphs, fitted curves are also plotted. The fitting ranges from 5 mm to 100 mm for $C = 50$ mm, and from 5 mm to 200 mm for $C = 200$ mm. As seen from these figures, the fitted curves are in good agreement with the measured data except the noise fluctuation. Table 1 lists resultant values of coefficients in the curve (8). As seen from the table, c_3 can be ignored so that the choice of (9) would be reasonable as our fitting curve.

4. Conclusion

To calibrate the probe for SAR evaluation in MHz band, it is convenient to express antenna factor in transmitting mode of small loop antenna as a function of the distance from the antenna. In this paper, an approximation of the above factor is analytically derived and validated by fitting corresponding measured data.

Acknowledgment

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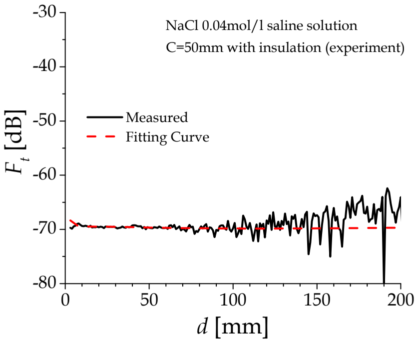
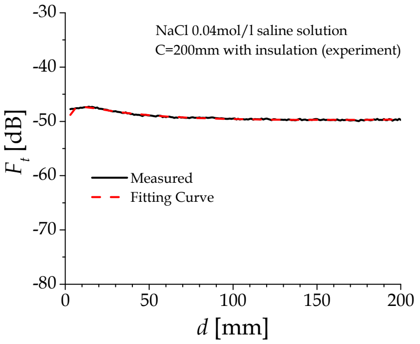


(a) $C = 50$mm



(b) $C = 200$mm

Fig. 2. Measured LAF and related fitted curves.

TABLE I
Values of coefficients for fitted curve in (8)

C [mm]	50	200
c_1 [mm]	1.57×10^{1}	9.79
c_2 [mm^2]	1.53×10^{-2}	3.93×10^{-1}
c_3	1.21×10^{-5}	1.25×10^{-5}
c_4 [mm]	2.85×10^{-3}	2.33×10^{-2}
c_5 [mm^2]	1.26×10^{-2}	-2.32×10^{-3}

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