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Effective medium parameters for distributions of clustering helices with structural deviations

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1. Introduction

Artificial complex media for radio waves such as chiral media [1], double negative materials [2] etc. have attracted attentions in recent years. The effective medium parameters have been analyzed especially for chiral ones by the quasi-static Lorentz approach [1],[3]-[6], the method of multiple scattering [7], methods of inverse scattering [8] etc. considering the particles' structures and mixing manners. In the fields of carbon microcoils (CMC), helical structures such as chiral materials interacting with radio waves have been of interest because of the fine absorption properties of CMC at frequencies of tens through hundreds GHz [9]-[10], although the bulk of CMC is usually composed of racemic mixture of coils. Sometimes they are used in clustering forms embedded into acrylic beads or foams. Even if coils are scattered in a homogeneous host materials, possibility of forming clusters here and there is to be admitted. Investigations of complex media composed of clustering particles including these cases are significant. Effects of electromagnetic coupling between clustering helices have been investigated [6]. For such structures to be realistic, dispersion of dimensions of particles and clusters should also be taken into account. The non-cluster cases of helices with their frequency characteristics variously shifted according to several kinds of distributions have been reported [5].

In this work, distributions of randomly oriented clusters of perfectly conducting helices considering deviations of structural dimensions in free space are analyzed by quasi-static Lorentz approach to determine the effective medium parameters. Currents on wires of clustering helices are calculated by the method of moments (MoM) with thin wire approximation utilizing Numerical Electromagnetics Code 2 (NEC2) [11] to obtain the polarizabilities. Sizes and spacings of clusters should be small compared to a wavelength for quasi-static approximation.

2. Description of the Problem

Only right or only left-handed N_{hlx} kinds of perfectly-conducting wire helices in free space are considered in this work. The l th kind of helices are assumed to be distributed with number density D_{nl} (helices/m^3) and have dimensional parameters of number of turns T_l, radius A_l, pitch P_l and radius and length of wire W_l and L_l respectively. The dimensions of the pitches are assumed to be deviated from the means $P_{l,0}$ as $P_l = P_{l,0} + \delta_l$, $|\delta_l| \le \delta_{\max}$ where $D_{nl} \cong D(P_l) = (1/\sigma\sqrt{2\pi})\exp(-\delta_l^2/2\sigma^2)$ ($l = 1,\cdots,N_{hlx}$), i.e. approximately obeying the Gaussian distributions with means $P_{l,0}$ and standard deviations σ. The radius A_l varies with the pitch P_l on condition that wire length L_l is fixed. Media realized by homogeneous distributions of randomly oriented N_{cls} kinds of clusters of different helices are considered. The number density of the k th kind of clusters is D_{nk}^{cls} (clusters/m^3). Randomly sampled N_k helices form each of the k th kind of clusters so that the external spherical regions of radius R_k are filld with N_k circumscribed spheres i.e. individual regions of helices to the utmost limit on condition that the distances between them are not shorter than the maximum wire gauge. They are rotated for random angles with respect to their axes which are also randomly oriented.

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3. The Method of Analysis

Time harmonic depencence $e^{j\omega t}$ is assumed, and the permittivity and the permeability in free space are denoted by ε_0 and μ_0 respectively hereafter. The effective relative permittivity $\varepsilon_{eff} = \varepsilon'_{eff} + j\varepsilon''_{eff}$, the effective relative permeability $\mu_{eff} = \mu'_{eff} + j\mu''_{eff}$ and the effective chirality (Lindell-Sihvola notation) $\kappa_{eff} = \kappa'_{eff} + j\kappa''_{eff}$ are assumed for the effective medium realized by homogeneous distributions of randomly-oriented clusters of helices. They are defined in the macroscopic constitutive relations between the average electric and magnetic flux densities and the fields $\boldsymbol{D}$, $\boldsymbol{B}$, $\boldsymbol{E}$ and $\boldsymbol{H}$ respectively:

$$\boldsymbol{D} = \varepsilon_0\varepsilon_{eff}\boldsymbol{E} - j\kappa_{eff}\sqrt{\varepsilon_0\mu_0}\boldsymbol{H}, \quad \boldsymbol{B} = \mu_0\mu_{eff}\boldsymbol{H} + j\kappa_{eff}\sqrt{\varepsilon_0\mu_0}\boldsymbol{E} \tag{1}$$

where the effective medium is assumed to be reciprocal. Flux densities can be expressed by electric and magnetic polarizations $\boldsymbol{P}_{ee}$, $\boldsymbol{P}_{em}$, $\boldsymbol{P}_{me}$ and $\boldsymbol{P}_{mm}$:

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}_{ee} + \boldsymbol{P}_{em}, \quad \boldsymbol{B} = \mu_0\boldsymbol{H} + \boldsymbol{P}_{me} + \boldsymbol{P}_{mm}, \quad \boldsymbol{P}_{ij} = \sum\nolimits_{k=1}^{N_{cls}} D_{nk}^{cls}\boldsymbol{p}_{ijk}. \tag{2}$$

$\boldsymbol{P}_{ij}$ $(i, j = e \text{ or } m)$ denotes electric $(i = e)$ or magnetic $(i = m)$ polarization i.e. dipole moment density due to electric $(j = e)$ or magnetic $(j = m)$ field. $\boldsymbol{p}_{ijk}$ $(k = 1,\cdots, N_{cls,}\ \ i, j = e \text{ or } m)$ denotes dipole moment of the k th kind of a single cluster and is related to the Lorentzian fields $\boldsymbol{E}_L$ or $\boldsymbol{H}_L$ by a polarizability tensor $\bar{\alpha}_{ijk}$:

$$\boldsymbol{p}_{iek} = \bar{\alpha}_{iek}\boldsymbol{E}_L, \quad \boldsymbol{p}_{imk} = \bar{\alpha}_{imk}\boldsymbol{H}_L, \\ \boldsymbol{E}_L = \boldsymbol{E} + (\boldsymbol{P}_{ee} + \boldsymbol{P}_{em})/3\varepsilon_0 \ , \quad \boldsymbol{H}_L = \boldsymbol{H} + (\boldsymbol{P}_{me} + \boldsymbol{P}_{mm})/3\mu_0 . \tag{3}$$

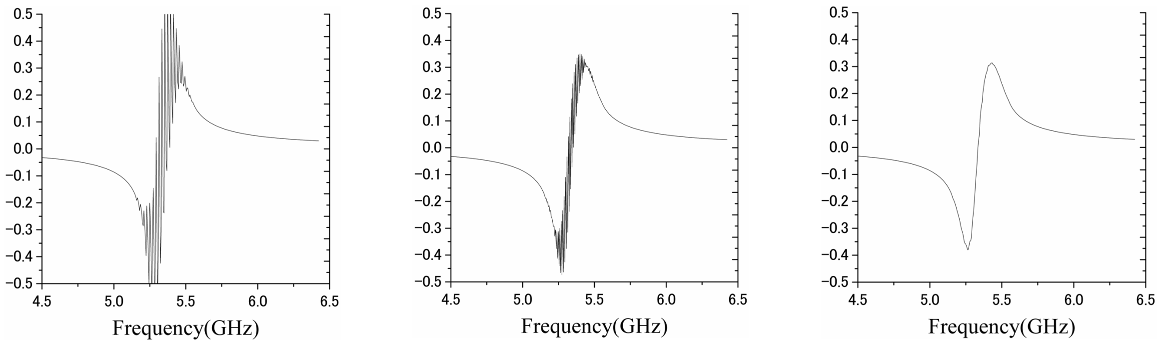
Dipole moments $\boldsymbol{p}_{ejk}$ and $\boldsymbol{p}_{mjk}$ of a cluster excited by $\boldsymbol{E}_L(j = e)$ and $\boldsymbol{H}_L(j = m)$ can be obtained from the currents $\boldsymbol{J}(\boldsymbol{r})$ on the wires of helices :

$$\boldsymbol{p}_{ejk} = \frac{1}{j\omega}\int \boldsymbol{J}(\boldsymbol{r})dv, \quad \boldsymbol{p}_{mjk} = \frac{1}{2}\int \boldsymbol{r}\times\boldsymbol{J}(\boldsymbol{r})dv \tag{4}$$

where $j = e$ or m. The elements of the 3×3 polarizability tensors can be obtained from Eq.(2) through Eq.(4). In this paper, scalar polarizabilities for isotropic media realized by randomly-oriented clusters are obtained by taking the averages of diagonal elements of the tensors, discarding the off-diagonal ones as in [4]. These polarizabilities are used to determine the effective medium parameters from the Lorentz-Lorentz formulae [1] derived from Eq.(1) through Eq.(3). Calculations of Eq.(4) in the Cartesian coordinate for the excitations of fields $\boldsymbol{E}_L = (E_x, E_y, E_z)$ and $\boldsymbol{H}_L = (H_x, H_y, H_z)$ are performed in the following procedure as in [3],[4]. The contribution of E_i in general is obtained by taking the average of the contributions of i-polarized waves propagating in $\pm j$ and $\pm k$ directions where i, j, k correspond to different characters of x, y and z. For the same meaning of i, j and k, the contribution of H_i is obtained by taking the average of half of the difference of the contributions of j-polarized waves propagating in $\pm k$ directions and that of k-polarized waves propagating in $\pm j$ directions. In this case, if (i, j, k) is either of (x, y, z), (y, z, x) and (z, x, y), the difference of the contributions of bilateral waves should be obtained by substracting those of positive directions from the negative ones, and otherwise vice versa. The size of a cluster should be less than around one tenth of a wavelength for a certain accuracy of solutions [3]. The currents on the wires of helices in Eq.(4) are calculated utilizing NEC2 code [11] where only axial components of currents on the wire surfaces are considered neglecting the circumferential variations for the thin-wire approximation in the moment method.

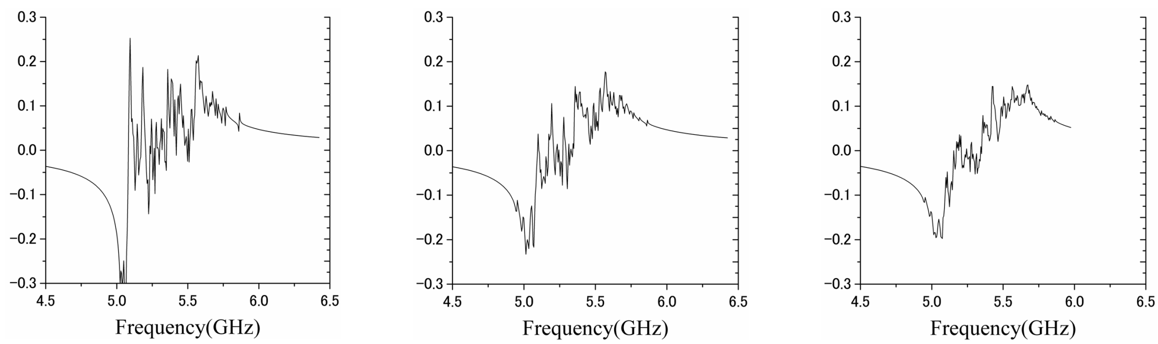
4. Numerical Study

In the following calculations, left-handed helices with T_l =7, W_l =0.0675(mm) and L_l =28.667(mm) $(l=1,\cdots,N_{hlx})$ are assumed. Angles of rotations of helices with respect to their axes, orientations of the axes are given by the uniform quasi-random numbers. Twenty basis functions are used for one turn of a helix in MoM. Dimension of the pitch of the lth kind of helices is assumed as $P_{l,0}$ =0.303(mm), $\delta_{\max}$ =0.1 $P_{l,0}$ and $\sigma=\delta_{\max}/3$ $(l=1,\cdots,N_{hlx})$. The number density of the lth kind of helices $D_{nl}\cong D(P_l)$ is given by the quasi-random numbers by Gaussian distributions with the exception of the non-cluster cases where the values of $D(P_l)$ = $(1/\sigma\sqrt{2\pi})\exp(-\delta_l^2/2\sigma^2)$, δ_l = $[2(l-1)/(N_{hlx}-1)-1]\delta_{\max}$, $l=1,\cdots,N_{hlx}$ are adopted. As for the number density of the k th kind of clusters, an identical value $D_{nk}^{cls}=\left(\sum_{l=1}^{N_{hlx}}D_{nl}\right)\Big/N_{cls}$ $(k=1,\cdots,N_{cls})$ is assumed. Figures 1 through 3 show the real part of the effective chiral parameters for the cases of separate helices (non-cluster cases), 3-helix clusters and 4-helix ones i.e. $N_k=N$=1, 3 and 4 ($k=1,\cdots,N_{cls}$) respectively for different values of N_{cls} and $N_{hlx}=N\cdot N_{cls}$ where $\sum_{l=1}^{N_{hlx}}D_{nl}$ =5×10^6 (helices/m^3). Results for non-cluster cases (Figure 1) converge to the curve of the authentic Cotton's effect as N_{cls} (and $N_{hlx}=N\cdot N_{cls}$) increases. As for the clustering cases (Figures 2 and 3), results gradually approach certain curves which could be superposition of effects of electromagnetic coupling between neighbouring helices [6] added on to the simple Cotton's effect. Calculations at the vicinity of or higher than resonant frequencies given by the lengths of wires of helices are less accurate than those at lower frequencies because of the limitation of the quasi-static approximation.



(a) (b) (c)

Figure 1: Real part of the effective chiral parameters for non-cluster cases ($N_{cls}=N_{hlx}$).
(a) N_{hlx}=20 (b) N_{hlx}=40 (c) N_{hlx}=100



(a) (b) (c)

Figure 2: Real part of the effective chiral parameters for 3-helix cluster cases. R_k =3.15(mm) ($k=1,\cdots,N_{cls}$), (a) N_{cls}=40 (N_{hlx}=120), (b) N_{cls}=120 (N_{hlx}=360), (c) N_{cls}=240 (N_{hlx}=720)

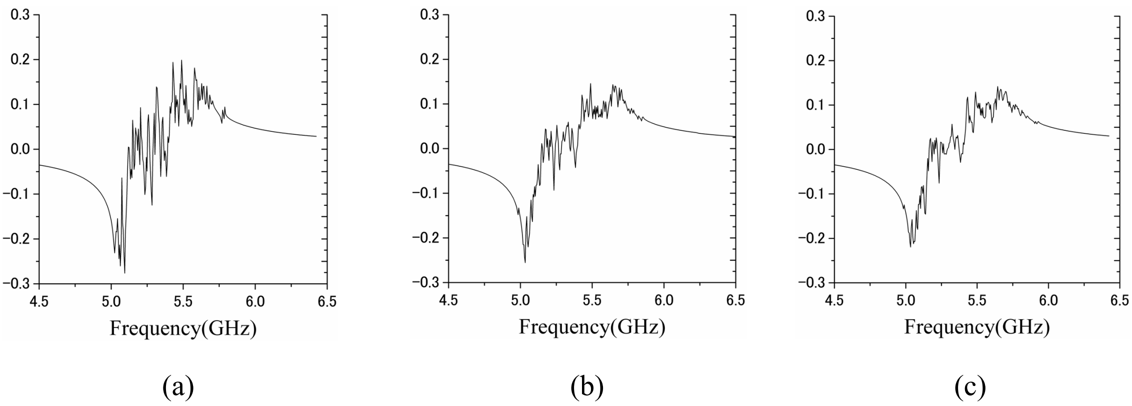


Figure 3: Real part of the effective chiral parameters for 4-helix cluster cases. R_k =3.3(mm) ($k = 1,\cdots,N_{cls}$), (a) N_{cls} =40 (N_{hlx} =160), (b) N_{cls} =120 (N_{hlx} =480), (c) N_{cls} =180 (N_{hlx} =720)

5. Conclusion

Calculations demonstrate that finer dispersion of deviations based on the increased kinds of clusters approach the authentic Cotton's effect for non-cluster cases and the transformations of that for clustering cases. Treatment of the models with greater variety woud contribute to the research on artificial media and the fields of biomimetics in material science. More rigorous analysis to avoid the dimensional limitations due to the quasi-static approximation is being considered.

References

[1] I. V. Lindell, A. H. Sihvola, S. A. Tretyakov and A. J. Viitanen, "Electromagnetic Waves in Chiral and Bi-Isotropic Media", Artech House, pp. 1-18, 193-243, 1994.

[2] *IEEE Trans. Antennas Propag., Special Issue on Metamaterials*, Vol. 51, No. 10, 2003.

[3] A. Ishimaru, S. Lee, Y. Kuga and V. Jandhyala, "Generalized constitutive relations for metamaterials based on the quasi-static Lorentz theory", *IEEE Trans. Antennas Propag., Special Issue on Metamaterials*, Vol. 51, No. 10, pp. 2550-2557, 2003.

[4] C. R. Brewitt-Taylor, P. G. Lederer F. C. Smith and S. Haq, "Measurement and prediction of helix-loaded chiral composites", *IEEE Trans. Antennas Propag.*, Vol. 47, No. 4, pp. 692-699, 1999.

[5] J. Reinert, J. Psilopoulos and A. F. Jacob, "On a statistical assessment of realistic chiral materials", *Electromagnetics*, Vol. 23, pp. 637-646, 2003.

[6] M. Asai and J. Yamakita, "Numerical study on waves from particles composed of bunches of helices", Proc. PIERS2006, Vol. 1, 2006

[7] Y. Nanbu, T. Matsuoka and M. Tateiba, "The effective constitutive parameters of a random medium containing small chiral spheres", *IEEJ Trans. FM*, Vol. 123, No. 3, pp. 259-264, 2003.

[8] C. Y. Chung and K. W. Whites, *J. Electro. Waves and Applic.*, Vol. 10, pp. 1363-1388, 1996.

[9] S. Motojima, *Papers of Technical Meeting on E.M.T.*, Vol. EMT-03-83, pp.65-, 2003.

[10] M. Asai, J. Yamakita, K. Matsumoto and H. Wakabayashi, "On electromagnetic chirality of helix-loaded materials", *Materials Integration*, Vol. 17, No. 7, pp. 27-33, 2004.

[11] G. J. Burke and A. J. Poggio, "Numerical electromagnetics code (NEC)- Method of moments, Part I-III", *Lawrence Livermore National Laboratory, Report*, UCID-18834, 1981.