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A Fast Algorithm of Green's Function for A Dielectric Layer with Ground Plane

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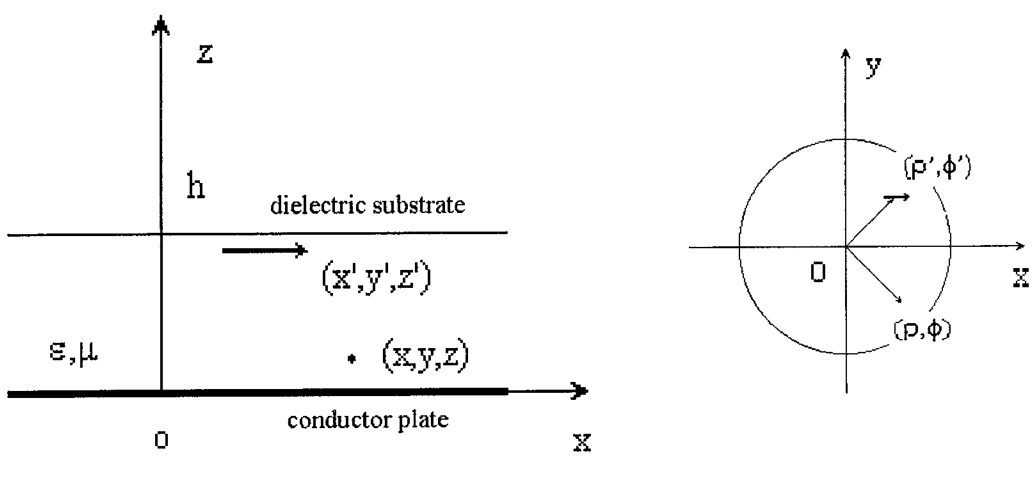
1. Introduction

A dielectric layer with ground plane is a typical structure for many electromagnetic problem, such as microstrip patch antenna, planar printed antenna, microwave circuits. When such structures are analyzed and designed, the computation of Green's function plays an important role. For complex structures or circuits printed in the substrate, the calculation in numeric methods is often time-consuming. Therefore, the fast computation of Green's function is indispensable. Many researchers pay attention in this problem.

Authors proposed a planar feeding circuit for CA-RLSA[1]. The circuit designed by microwave circuit principles has many advantages. Further researches are necessary to improve its performance, such as the analysis and design based on full-wave analysis results. Unfortunately, the Green's function, expressed in terms of either an infinite image expansion[2] or an infinite modal expansion[3], converges very slowly. Y.Rahmat-sammi proposed a way to speed up the convergence of such kind of series. But the result is only suitable for near field case. Authors also presented an accelerant method to calculate the Green's function of a horizontal dipole[4]. However, the result only can be used in the special cases for parallel plate waveguides. Spectral domain approach is powerful for multilayer, planar geometries. However, calculation of the inner products in spectral domain is also excessively time-consuming to be applied to complex structures. The complex image method[5][6] is good choice to overcome above disadvantages. The



(a) (b)

Fig.1 A horizontal dipole in a substrate with ground plane drawback of the method is that the complex mathematical pretreatment is required.

In this paper, a fast algorithm of Green's function for a dielectric layer with ground plane is proposed. The Green's function is expressed in terms of a common infinite image expansion in the spatial domain. An approximated closed-form of the infinite series is derived. The expression is simple and easy to be coded. Any complex mathematical pretreatment is unnecessary.

2. The expression

The structure and the coordinates used for driving the expression are shown in Fig.1. A horizontal dipole, which can be either an electric dipole or an equivalent magnetic dipole, is regarded as the exciting source. For an electric dipole, the Green's function expressed in terms of an infinite image expansion is:

$$\frac{4\pi}{\mu_0}G_A^{xx} = 4\pi\varepsilon_0 G_q = \sum_{n=0}^{\infty}\left[\frac{K^n e^{-jk_0 r_{n1}}}{r_{n1}} - \frac{K^n e^{-jk_0 r_{n2}}}{r_{n2}} - \frac{K^{n+1} e^{-jk_0 r_{n3}}}{r_{n3}} + \frac{K^{n+1} e^{-jk_0 r_{n4}}}{r_{n4}}\right] \quad (1)$$

where

$$\begin{aligned} r_{n1} &= \sqrt{|\vec{\rho}-\vec{\rho}'|^2 + (2nh+z-z_0')^2} \\ r_{n2} &= \sqrt{|\vec{\rho}-\vec{\rho}'|^2 + (2nh+z+z_0')^2} \\ r_{n3} &= \sqrt{|\vec{\rho}-\vec{\rho}'|^2 + [2(n+1)h-z-z_0']^2} \\ r_{n4} &= \sqrt{|\vec{\rho}-\vec{\rho}'|^2 + [2(n+1)h-z+z_0']^2} \end{aligned} \quad (2)$$

$$K = \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \quad (3)$$

where (ρ,φ,z) and (ρ',φ',z') are the position point and the source point respectively, and $\vec{\rho}$ and $\vec{\rho}'$ are the position vectors for them. ε_r is the relative dielectric constant of the substrate.

For an equivalent magnetic dipole, the result is:

$$\frac{4\pi}{\varepsilon_0}G_F^{xx} = 4\pi\mu_0 G_{qm} = \sum_{n=0}^{\infty}\left[\frac{K^n e^{-jk_0 r_{n1}}}{r_{n1}} + \frac{K^n e^{-jk_0 r_{n2}}}{r_{n2}} + \frac{K^{n+1} e^{-jk_0 r_{n3}}}{r_{n3}} + \frac{K^{n+1} e^{-jk_0 r_{n4}}}{r_{n4}}\right] \quad (4)$$

It can be seen from eq.(1) and eq.(4) that the convergent speed of those series is dependent on the feature of term $K^n e^{-jk_0 r_n}/r_n$ for both the electric and magnetic dipoles. So in the following part of the paper, our attention is focused on the problem of the convergence of the series which is defined again as:

$$g(\vec{r}'/\vec{r}) = \sum_{n=0}^{\infty}\frac{K^n e^{-jk_0 r_n}}{r_n} = \sum_{n=0}^{L}\frac{K^n e^{-jk_0 r_n}}{r_n} + R_L \quad (5)$$

where

$$R_L = \sum_{n=L+1}^{\infty}\frac{K^n e^{-jk_0 r_n}}{r_n} \quad (6a)$$

$$r_n = \sqrt{|\vec{\rho}-\vec{\rho}'|^2 + (2nh)^2} \quad (6b)$$

In r_n, $|z \pm z'|$ has been regarded as zero. The assumption would not disturb the availability of results because $|z \pm z'| \le 2h$. The index L is taken under the condition of $2Lh >> |\vec{\rho}-\vec{\rho}'|$, and then in the denominator of R_L , $r_n \cong 2nh$, but in the numerator,

$$r_n \cong r_L + (n-L)2h\cos\theta_L, \qquad n > L \qquad (7)$$

The definition of r_L, θ_L is same as that in [1]. Then R_L in eq.(6a) reduces to

$$R_L = \frac{1}{2h} e^{-jk_0(r_L - 2Lh\cos\theta_L)} \sum_{n=L+1}^{\infty} \frac{K^n e^{-j2nhk_0\cos\theta_L}}{n} \qquad (8)$$

The following integral can be proved easily:

$$-\frac{1}{2\pi}\int_0^{2\pi} \ln(1-Ke^{-jx})e^{jnx}dx = \frac{K^n}{n} \qquad (9)$$

From characteristics of Fourier Transform, the following formula can be obtained:

$$\sum_{n=1}^{\infty} \frac{K^n e^{-jnx}}{n} = -\ln(1-Ke^{-jx}) \qquad (10)$$

Then the Green's function $g(\vec{r}'/\vec{r})$ in eq.(5) is finally approximated in the following form:

$$g(\vec{r}'/\vec{r}) \cong g_L(\vec{r}'/\vec{r})$$
$$= \sum_{n=0}^{L} \frac{K^n e^{-jk_0 r_n}}{r_n} - \frac{1}{2h} e^{-jk_0(r_L - 2Lh\cos\theta_L)} [\ln(1-Ke^{-j2hk_0\cos\theta_L}) + \sum_{n=1}^{L} \frac{K^n e^{-j2nhk_0\cos\theta_L}}{n}] \qquad (11)$$

This equation is the final form of the approximation of the Green's function defined in eq.(5). The result approaches the exact form for $L \to \infty$. However, for practical applications, the smaller the parameter L, the more effective this approximation becomes. The truncation number L and the associated error are assessed below.

3. Discussions

In order to use the result obtained above, the truncation error should be assessed and the truncation number L should be determined. An error function is defined first as:

$$e(|\vec{\rho}-\vec{\rho}'|, h, L) = |[g_L(\vec{r}'/\vec{r}) - g(\vec{r}'/\vec{r})]/g(\vec{r}'/\vec{r})| \qquad (12)$$

Where $g_L(\vec{r}'/\vec{r})$ is the result calculated by eq.(11) and $g(\vec{r}'/\vec{r})$ is the accurate one obtained by letting L approach infinite.

Fig. 2 and Fig. 3 show the relative error for small ratios and large ratios of horizontal distance and height respectively. In Fig.2, the results against L taken in two ways are given. One way is that L is taken a different value in different sub-regions, namely, regarded as a segment line. In this example, the region in which ratios are from 0 to 10 is divided into six sub-regions. From left to right, L is taken as 5,12,18,21,24 and 27 respectively. The other way is to determine L by an empirical relationship between L and t which is

$$L = \text{int}[a_3 t^3 + a_2 t^2 + a_1 t + a_0] \qquad (13)$$

where $t = |\vec{\rho}-\vec{\rho}'|/h$. The function int[x] indicates the largest integer which does not exceed x. The relationship is determined by test in order to make the envelop of the error in the region smooth so that L is smaller for given errors in practical applications, namely, the efficiency of the computation is higher. Changing the coefficients in the equation can satisfy different requirements for uses in error. In this example, coefficients are 0.019, -0.65,6.74 and 4 respectively.

For large ratios, a similar relationship is also obtained, which is:

$$L = \text{int}[a * \log_b(t)] \qquad (14)$$

In Fig.3, several results are given for different parameters a and b. One may choose other values of the parameters for requirements in error in applications.

For all results, other parameters are: $\varepsilon_r = 9.6$, f=12GHz, h=0.6mm.

4. Conclusions

A fast algorithm of Green's function for a dielectric layer with ground plane has been proposed. The simple closed-form would make not only the mathematical pretreatment simple, but also code easy in use. So the algorithm is useful in engineering applications. The effective empirical relationships to take the truncation number L are also given for two cases. The desired accuracy can be obtained by simply changing the parameters in those relationships.

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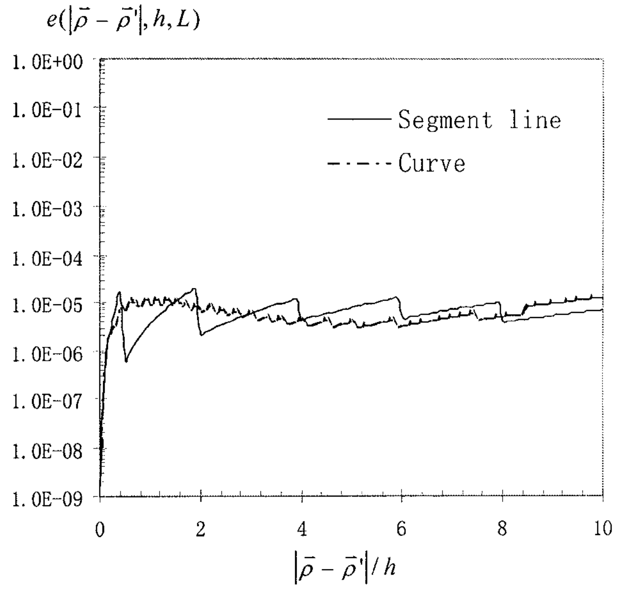

Fig. 2 The relative error for small ratios of distance and height

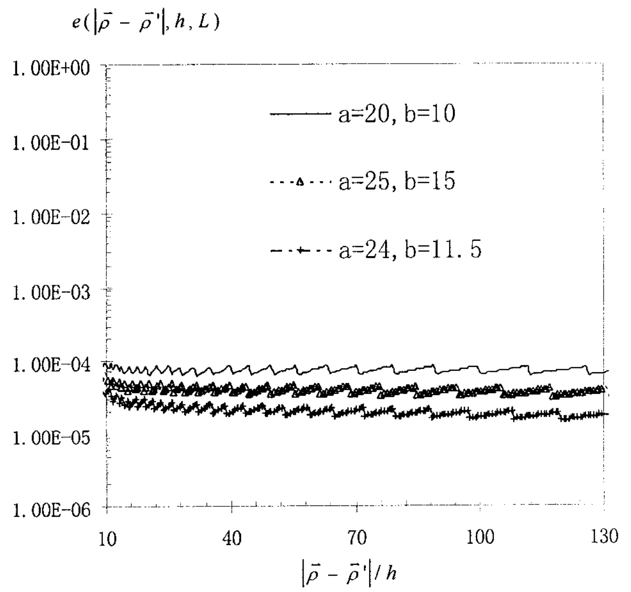

Fig. 3 The relative error for large ratios of distance and height