MEAN DYADIC GREEN'S FUNCTION FOR A TWO-LAYER ANISOTROPIC RANDOM MEDIUM : NONLINEAR APPROXIMATION TO THE DYSON EQUATION

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Abstract

The mean dyadic Green's function for a two-layer anisotropic random medium with arbitrary three-dimensional correlation functions has been calculated with the zeroth-order solutions to the Dyson equation under the nonlinear approximation. The effective propagation constants are determined for the coherent vector fields propagating in the anisotropic random medium layer. There are four characteristic waves corresponding to the ordinary and extraordinary waves with upward and downward propagating vectors.

Introduction

The study of the coherent (or mean) wave propagation in continuous random medium has been of great interest in the fields of microwave remote sensing of earth environment and optical communications in the atmosphere. To understand the coherent wave motion in random medium, we must solve the Dyson equation [1], which is an exact equation for the mean field or the mean dyadic Green's function. The mass operator that appears in the Dyson equation is approximated in the form of an infinite series. The most commonly used bilocal approximation leads to solutions with potentially severe range restrictions and it also leads to solutions that are energetically inconsistent with the Bethe-Salpeter equation under the ladder approximation. A better approximation which circumvents these difficulties is the nonlinear approximation in which more multiple scattering terms are included. The mean Green's function with the nonlinear approximation was first calculated by Rosenbaum [2] for the unbounded random medium using the Fourier transform method. Tsang and Kong [3-4] introduced the two-variable expansion technique to find the nonlinearly approximated mean Green's functions for the two-layer random medium with one-dimensional fluctuation [3] and for the half-space random medium with three-dimensional fluctuation [4]. The vector problem has been solved for the mean dyadic Green's function (MDGF) with the nonlinear approximation by Zuniga and Kong [5] for the two-layer case.

While extensive efforts are being made in the calculation of the mean Green's function for an isotropic random medium, the MDGF for an anisotropic random medium has not been fully developed. Liu [6] calculated the effective dielectric tensor and propagation constant of mean wave in a turbulent magnetoactive plasma medium by the first-order iteration which corresponds to the bilocal approximation. Dence and Spence [7] investigated the coherent wave motion in random anisotropic media and evaluated the effective dielectric tensor and the MDGF for an unbounded, uniaxially anisotropic random medium by solving the bilocally approximated Dyson equation.

In this paper, we consider a bounded, anisotropic random medium and solve the Dyson equation with the nonlinear approximation. The permittivity tensor of the random medium is assumed to be uniaxial with an optic axis tilted off the z-axis by some angle both for its mean and for randomly fluctuating part. We employ the two-variable expansion technique to obtain a zero-order solution for the mean dyadic Green's functions of a two-layer anisotropic random medium with arbitrary three-dimensional correlation functions.

Formulation

Consider an anisotropic random medium layer with boundaries at $z = 0$ and $z = -d$ as shown in Figure 1. The anisotropic random medium is characterized by its permittivity tensor

$$\bar{\bar{\epsilon}}_1(\bar{r}) = \bar{\bar{\epsilon}}_{1m} + \bar{\bar{\epsilon}}_{1f}(\bar{r}) \tag{1}$$

where $\bar{\bar{\epsilon}}_{1m} \equiv < \bar{\bar{\epsilon}}_1(\bar{r}) >$ is the mean part and $\bar{\bar{\epsilon}}_{1f}(\bar{r})$ represents the randomly fluctuating part whose ensemble average vanishes. The upper region is free space and isotropic with permittivity ϵ_o and the lower region is homogeneous and isotropic with permittivity ϵ_2. All three regions are assumed to have the same permeability μ. In general, $\bar{\bar{\epsilon}}_{1m}$ and $\bar{\bar{\epsilon}}_{1f}(r)$ are taken to be uniaxial with an optic axis tilted off the z-axis by angles ψ and ψ_f, respectively, as shown in Figure 2.

The mean dyadic Green's function (DGF) of a point source imbedded in an anisotropic random medium satisfies the Dyson equation which under the nonlinear approximation takes the following form [8]

$$\begin{aligned}\nabla \times \nabla \times \bar{\bar{G}}_{11m}(\bar{r},\bar{r}_o) - \omega^2\mu\bar{\bar{\epsilon}}_{1m} \cdot \bar{\bar{G}}_{11m}(\bar{r},\bar{r}_o)\\ = \bar{\bar{I}}\delta(\bar{r}-\bar{r}_o) + \int_V d^3\bar{r}_2 \ < \bar{\bar{Q}}(\bar{r}) \cdot \bar{\bar{G}}_{11m}(\bar{r},\bar{r}_2) \cdot \bar{\bar{Q}}(\bar{r}_2) > \cdot \bar{\bar{G}}_{11m}(\bar{r}_2,\bar{r}_o)\end{aligned} \tag{2}$$

where $\bar{\bar{Q}}(\bar{r}) = \omega^2\mu\bar{\bar{\epsilon}}_{1f}(\bar{r})$ and the spatial integration extends over the layer of the anisotropic random medium. The first and second subscripts of the DGF indicate the regions containing the observation point and the source point, respectively. The third subscript m indicates that the DGF is the mean dyadic Green's function (MDGF). It is assumed that $\bar{\bar{Q}}(\bar{r})$ takes the following form:

$$\bar{\bar{Q}}(\bar{r}) = Q(\bar{r})\,\bar{\bar{q}} \tag{3}$$

where $Q(\bar{r}) = \omega^2\mu\epsilon_{1f}(\bar{r})$ and $\bar{\bar{q}}$ is a dielectric tensor of the permittivity fluctuatation associated with the geometry. We define the correlation function as

$$C(\bar{r}-\bar{r}_2) \equiv < Q(\bar{r})Q(\bar{r}_2) > \tag{4}$$

and introduce the Fourier transforms of the mean DGF and the correlation function:

$$\bar{\bar{G}}_{11m}(\bar{r},\bar{r}_o) = \frac{1}{(2\pi)^2}\int d^2\bar{k}_\rho\ \bar{\bar{g}}_{11m}(\bar{k}_\rho,z,z_o)e^{i\bar{k}_\rho\cdot(\bar{\rho}-\bar{\rho}_o)} \tag{5}$$

$$C(\bar{r}-\bar{r}_2) = \delta\int d^3\bar{\alpha}\ \Phi(\bar{\alpha})e^{-i\bar{\alpha}\cdot(\bar{r}-\bar{r}_2)} \tag{6}$$

where $\bar{k}_\rho \equiv \hat{x}k_x + \hat{y}k_y$ and $d^2\bar{k}_\rho \equiv dk_xdk_y$. Substituting (4)-(6) into (2) and performing the transverse integrations ($d^2\bar{\rho}_2 = dx_2dy_2$ and $d^2\bar{\alpha}_\perp = d\alpha_xd\alpha_y$), we obtain

$$\begin{aligned}\frac{d^2}{dz^2}\bar{\bar{g}}_{11m}(\bar{k}_\rho,z,z_o) + (\omega^2\mu\bar{\bar{\epsilon}}_{1m} - \bar{\bar{I}}k_\rho^2)\ \bar{\bar{g}}_{11m}(\bar{k}_\rho,z,z_o)\\ - (i\bar{k}_\rho + \hat{z}\frac{d}{dz})(i\bar{k}_\rho + \hat{z}\frac{d}{dz}) \cdot \bar{\bar{g}}_{11m}(\bar{k}_\rho,z,z_o)\\ = -\bar{\bar{I}}\delta(z-z_o) - \delta\int_{-\infty}^{\infty} d\bar{k}'_\rho \int_{-\infty}^{\infty} d\alpha_z \int_{-d}^{0} dz_2\ \Phi(\bar{k}'_\rho - \bar{k}_\rho,\alpha_z)e^{-i\alpha_z(z-z_2)}\\ \bar{\bar{q}} \cdot \bar{\bar{g}}_{11m}(\bar{k}'_\rho,z,z_2) \cdot \bar{\bar{q}} \cdot \bar{\bar{g}}_{11m}(\bar{k}_\rho,z_2,z_o)\end{aligned} \tag{7}$$

To solve (7), we make use of the *two-variable expansion technique* (or the *multiple scale analysis*) to handle the secular terms that arise, which has been used to study the long-distance behavior of wave propagation in an isotropic random medium [3-5]. With the variance δ to be a small parameter, we introduce the long-distance scales as

$$\xi = \delta z, \quad \xi_o = \delta z_o \quad \xi_2 = \delta z_2 \tag{8}$$

and expand the Fourier transformed mean DGF in a perturbation series

$$\bar{\bar{g}}_{11m}(\bar{k}_\rho, z, z_0) = \bar{\bar{g}}^{(0)}_{11m}(\bar{k}_\rho, z, \xi; z_0, \xi_0) + \delta \bar{\bar{g}}^{(1)}_{11m}(\bar{k}_\rho, z, \xi; z_0, \xi_0) + \cdots \tag{9}$$

where the superscript denotes the order of the solution for the MDGF. Here small δ physically corresponds to weak fluctuations. Following the procedures of the two-variable expansion technique to zeroth order, we calculate the corrections to the propagation constants which accounts for the multiple scattering due to random fluctuations.

Results and Discussions

The resultant expressions for the zeroth-order MDGF in region 1 are shown to be

$$\begin{aligned}
\overline{\overline{G}}^{>}_{11m}(\bar{r}, \bar{r}_0) = \frac{1}{(2\pi)^2} \int d\bar{k}_\rho e^{i\bar{k}_\rho \cdot (\bar{r} - \bar{r}_0)} \\
\Big\{ \Big[\hat{o}(k^{o}_{1z}) e^{i\eta_{ou} z} + A_2(\bar{k}_\rho) \hat{o}(-k^{o}_{1z}) e^{i\eta_{od} z} \\
+ A_3(\bar{k}_\rho) \hat{e}(k^{eu}_{1z}) e^{i\eta_{eu} z} + A_4(\bar{k}_\rho) \hat{e}(k^{ed}_{1z}) e^{i\eta_{ed} z} \Big] \\
\Big[B_1(\bar{k}_\rho) \hat{o}(k^{o}_{1z}) e^{-i\eta_{ou} z_0} + B_2(\bar{k}_\rho) \hat{o}(-k^{o}_{1z}) e^{-i\eta_{od} z_0} \\
+ B_3(\bar{k}_\rho) \hat{e}(k^{eu}_{1z}) e^{-i\eta_{eu} z_0} + B_4(\bar{k}_\rho) \hat{e}(k^{ed}_{1z}) e^{-i\eta_{ed} z_0} \Big] \\
+ \Big[C_1(\bar{k}_\rho) \hat{o}(k^{o}_{1z}) e^{i\eta_{ou} z} + C_2(\bar{k}_\rho) \hat{o}(-k^{o}_{1z}) e^{i\eta_{od} z} \\
+ \hat{e}(k^{eu}_{1z}) e^{i\eta_{eu} z} + C_4(\bar{k}_\rho) \hat{e}(k^{ed}_{1z}) e^{i\eta_{ed} z} \Big] \\
\Big[D_1(\bar{k}_\rho) \hat{o}(k^{o}_{1z}) e^{-i\eta_{ou} z_0} + D_2(\bar{k}_\rho) \hat{o}(-k^{o}_{1z}) e^{-i\eta_{od} z_0} \\
+ D_3(\bar{k}_\rho) \hat{e}(k^{eu}_{1z}) e^{-i\eta_{eu} z_0} + D_4(\bar{k}_\rho) \hat{e}(k^{ed}_{1z}) e^{-i\eta_{ed} z_0} \Big] \Big\}, \qquad z > z_0^+
\end{aligned} \tag{10}$$

where

$$\eta_{ou}(\bar{k}_\rho) = k^{o}_{1z} + \delta \lambda^{ou}(\bar{k}_\rho) \tag{11.1}$$
$$\eta_{od}(\bar{k}_\rho) = -k^{o}_{1z} + \delta \lambda^{od}(\bar{k}_\rho) \tag{11.2}$$
$$\eta_{eu}(\bar{k}_\rho) = k^{eu}_{1z} + \delta \lambda^{eu}(\bar{k}_\rho) \tag{11.3}$$
$$\eta_{ed}(\bar{k}_\rho) = k^{ed}_{1z} + \delta \lambda^{ed}(\bar{k}_\rho) \tag{11.4}$$

are the effective propagation constants of the characteristic waves propagating in an anisotropic random medium layer. All the variables and the coefficients depend on the angles of propagation. The first and second parts in (10) match with the horizontal and vertical polarization parts of the MDGF in region 0, respectively. The MDGF in region 1 for $z < z_0^-$ can be obtained by using the symmetrical property of the DGF. The MDGF's in regions 0 and 2 are obtained by matching the boundary conditions at $z = 0$ and $z = -d$.

As seen from the results for the MDGF given by (10), four characteristic waves propagate in an anisotropic random medium layer. They are the ordinary and extraordinary; the upward and downward propagating waves. These four coherent waves propagate with four distinctive effective propagation constants given by (11.1)-(11.4).

The correction to the propagation constants caused by the anisotropic random inhomogeneities, $\delta\lambda^{(p)}(\bar{k}_\rho)$, $(p) = ou, od, eu, ed$, is shown to be complex in general even when the mean permittivities, ϵ_1 and ϵ_{1z}, are purely real. It indicates that the coherent ordinary and extraordinary waves experience an exponential decay, caused by scattering of waves as they propagate in an anisotropic random medium, even though the medium is lossless. In addition the coherent waves can be slowed down or speeded up, again as a consequence of the random inhomogeneities. Typical numerical results for the corrections to the propagation constants can be obtained as a function of propagation angles and polarizations.

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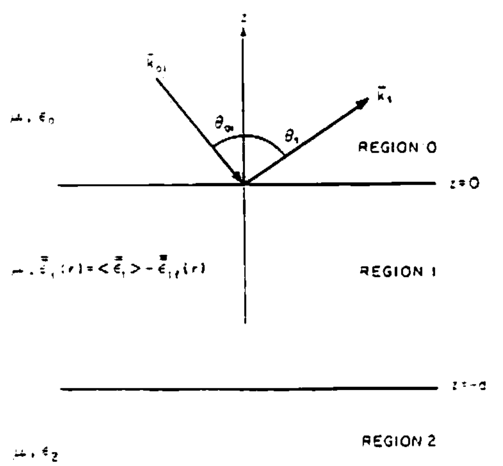

Figure 1.

Scattering geometry of a two-layer anisotropic random medium

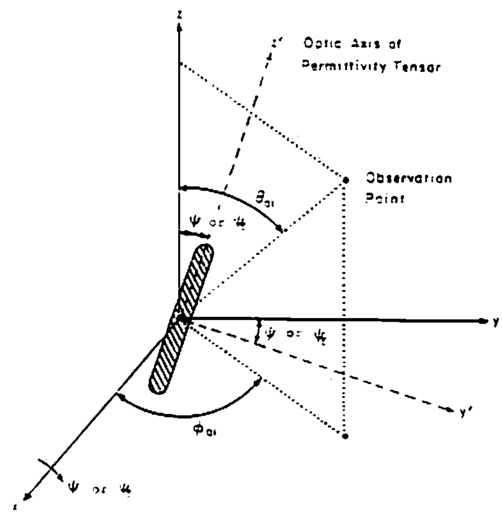

Figure 2.

Geometrical configuration of permittivity tensor in an anisotropic random medium layer.