Effect of Mutual Coupling on the Performance of Multielement Antenna Systems

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Abstract

In this paper, we investigate the signal-to-noise ratio (SNR) in the palm-sized multi-antenna communication systems. The presented analysis includes the mutual coupling on both, signal and thermal noise. We apply the Nyquist's thermal noise theorem to determine the thermal noise behavior in the multi-antenna system and to confirm the partial correlation of thermal noise for antenna spacing lower then a one wavelength. Then, we develop a method for thermal noise power calculation for the multi-antenna system with coupled antennae. Simulation results shows that SNR is underestimated if the effect of mutual coupling for thermal noise is not accounted.

I. Introduction

MULTIPLE-INPUT multiple output (MIMO) wireless systems, characterized by multiple antennas at the transmitter and receiver, have demonstrated the potential for increased capacity by exploiting the spatial properties of the multipath channel [1]. If the channel matrix coefficients are i.i.d (independent identically distributed) complex Gaussian variables, then linear increase in capacity with the number of antenna is possible. The mutual independence of channel coefficients is generally achieved by wide inter-element spacing in the multi-antenna system. However, in portable sized wireless communication systems, a wide antenna spacing is often unrealistic.

For closely spaced antennae, the resulting antenna mutual coupling significantly affects the communication system performance. The impact of antenna mutual coupling on the antenna arrays has been evaluated by examining the influence of the coupled antennae on the signal correlation and on that way on signal-to-interference-plus-noise ratio(SINR) [2]. Then, the model for evaluation the mutual coupling effect on MIMO channel capacity was presented in [3]. In [4], the mutual coupling effect on the radiated power at the transmitter and the power collection capability is assessed in the multi-antenna systems.

While above studies present important contributions concerning the effect of array mutual coupling on MIMO system performance, they neglect mutual coupling effect on thermal noise. On that way, the accurate analysis of the signal-to-noise(SNR) ratio is omitted. The analysis of thermal noise behavior due to the mutual coupling effect in the multi-antenna system for antenna spacings$[0, 1\lambda]$ is presented in [5].

In this paper, we analyze the SNR of the multi-antenna systems when mutual coupling affects both signal and noise. We reveal that the SNR is underestimated if the effect of mutual coupling on thermal noise is not considered. The Nyquist's thermal noise theorem [6] enables to identify the correlated part of the thermal noise from the total thermal noise and to confirm the correlation of noise for antenna spacing below one wavelength. We calculate noise correlation coefficients to additionally assure the existence of the noise correlation. Simulation results show that the SNR underestimation is about $0.5\ dB$ for two-dipole array and $1dB$ for three-dipole array, for antenna spacings below $0.5\ \lambda$. Finally, we compare the error made by classical thermal noise consideration with the presented method.

The rest of this paper is organized as follows. Section II describes the multi-antenna system representation for SNR and thermal noise analysis. In Section III, the mutual coupling on thermal noise is elaborated. Sections IV is devoted to thermal noise power evaluation in two-dipole array. Simulation results are given in section V. The concluding remarks are given in Section VII.

II. Multielement Antenna System Representation

The multi-antenna system can be represented by a general linear network using a generalized form of Thevenin's theorem. The generalization of the Thevenin's theorem holds true not only for coherent sources but also for thermal noise sources [7]. Thus, the multi-antenna system with $N = n_R$ antenna elements which can be represented by a linear n_R-terminal-pair network containing noise current generators. With respect to its terminal pairs, the network is completely specified by its admittance matrix $\mathbf{Y}$ and a set of n_R nodal current generator $i_1, i_2, ..., i_{n_R}$.

The admittance matrix $\mathbf{Y}$ is squared matrix of order n_R with the following form:

$$\mathbf{Y} = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n_R} \\ y_{21} & y_{22} & \cdots & y_{2n_R} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n_R1} & y_{n_R2} & \cdots & y_{n_Rn_R} \end{pmatrix} \tag{1}$$

The nodal current generators are represented by column vector $\mathbf{i}$:

$$\mathbf{i} = \begin{pmatrix} i_1 \\ i_2 \\ \cdot \\ i_{n_R} \end{pmatrix} \tag{2}$$

where the nodal current generator $i_1, i_2, ..., i_{n_R}$ are not in general independent. In addition, the squared currents can be written in the matrix form:

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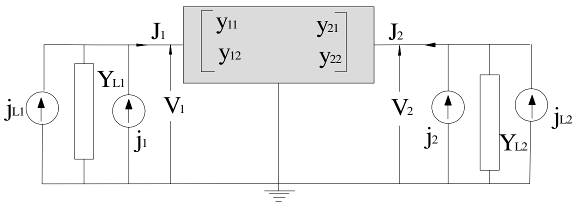

Fig. 1. Nodal network represention for two antenna array

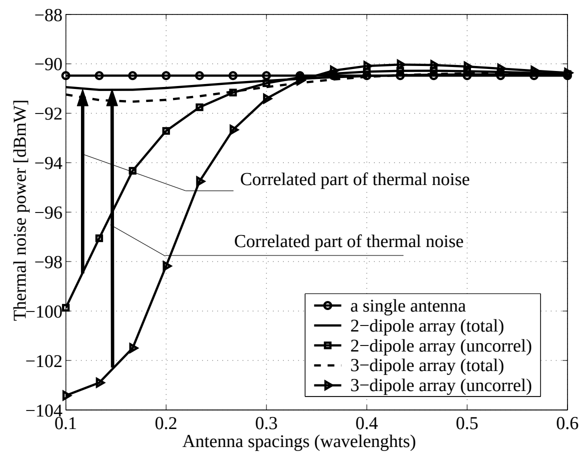

Fig. 2. Correlated part of thermal noise power versus antenna spacings for different number of dipoles in the antenna array

$$\overline{\mathbf{ii}^{\dagger}} = \begin{pmatrix} \overline{i_1 i_1^*} & \overline{i_1 i_2^*} & \cdots & \overline{i_1 i_{n_R}^*} \\ \overline{i_2 i_1^*} & \overline{i_2 i_2^*} & \cdots & \overline{i_2 i_{n_R^*}} \\ \cdots & \cdots & \cdots & \cdots \\ \overline{i_{n_R} i_1^*} & \overline{i_{n_R} i_2^*} & \cdots & \overline{i_{n_R} i_{n_R}^*} \end{pmatrix} \tag{3}$$

where the subscript † indicates the Hermitian transpose (complex conjugate transpose).

The internal sources may be alternatively represented by the set of n_R nodal voltage generators of infinite internal impedance.

III. Nyquist Thermal Noise Theorem for Coupled Antennae

The isolated receivers of two closely spaced antennae receive partially correlated noise [8]. The magnitude of correlation is calculated using a generalized form of Nyquist's thermal noise theorem given in [6]. It was shown that general nonreciprocal network with a system of internal thermal generators all absolute temperature T is equivalent to the source-free network together with a system of noise current generators I_r and I_s with infinite internal impedance [8]. Noise currents are correlated and their cross-correlation is given by:

$$\overline{I_s I_r} df = 2kT(Y_{sr} + Y_{sr}^*)df \tag{4}$$

where Y_{rs} are the mutual admittance and k is Boltzmann constant. Correlation is zero when the mutual coupling is purely reactive.

IV. Thermal noise power

In this Section, we calculate the thermal noise power for two-antenna array by taking into consideration the mutual coupling effect. Although, two-antenna array is the simplest case, it enables to draw important conclusions about the mutual coupling effect on thermal noise.

The generalized Nyquist's thermal noise theorem allows us to determine thermal noise power of coupled antennae in the multi-antenna system. The theorem states that for passive network in thermal equilibrium it would be appear possible to represent the complete thermal-noise behavior by applying Nyquist's theorem independently to each element of the network. In the case of the multi-antenna system these elements are self-impedances and mutual-impedances.

On can write the noise current for two-antenna array from Fig 1 in terms of its spectral density by:

$$\begin{aligned} J_1 = y_{11}V_1 + y_{12}V_2 = j_{L1} + j_1 - Y_{L1}V_1 \\ J_2 = y_{21}V_1 + y_{22}V_2 = j_{L2} + j_2 - Y_{L2}V_2 \end{aligned} \tag{5}$$

where $J_i,\ i = 1,2$ are the total noise current spectrum and $V_i,\ i = 1,2$ associated noise voltage spectrum of i^{th} antenna element. $j_i,\ i = 1,2$ are the nodal noise current spectrum of i^{th} port, $j_{Li},\ i = 1,2$ are the noise current spectrum associated with load admittance of receiver $Y_{Li},\ i = 1,2$ of i^{th} antenna elements.

Furthermore:

$$\begin{aligned} j_{L1} + j_1 = (y_{11} + Y_{L1})V_1 + y_{12}V_2 \\ j_{L2} + j_2 = y_{21}V_1 + (y_{22} + Y_{L2})V_2 \end{aligned} \tag{6}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{pmatrix} y_{11} + Y_{L1} & y_{12} \\ y_{21} & y_{22} + Y_{L2} \end{pmatrix}^{-1} \begin{bmatrix} j_{L1} + j_1 \\ j_{L2} + j_2 \end{bmatrix} \tag{7}$$

$$V_1 = \tfrac{1}{|D|}[(y_{22} + Y_{L2})(j_{L1} + j_1) - y_{21}(j_{L2} + j_1)] \quad V_2 = \tfrac{1}{|D|}[(y_{11} + Y_{L1})(j_{L2} + j_2) - y_{12}(j_{L1} + j_1)] \tag{8}$$

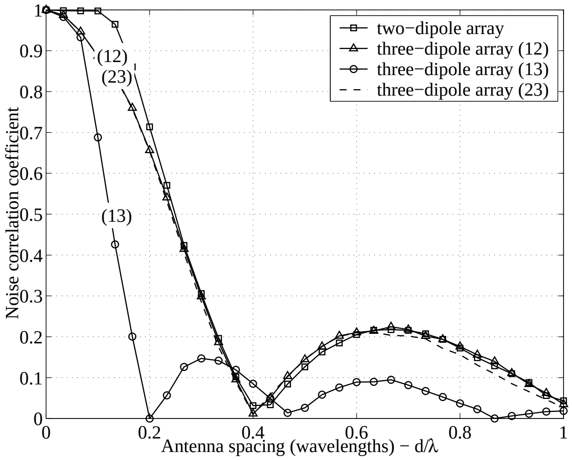

Fig. 3. Correlation coefficinet of thermal noise voltages in two- and three-dipole array due to mutual coupling effect

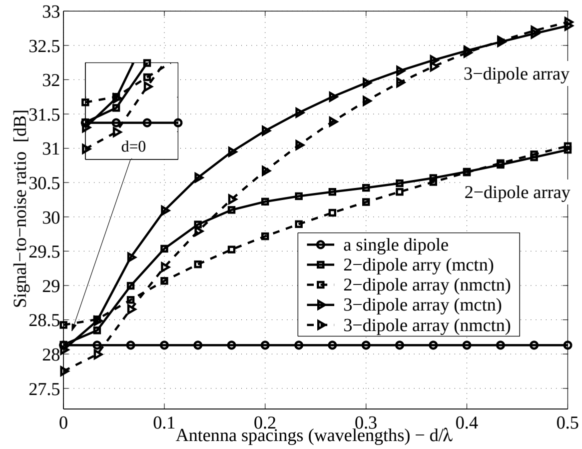

Fig. 4. The SNR value verus antenna spacing for two-dipole and three-dipole array assuming mutual coupling for both signal and noise

where $|D|$ is determinant of the following matrix

$$\begin{pmatrix} y_{11}+Y_{L1} & y_{12} \\ y_{21} & y_{22}+Y_{L2} \end{pmatrix} \tag{9}$$

The average powers absorbed in the receiver loads of the first and second antenna are proportional to P_{L1} and P_{L2}, respectively:

$$P_{L1}=\tfrac{1}{2}(Y_{L1}+Y_{L1}^*)\overline{V_1V_1^*} \quad P_{L2}=\tfrac{1}{2}(Y_{L2}+Y_{L2}^*)\overline{V_2V_2^*} \tag{10}$$

Substituting the expression (8) for first antenna in corresponding expression (10) yields

$$P_{L1}=\frac{(Y_{L1}+Y_{L1}^*)}{2|D||D^*|}[(y_{22}+Y_{L2})(y_{22}^*+Y_{L2}^*)(\overline{i_{L1}i_{L1}^*}+\overline{i_1i_1^*})-y_{21}(y_{22}^*+Y_{L2}^*)\overline{i_1i_2^*}$$
$$-y_{21}^*(y_{22}+Y_{L2})\overline{i_1^*i_2}+y_{21}y_{21}^*(\overline{i_{L2}i_{L2}^*}+\overline{i_2i_2^*})] \tag{11}$$

Using (4) for nodal current correlation, expression (11) becomes:

$$P_{L1}=2kT\frac{(Y_{L1}+Y_{L1}^*)}{2|D||D^*|}[(y_{22}+Y_{L2})(y_{22}^*+Y_{L2}^*)((Y_{L1}+Y_{L1}^*)+(y_{11}+y_{11}^*))$$
$$-y_{21}(y_{22}^*+Y_{L2}^*)(y_{12}+y_{12}^*)-y_{21}^*(y_{22}+Y_{L2})(y_{12}+y_{12}^*)+y_{21}y_{21}^*((Y_{L2}+Y_{L2}^*)+(y_{22}+y_{22}^*))] \tag{12}$$

Similarly, for second antenna:

$$P_{L2}=2kT\frac{(Y_{L2}+Y_{L2}^*)}{2|D||D^*|}[(y_{11}+Y_{L1})(y_{11}^*+Y_{L1}^*)((Y_{L2}+Y_{L2}^*)+(y_{22}+y_{22}^*))-y_{12}(y_{11}^*+Y_{L1}^*)(y_{21}+y_{21}^*)$$
$$-y_{12}^*(y_{11}+Y_{L1})(y_{21}+y_{21}^*)+y_{12}y_{12}^*((Y_{L1}+Y_{L1}^*)+(y_{11}+y_{11}^*))] \tag{13}$$

where k is Boltzmann constant and T is absolute temperature.

Finally, the total thermal noise power for two coupled antenna elements, for frequency band B can be calculated as:

$$N_{total}=\int_B P_{L1}df+\int_B P_{L2}df \tag{14}$$

V. SNR ANALYSIS

In this section we analyze the output SNR of the multi-antenna system under the assumption that mutual coupling affects both, signal and thermal noise.

The fair comparison of the antenna arrays with different number of antenna elements in terms of the signal-to-noise ratio is provided by assuming that incident fields have the same limited powers. The incident field is given by $E=V_{sig}*h$, where V_{sig} and h are induced voltage in the antenna elements and the height of antenna elements, respectively.

Induced signal currents in the antenna elements can expressed in the following form:

$$\mathbf{I}_{sig}=(\mathbf{Y}+\mathbf{Y_L})\times\mathbf{V}_{sig} \tag{15}$$

where $\mathbf{Y}$ is admittance matrix of the multi-antenna array (1), $\mathbf{Y}_L$ is diagonal matrix of received load admittances and $\mathbf{V}_{sig}$ are signal voltage column vector.

Now, the signal power collected by the multi-antenna system is:

$$\mathbf{P}_s = \frac{1}{2}\mathbf{R}_L \times (\mathbf{I}_{sig} \times \mathbf{I}_{sig}^{\dagger}) \quad (16)$$

where $\mathbf{R}_L$ is diagonal matrix of received load resistances.

Finally, the received SNR is given by:

$$SNR = \frac{trace(\mathbf{P}_s)}{N_{total}} \quad (17)$$

VI. Simulation results

To confirm results of the presented analytical analysis, we use the simulation models consist of the uniform linear arrays (ULA) with two and three half-wave dipoles in multi-antenna system. This simplified models still enable significant conceptual insight to be gained into SNR behavior of the multi-antenna system due to the mutual coupling effects on thermal noise.

Fig. 2 confirms the existence of correlated part of thermal noise. Nyquist's thermal theorem enables the identification of correlated part from total thermal noise of any dipole in dipole array. In such a way, thermal noise of one antenna element that originates from self-impedance of the antenna element is identified as uncorrelated part of thermal noise, since it comes from its own antenna elements. On the other hand, the correlated part of thermal noise which originates from mutual impedances is induced thermal noise from adjacent antennae. The simulation results from Fig. 2 show that the correlated noise power collected from antennae increases as the spacing between antenna decreases. Also, one dipole in three-antenna array will collect more correlated noise power then in two-dipole array, for the same antenna spacing.

Additionally, we estimate the noise correlation coefficients in order to drawn conclusions about MIMO system SNR performance, since it appears that the antenna array with correlated noise exhibits higher SNR then when noise correlation is neglected. Therefore, we analyze a two-antenna array and estimate the correlation coefficients of complex thermal noise voltages within antenna spacing range $[0, 1\lambda]$. The voltage correlation coefficient is computed as $\rho_{12} = < V_1, V_2 >$,where $V_i,\ i = 1, 2$ is the voltage at the output port of i^{th} antenna element. Operation $< a, b >$ computes the complex correlation coefficient between a and b as$< a, b >= \frac{E\{[a-E\{a\}][b-E\{b\}]^*\}}{\sqrt{E\{|a-E\{a\}|^2\}E\{|b-E\{b\}|^2\}}}$ We calculate thermal noise voltages for two-antenna array by using (8).

Fig. 3 plots the resulting magnitude of the correlation coefficients versus antenna spacing, for both, two-dipole and three-dipole arrays. The correlation coefficients are calculated for the adjacent dipoles (12),(23) and dipoles set $2 * d$ apart(13). The results from Fig.3 confirm that thermal noise between the adjacent antenna elements in the multi-antenna system is highly correlated for antenna spacing up to 0.5λ.

The uncorrelated white noise is usually presupposed in antenna array applications, neglecting the radiation characteristics of noise. However, the results from Fig.3 show that the mutual coupling strongly correlates thermal noise in the closely spaced antenna elements. The noise correlation depends of a antenna spacing, but also of the position of antenna elements in antenna array.

Fig. 4 depicts combined effects upon the SNR of electromagnetic coupling for signal and for thermal noise in the multi-antenna system. Simulation analysis confirms that a higher SNR can be obtained if mutual coupling of thermal noise (mctn) is incorporated in compression with traditional approach which ignores the coupling interaction for thermal noise(nmctn). Thus, disregarding of mutual coupling of thermal noise leads to SNR underestimation which is about $0.5dB$ and $1dB$ for the two-dipole and three-dipole array, respectively, for antenna spacing below 0.5λ. The simulation results point on the trend that the underestimation will grow as the number of antenna in the multi-antenna elements increases. With Fig. 4, we also confirm that as antennae approach to each other, the multi-antenna system starts to act as a single antenna with an equivalent resistance.

VII. Conclusion

This paper outlines a procedure for SNR analysis of the multi-antenna system with coupled antennae. We present a method for thermal noise power calculation in the multi-antenna system, which can be used to determine thermal noise behavior of the multi-antenna system with small antenna spacing. We confirm the partial correlation of thermal noise for antenna spacing below a wavelength. We show and enumerate that the SNR is underestimated if the effect of mutual coupling on thermal noise is neglected. Additionally, our simulation results reveal that the multi-antenna system starts to act as a single antenna with an equivalent radiated resistance if antenna spacings drop to zero.

References

[1] G. Foschini, "On limits of wireless communication in fading environment when using multiple antennas," *Wireless Personal Communication*, no. 6, pp. 311–335, march 1998.

[2] I.J.Gupta and A.K.Ksienski, "Effect of the mutual coupling on the performance of the adaptive arrays," *IEEE Trans. Antennas Propagat.*, vol. 31, no. 5, pp. 785–791, sep 1983.

[3] T. Sventenson and A.Ranheim, "Mutual coupling effects on the capacity of the multielement antenna system," in *Acoustic, Speech and Signal Processing,Proceedings (ICASSP'01)*, 2001.

[4] J.W.Wallace and M.A.Jensen, "Mutual coupling in mimo wireless systems: A rigorous network theory analysis," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1317–1325, 2004.

[5] S. Krusevac, P. Rapajic, and R. Kennedy, "Mutual coupling effect on thermal noise in multi-antenna wireless communication systems," in *AusCTW 2005*, Feb. 2005.

[6] E. Schremp, *MIT Rad. Lab. Series 18,Valley and Wallman: Vacuum Tube Amplifier*. New York: McGraw-Hill,Inc., 1949.

[7] A. Starr, *Electric Circuits and Wave Filters*, 2nd ed. London: Pitman, 1946.

[8] R. Twiss, "Nyquist's and thevenin's generalized for nonreciprocal linear nwrworks," *J. Applied Phys.*, vol. 26, pp. 559–602, may 1955.