ELECTROMAGNETIC IMPULSE RETURN FROM A CONDUCTIVE MEDIUM WHICH HAS COME INTO BEING

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According to the general property [Morgenthaler F.R., IRE Trans. on microw. theory and techn., 1958, MTT-6, 167-172] an electromagnetic signal splitting also occurs in a conductive medium when its conductivity has a time abrupt change [Borisov V.V., Geomagnetism and Aeronomy, 1989, V.29, №5, 730- 737]. Owing to existance of wave dispersion the medium transient leads to a continuous wave spectrum formation in case of a monochromatic original signal with a rectangular front. Nevertheless the continuous wave spectrum formation also takes place in the case of the original harmonic wave but the transient medium is restricted.

In this communication the influence of the both restrictions is studied, i.e. an interaction of the electromagnetic signal with the conductive medium which comes into being at zero moment in a half-space $x>0$ is considered. The signal has a form of the rectangular impulse of a finite duration

$$E_1(t,x) = E_0\theta(vt+vt_0-x)\theta(x-vt-vt_0+vT),$$

where $v = c/\sqrt{\varepsilon}$ is a pulse velocity in the background, ε - the medium permittivity, T is the pulse duration, t_0 is a fixed moment, θ - the Heaviside unit function.

The situation when the impulse completely penetrates into the region of the conductive medium till the moment when the conductivity abruptly changes from zero to σ' is considered. As a result of the conductivity abrupt change the reverse impulse appears. When crossing the medium boundary this impulse initiates an electromagnetic field in the region of the background $x<0$ too. It should be noted that this field is not a result of an impulse reflection from the medium boundary, but the partial return of the original impulse that was caused by the transient medium. It is essential that till the moment of the conductive medium formation the impulse already passed the boundary completely.

It is suitable to describe the present situation by the evolutionary approach [A.G.Nerukh, N.A.Khizhnjak. The modern problems of the transient macroscopic electrodynamics. Kharkov.: "Test-radio", 1991, 280p.], which is based on the integral expression for an electric intensity of the electromagnetic field. If the original field has only a component normal to the axis X being not dependent on Y and Z then such an expression has the form

$$E(t,x) = E_1(t,x) - \frac{\sigma}{v}\int_0^\infty dt'\int_0^\infty dx'\delta(t-t'-\frac{|x-x'|}{v})E(t',x'), \qquad (1)$$

where $\sigma = (2\pi\sigma')/\varepsilon$, $\delta(t)$ - the delta-function. Within the conductive medium this expression is the Volterra integral equation. Its solution is constructed by the resolvent

$$R(t,t',x,x') = -\frac{\sigma}{v}\int_{\alpha-i\infty}^{\alpha+i\infty} S(p)e^{(p-\sigma)(t-t')}\left\{e^{-\frac{|x-x'|}{v}(p^2-\sigma^2)^{1/2}} + \right.$$

$$\left. + \frac{1-S(p)}{1+S(p)}e^{-\frac{x+x'}{v}(p^2-\sigma^2)^{1/2}}\right\}\frac{dp}{2\pi i}, \qquad (2)$$

where $S(p) = (p-\sigma)^{1/2}(p+\sigma)^{-1/2}$, $\alpha \geqslant \sigma$, by means of the expression

$$E_t = E_1 + \int_0^\infty dt' \int_0^\infty dx' R(t,t',x,x')E_1(t',x').$$

The substitution of E_1 into this expression gives

$$E_t = E_0\theta(vt_0-x)\theta(x-vt_0+vT)e^{-2\sigma t} +$$

$$+ \sum_{m=0}^{1}(-1)^m \left[\theta(t-|\frac{x}{v} - t_0 + mT|)E_2(t,x,t_0-mT) - \right.$$

$$\left. - \theta(t-t_0+mT-\frac{x}{v})E_3(t,x,t_0-mT)\right]. \qquad (3)$$

In this formula the first item is the original impulse relic, and the sum gives the secondary waves radiation field that is determined by the expression

$$E_2(t,x,\xi) = \frac{1}{2}E_0e^{-\sigma t}\left\{I_0(\sigma\sqrt{t^2-(\frac{x}{v}-\xi)^2}) + \right.$$

$$\left. + \sigma(\frac{x}{v}-\xi)\int_{\frac{x}{v}-\xi}^{t}(u^2-(\frac{x}{v}-\xi)^2)^{-1/2} I_1(\sigma\sqrt{u^2-(\frac{x}{v}-\xi)^2})e^{\sigma(u-t)}du\right\} +$$

$$+ \frac{1}{2}E_0e^{-2\sigma t}\begin{cases} e^{\sigma(\frac{x}{v}-\xi)}, & \xi \leqslant \frac{x}{v} \leqslant t+\xi, \\ & \text{if} \\ -e^{-\sigma(\frac{x}{v}-\xi)}, & -t+\xi \leqslant \frac{x}{v} < \xi; \end{cases} \qquad (4)$$

$$E_3(t,x,\xi) = \frac{1}{2}E_o e^{-\sigma t}\left\{\sqrt{\frac{vt-v\xi-x}{vt+v\xi+x}}\, I_1\left(\sigma\sqrt{t^2-\left(\frac{x}{v}+\xi\right)^2}\right) + \right. \tag{5}$$

$$\left. + \int_{-\frac{x}{v}-\xi}^{t} e^{\sigma u}\left[\sigma\left(\frac{x}{v}+\xi\right)\frac{I_o\left(\sigma\sqrt{u^2-\left(\frac{x}{v}+\xi\right)^2}\right)}{u-\frac{x}{v}-\xi} - \frac{I_1\left(\sigma\sqrt{u^2-\left(\frac{x}{v}+\xi\right)^2}\right)}{\sqrt{u^2-\left(\frac{x}{v}+\xi\right)^2}}\right]du\right\},$$

$I_{o,1}$ - the modified Bessel function.

The space and time field distribution is shown on Fig.1. The lines with arrows are front world-lines of the direct and reversed impulses and the impulse reflected from the boundary . These fronts are sources of the secondary waves which phase velocities are equal to $v_{ph}=vt(x/v-t_o+mT)^{-1}$, where m=0 correspondes to the impulse fore- front, and m=1 - to the impulse back-front. The phase velosity inhomogeneity leads to "running out" of the field through the back fronts of the impulses and the formation of traces that are hatched on Fig. 1. The field spatial distribution at several moments is presented on Fig. 2. There is such a distribution of an energy between the direct and revers impulses that in time their decreasing amplitudes become closer in value.

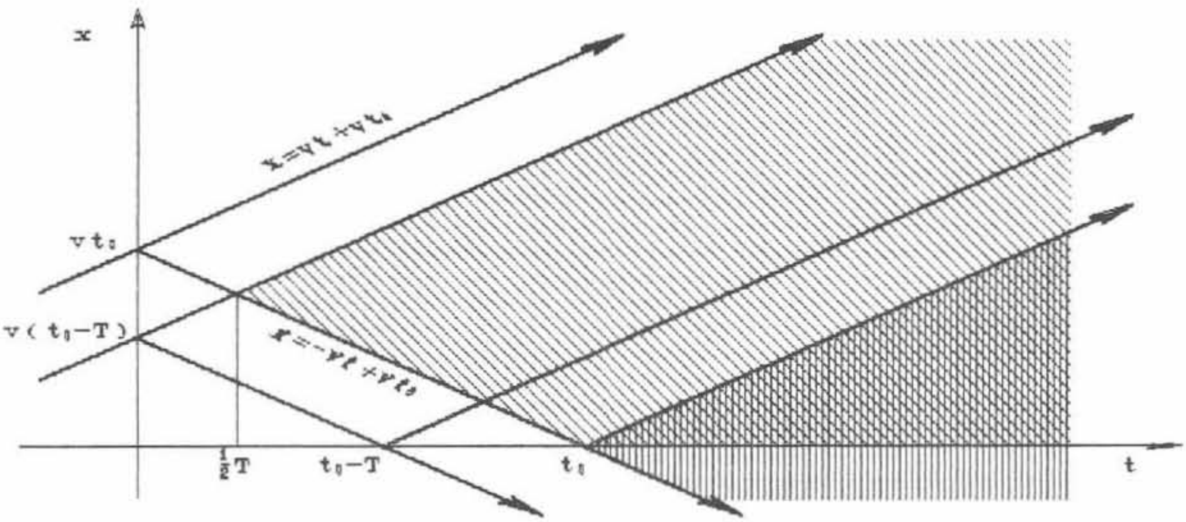
The field which comes out from the formed conductive medium is determined by the form which one gets substituting (1) into (3) :

$$E_r=E_o e^{-\sigma t}\sum_{m=0}^{1}\theta\left(t-t_o+mT+\frac{x}{v}\right)\sqrt{\frac{vt-vt_o+mTv+x}{vt+vt_o-mTv+x}}\, I_1\left(\sigma\sqrt{\left(t+\frac{x}{v}\right)^2-(t_o-mT)^2}\right).$$

As follows from the expression a time asymptote of the field in the impulse coming out from the conductor is determined as an exponent $E_r \approx \sigma/2(t+x/v+t_o-T)\exp(-\sigma t)$, whereas the field within the trace, in particular near the boundary (x<0), is changing as a descending power of time

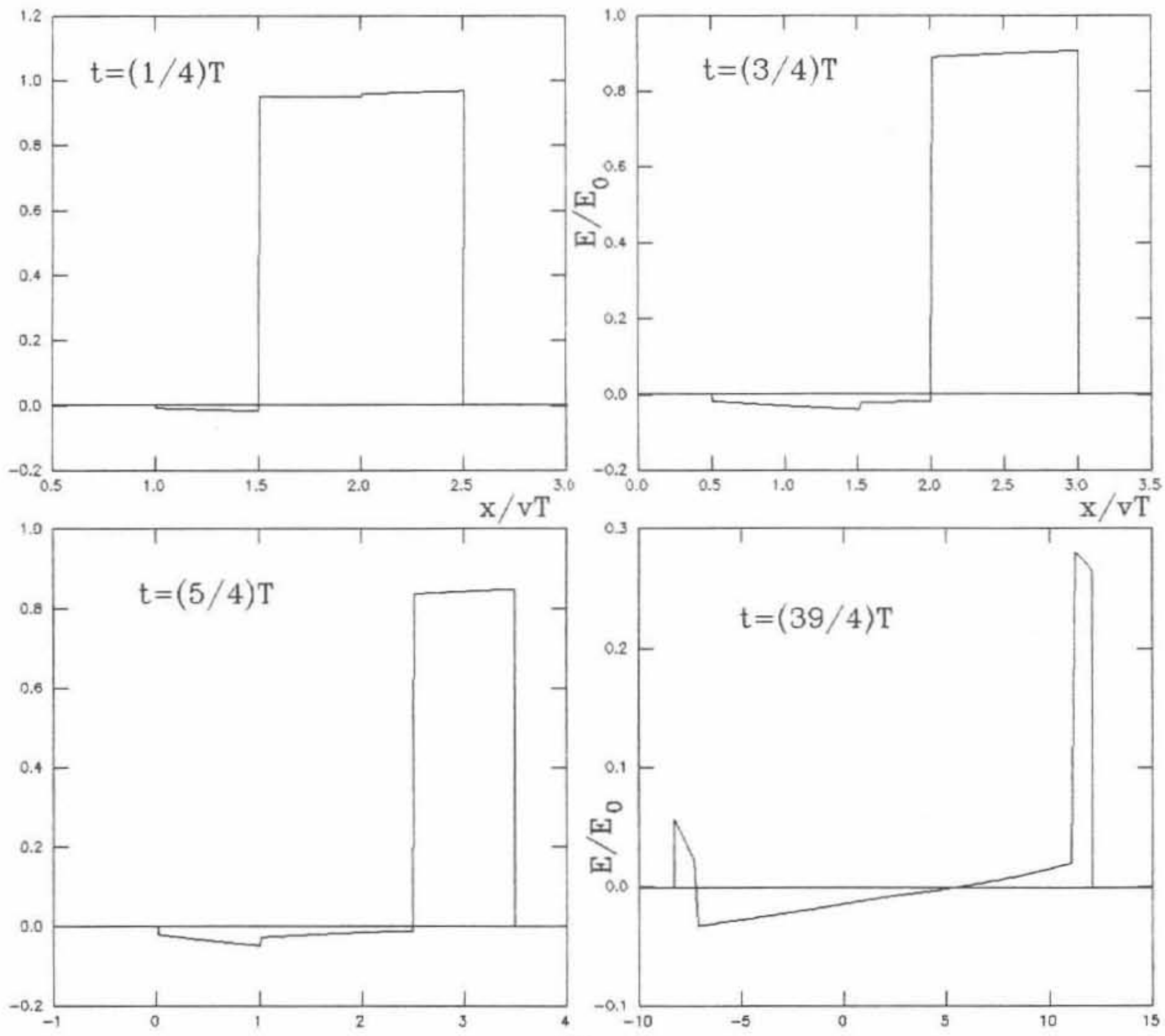
$$E_r \approx [1+\sigma(t_o-1/2T)]\exp(\sigma x/v)[2\pi\sigma(t+x/v)]^{-3/2} .$$

Thus the abrupt time changing of conductivity in a half-space leads to the splitting of an electromagnetic field impulse which penetrates into this half-space before the conductivity changes. Besides the reversed impulse partly comes out of the conductor and leaves the trace. The trace field changes as the descending power of time whereas the impulse field is diminishing exponentially.



x
x=vt+vt₀
vt₀
v(t₀−T)
x=−vt+vt₀
½T
t₀−T
t₀
t

Fig. 1



t=(1/4)T
t=(3/4)T
t=(5/4)T
t=(39/4)T
E/E0
x/vT

Fig. 2