ELECTROMAGNETIC POLARIZATIONS IN RANDOM MEDIA AND GENERALIZED
VECTOR RADAR EQUATIONS

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ABSTRACT

In microwave and millimeter wave propagation and scattering in
random discrete scatterers, the cross polarization effects due to
nonspherical particles such as rain are included in the description of
the coherent field. However, general polarization effects on
partially coherent waves caused by random media have not received much
attention. This paper presents a general Stokes vector representation
of the propagation and scattering characteristics of waves in random
media and rough surfaces including the transmission and receiving
characteristics of antennas and generalized vector radar equations to
include polarizations, partial coherence, and random media effects.

Introduction

In recent years, the propagation and scattering of microwave and
millimeter waves through random scatterers such as rain, ice, clouds,
snow, hail, smoke, and dust have received considerable attention
[1-9]. Most studies, however, make use of the scalar wave
approximation and the scalar radiative transfer theory. Recently,
there has been an increasing interest in the study of polarization
characteristics for use in communication and radar applications
[2,5-13], and there is a need to formulate the propagation and
scattering problems including complete Stokes vector representations.
This paper gives a generalization of propagation effects and vector
radar equations for random media. Included in this study are a vector
description of clutter, doppler, image resolution, and contrast.
Modifications of the radiative transfer theory for high density media
and backscattering enhancement are also discussed.

Stokes Vector Radar Equations

A conventional radar equation is a scalar equation relating the
received power to the transmitting power. The polarization effects
are usually included in the form of a mismatch factor or polarization
loss factor. We extend the conventional radar equation to include the
partial coherence, the propagation effects, and the deterministic as
well as the statistical properties of the target. First the incident
flux density Stokes vector $[S_i]$ is defined using the Stokes vector
$[I_t]$.

$$[S_i] = \frac{G_t P_t}{4\pi R_1^2} [T_1] [I_t] \quad ,$$

where
$$[I_t] = \begin{bmatrix} I_{tv} \\ I_{th} \\ U_t \\ V_t \end{bmatrix} = \begin{bmatrix} \langle E_v E_v^* \rangle \\ \langle E_h E_h^* \rangle \\ 2\mathrm{Re}\langle E_v E_h^* \rangle \\ 2\mathrm{Im}\langle E_v E_h^* \rangle \end{bmatrix}$$

and E_v and E_h are the vertical and horizontal components of the normalized transmitted electric field. The 4 x 4 matrix $[T_1]$ represents the propagation characteristics of the medium from the transmitter to the target to be described in the next section. We then make use of the 4 x 4 Mueller scattering matrix $[\sigma]$ of the target and the 4 x 4 propagation matrix $[T_2]$ from the target to the receiver and obtain the following vector radar equation:

$$\frac{[P_r]}{P_t} = \frac{\lambda^2}{(4\pi)^3} \frac{G_t G_r}{R_1^2 R_2^2} [T_2][\sigma][T_1][I_t] \quad .$$

If the receiving antenna has the normalized transmitting Stokes vector $[I_r]$, then the received power P_r is given by

$$P_r = [\bar{I}_r]^* [P_r] \quad ,$$

where $[\bar{I}_r]^* = [I_{rv} I_{rh} U_r -V_r]$.

Propagation Effects in Random Scatterers

If the medium is not free space and contains random media such as turbulence and scatterers, then we need to consider the propagation of the Stokes vectors. The Stokes vector $[I]$ can be divided into the coherent vector $[I_c]$ and the incoherent (diffuse) vector (I_d). The propagation characteristics of the coherent Stokes vector $[I_c]$ is given by

$$\frac{d}{ds} [I_c] = - [\gamma][I_c] \quad ,$$

where $[\gamma]$ is the 4 x 4 extinction matrix which includes depolarization effects due to nonspherical particles. The extinction matrix is reduced to a scalar extinction coefficient for spherical particles. The diffuse vector $[I_d]$ satisfies the vector radiative transfer equation with the extinction matrix $[\gamma]$ and the Mueller matrix $[S]$.

$$\frac{d}{ds} [I] = - [\gamma][I] + \int[S][I'] \, d\omega' \quad .$$

Solutions of the vector radiative transfer equation are obtained for a limited number of cases including the plane wave incident on a medium containing axially symmetric particles and the first-order scattering solutions [4-9].

Stokes Vectors for Pulse and Doppler Shift

For a narrow band transient problem in random media, a cw formulation of the radiative transfer equation can be extended to include the pulse broadening and doppler shift. For a wide band problem, however, the problem must be formulated in terms of the two-frequency Stokes vector and the two-frequency radiative transfer equation. Some recent developments in this area will be discussed [14-16].

Vector Image Resolution and Contrast

The image resolution of the target is expressed in terms of the modulation transfer function of the random medium and the receiving aperture system [1,17]. We have conducted an experimental study of the modulation transfer function of a discrete scattering medium. The coherent component depends on the aperture size, and when the scattering optical depth is increased, the incoherent component becomes dominant. The cutoff spatial frequency for the modulation transfer function decreases with the optical distance if the particle sizes are greater than a wavelength, while the cutoff frequency is less affected for smaller particles.

Dense Media and Backscattering Enhancement

When the particle density exceeds about 1%, the particles can no longer be considered point scatterers, and the pair-correlation function needs to be included. The extinction coefficient then departs from the low density Foldy's formula. It decreases with the density if the particle sizes are small, while it increases slightly with the density if the particle sizes are large compared with a wavelength. Experimental and theoretical studies are presented to verify these results [18-19]. The backscattering from random scatterers is enhanced because of the multiple scattering effects giving rise to a sharp peak of the angular width of the order of $\rho\sigma_t\lambda$. The theoretical and experimental aspects of this enhancement will be discussed [20,21].

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Acknowledgment