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Method of Moment for Local Correction of Physical Optics

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1. Introduction

Diffraction analyses for electromagnetic waves are generally reduced to the problems to derive secondary sources on the scatterers. In low frequency, it's well known that Method of Moment (MoM) is effective and has high accuracy [1]. However, in high frequency, the size of matrix is so large that we have difficulties in obtaining the current distributions. On the other hand, when we deal with high frequency problems, we can use locality to approximate currents. Typical example of this technique is Physical Optics (PO) [2]. When using PO, we determine the currents at the point of interest by assuming the tangential infinite plane and independent from macroscopical shape of the scatterer; we assumed the "locality" of the phenomena. The accuracy of PO current is high except the edges, corners, and surfaces with small curvature; the errors are localized at these "critical regions".

In this report, we consider the minor and local corrections to PO where the currents only at the critical regions are derived by MoM after defining PO currents in other regions. Since the unknown currents are assigned not the entire but only the critical regions, computational load is not increasing so fast with the frequency[3]. The scattering from 2D corner reflector is analyzed by this method. We compare the necessary widths for MoM region at the center corner and the edge. It is found that the perturbation to PO is larger at the center corner and the MoM region should be wider there than in the edge.

2. Concept of Hybrid Method

Fig.1 shows the analysis model, 2D strip. We compare the analysis results of current distribution by PO and MoM for the model. The difference between PO current and the result of MoM is small in the center of the strip, but is very large near the edges because of the perturbation. From this phenomenon, we assume simple PO currents in the area where PO currents are accurate while we assign the unknown currents elsewhere, such as the edges, corners and curved surfaces, in order that we can reduce the calculation time without degrading the accuracy. Here, we show the mathematical derivation below.

In MoM, we divide the scatterer into many pieces and assume the unknown currents on each of them. Then we set up the matrix equation as regard to it and calculate the inverse matrix of this equation for the unknown currents.

$$[Z_{NN}][I_N]=[V_N] \tag{1}$$

$$\Rightarrow [I_N]=[Z_{NN}]^{-1}[V_N] \tag{2}$$

When executing the program, it takes too long time to calculate the inverse matrix in the high frequency. The size of the matrix in the equation (2) is $N \times N$. In proposed method, the unknown

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currents in the equation (1) are substituted by the PO currents, that is $2\boldsymbol{n}\times\boldsymbol{H}$, only in the areas where PO currents are reliable. If $M(<N)$ unknown currents out of total N are replaced by PO currents, we can get equation (3).

$$\left[Z_{N,N-M}\right]\left[I_{N-M}\right]=\left.\begin{bmatrix} V_1-\sum_i z_{1i}I_i^{PO} \\ \vdots \\ V_N-\sum_i z_{N,i}I_i^{PO} \end{bmatrix}\right\} N \tag{3}$$

where $\sum_i$ indicates the sum for $1\le i\le M$. These are N equations for $(N-M)$ of unknowns. Mathematically, we should approximately satisfy this by least-square method. But we have calculation errors when we use this method. The reason for this is as follows.

z_{mn}, which is in the left side of equation (3), is expressed as equations (4) and (5), when we put the parameters as Fig.3.

$$z_{mn}=-\int_{-w/2}^{w/2}H_0^{(2)}(\beta\left|\rho_m-\rho_n\right|)dx_s \tag{4}$$

$$\propto\frac{1}{\sqrt{\left(x_m-x_n\right)^2+\left(\frac{t}{2}\right)^2}}\qquad (t\ll\rho_m,\rho_n) \tag{5}$$

When we consider as $m\approx n$, which means that the equivalent source and the point of observation on the scatter is close each other, the value of z_{mn} becomes very large. On the other hand, if we calculate this equation in the whole part of the scatterer, or both the area for PO currents and that for the unknown currents, the change in the magnitude of z_{mn} is very large, which causes serious numerical errors. Therefore, we solve $(N-M)$'s equations only in the area for $(N-M)$ unknown currents.

$$\left[Z_{N-M,N-M}\right]\left[I_{N-M}\right]=\left.\begin{bmatrix} V_1-\sum_i z_{1i}I_i^{PO} \\ \vdots \\ V_N-\sum_i z_{N,i}I_i^{PO} \end{bmatrix}\right\} N-M$$

$$\Rightarrow\left[I_{N-M}\right]=\left[Z_{N-M,N-M}\right]^{-1}\begin{bmatrix} V_1-\sum_i z_{1i}I_i^{PO} \\ \vdots \\ V_N-\sum_i z_{N,i}I_i^{PO} \end{bmatrix}$$

The size of the matrix is reduced from $(N\times N)$ to $(N-M)\times(N-M)$, which enable us to ease the calculation load. As the frequency becomes higher, the critical region is much smaller than the whole area and $N-M<<N$, the reduction is significant.

3. Numerical results and discussion

We analyze the 2D corner reflector as is shown in Fig.4. We assign the unknown currents near edges and corners only where PO's calculation errors are notable as is shown in Fig.4. In this case, N=800 and M=320 are used. We show the analysis results of current distribution in Fig.5. Good agreement of hybrid method and MoM is seen everywhere on the scatters. The smooth switch of the two methods is also confirmed as shown in Fig.5-(b). Fig.6 is the radiation pattern of this model. We can observe good agreement of hybrid method and MoM, though result of PO is different from the others especially in shadow region of the scatterer.

As the area of PO currents is increased, the computation time is reduced but errors become large. On the contrary, as the area of it is small, errors become small but the calculation time is increased. Now, we consider the minimum width of MoM regions. Fig.7 shows the dependence of errors for different width of MoM regions. The errors between MoM and PO/MoM in radiation pattern, normalized by the incidence, are plotted as functions of width for MoM regions. When the dependence upon the width at the corner is calculated, the width of the MoM region from the edge is fixed at 2 λ , and vice versa. The minimum width for the acceptable error is larger at the central corner than around edge. This means that we should allocate more unknown currents around the corner.

4. Conclusions

In this report, local correction of Physical Optics by MoM is proposed. The satisfactory agreement is observed for 2D corner reflectors. The minimum width for MoM region is larger for the corner than the edge in this specific problem. The application to curved surfaces and three dimensional scatterers are important topics in the future.

References
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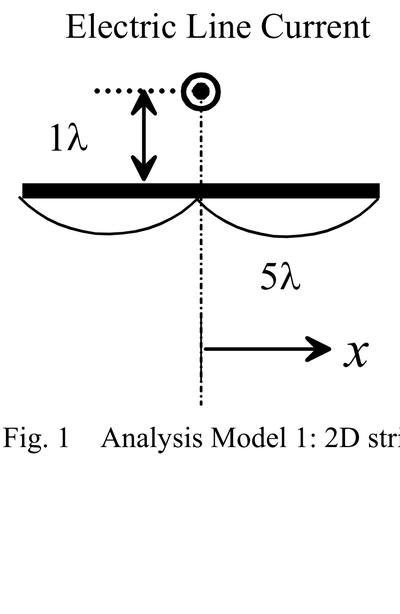

Fig. 1 Analysis Model 1: 2D strip

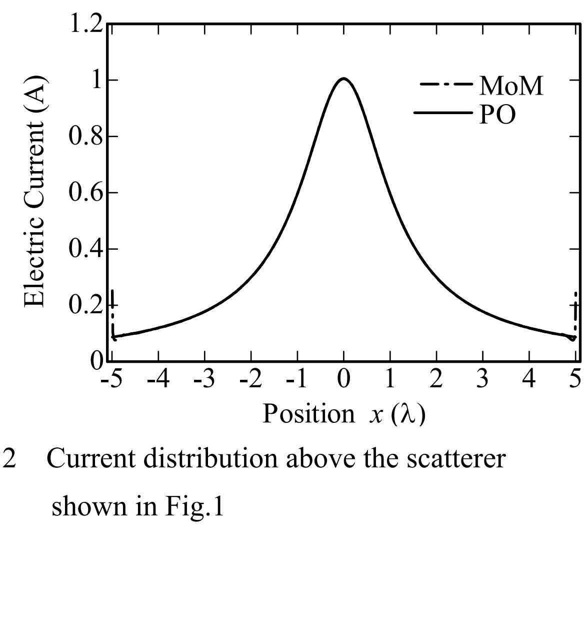

Fig. 2 Current distribution above the scatterer shown in Fig.1

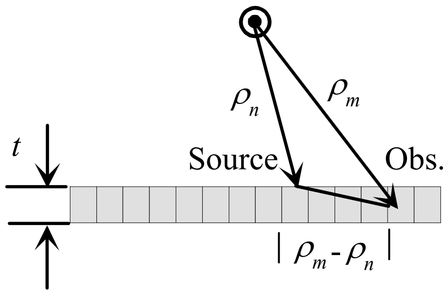

Fig. 3 Geometry of a line source above a 2D finite width strip

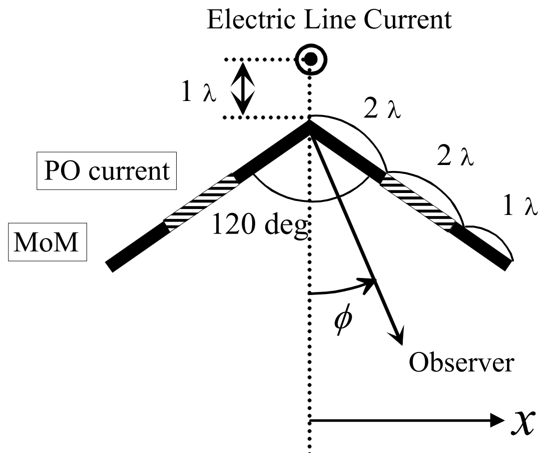

Fig. 4 Analysis Model 2: 2D corner reflector

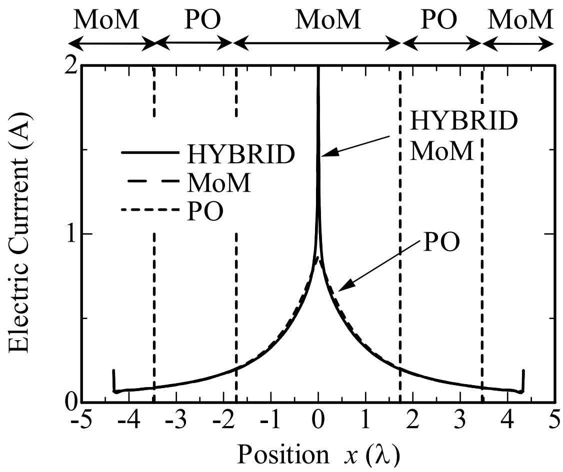

Fig.5-(a) Current distribution on 2D corner reflector

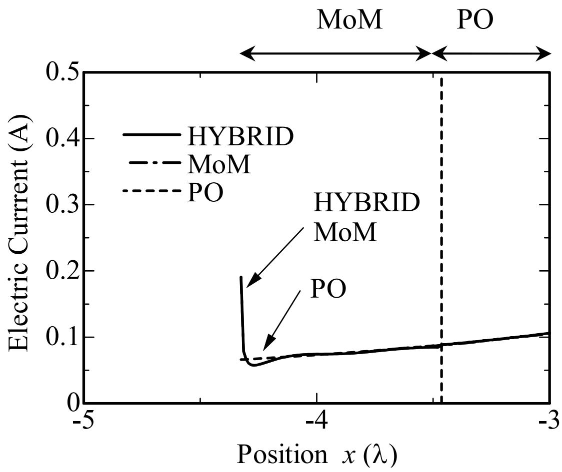

Fig.5-(b) Enlarged figure of Fig5-(a).

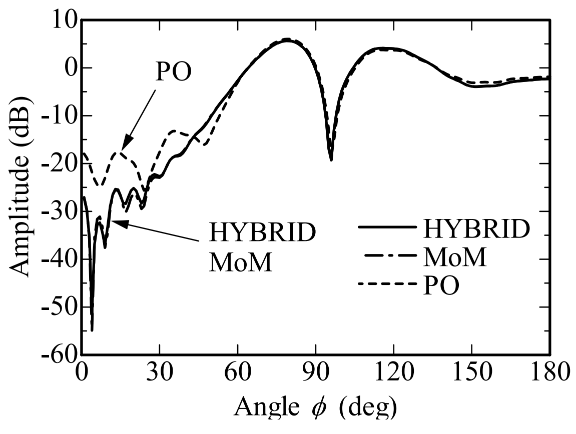

Fig.6 Radiation pattern for 2D corner reflector

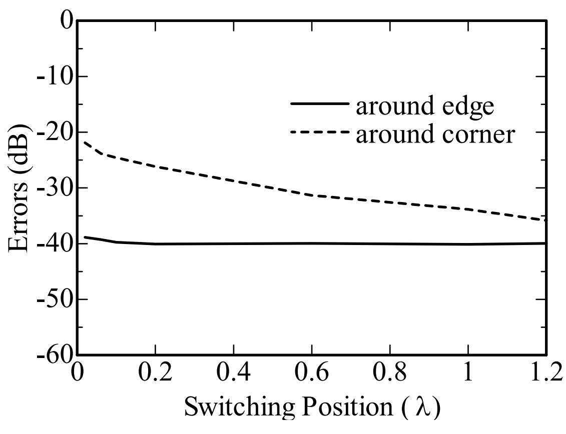

Fig.7 Width of MoM region in Hybrid Method