# FDTD ANALYSIS OF NONLINEAR DIELECTRIC WAVEGUIDE WITH RECTANGULAR GRATING STRUCTURE 

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## 1 Introduction

For high bit-rate data transmission in optical communication, wavelength division multiplexing(WDM) technique is considerably studied and has been introduced to actual systems. To obtain higher bit-rate, pulse amplitude modulation with multi intensity levels may be one of the solutions. Of course, the nonlinearity in optical fiber cable must be suppressed, as well as the compensator and repeater which are independent to the signal amplitude will also be required. In such a communication system, the dividing switch to drop off the desired intensity signal will be needed.

In the past decade, the Finite-Difference Time-Domain(FDTD) method[1] for dispersive material have been proposed by some authors [2]-[10]. These works enables us to visualize the dynamic behavior of electromagnetic wave in linear and nonlinear dielectric media. However, the most of the reported examples treated one-dimensional (1D) problems and the size of computational region was small in comparison with the electromagnetic wavelength, mostly due to the lack of large computer memory and high-speed CPU. Refs.[2]-[6] and [8]-[10] treated the 1D problem. In Ref.[7], two-dimensional(2D) corrugated waveguide was analyzed but the computational region was small $(1.5 \mu \mathrm{~m} \times 1.8 \mu \mathrm{~m})$. Sullivan formulated the FDTD scheme utilizing Z-transform technique[8]-[10] to include linear and nonlinear dispersions for 1D problem. Although some static analysis of nonlinear waveguide by the finite element method or the beam propagation method were reported, the dynamic behavior of optical wave in 2D waveguide is not known. These FDTD formulations are extendable from 1D to 2D problems.

In this paper, a switch for multi-amplitude optical pulse communication systems is proposed and analyzed by FDTD method. The switch is composed of nonlinear dielectric material and rectangular grating structure. We used Sullivan's Z-transform formulation[8]-[10] for linear dispersion, Raman scattering and Kerr effect. Some of the radiation characteristics are shown and the possibility as a dividing switch is discussed.

## 2 Finite-Difference Time-Domain Method

According to Yee's FDTD scheme[1], the two dimensional (i.e. $\partial / \partial y=0$ ), discretized Maxwell's equations for transverse electric (TE) mode, which propagates to $z$ axis, are described as follows;

$$
\begin{align*}
H_{x}^{n+1 / 2}\left(i+\frac{1}{2}, k\right)=H_{x}^{n-1 / 2}\left(i+\frac{1}{2}, k\right)+\frac{\Delta t}{\mu_{0} \Delta z} \cdot & {\left[E_{y}^{n}\left(i+\frac{1}{2}, k+\frac{1}{2}\right)-E_{y}^{n}\left(i+\frac{1}{2}, k-\frac{1}{2}\right)\right], }  \tag{1}\\
H_{z}^{n+1 / 2}\left(i, k+\frac{1}{2}\right)=H_{z}^{n-1 / 2}\left(i, k+\frac{1}{2}\right)-\frac{\Delta t}{\mu_{0} \Delta x} \cdot & {\left[E_{y}^{n}\left(i+\frac{1}{2}, k+\frac{1}{2}\right)-E_{y}^{n}\left(i-\frac{1}{2}, k-\frac{1}{2}\right)\right], }  \tag{2}\\
D_{y}^{n+1}\left(i+\frac{1}{2}, k+\frac{1}{2}\right)=D_{y}^{n}\left(i+\frac{1}{2}, k+\frac{1}{2}\right)+\Delta t \cdot & {\left[\frac{H_{x}^{n+1 / 2}\left(i+\frac{1}{2}, k+1\right)-H_{x}^{n+1 / 2}\left(i+\frac{1}{2}, k\right)}{\Delta z}\right.} \\
& \left.-\frac{H_{z}^{n+1 / 2}\left(i+1, k+\frac{1}{2}\right)-H_{z}^{n+1 / 2}\left(i, k+\frac{1}{2}\right)}{\Delta x}\right], \tag{3}
\end{align*}
$$

where $E$ and $H$ are electric and magnetic field and $D$ is the electric flux density. The space and time discretization steps are $\Delta x, \Delta z, \Delta t$ and $i, k, n$ are the number of the corresponding grid. Giving a set of suitable initial conditions and a constitutive equation between $D$ and $E$ to above equations, the latest field is calculated successively as the increase of time step number.

We consider lossless, isotropic and dispersive nonlinear dielectric material. The electric field, the polarization terms and the flux density of the material holds the following relation;

$$
\begin{equation*}
\varepsilon_{0} \varepsilon_{\infty} E_{y}=D_{y}-P_{L}-P_{R}-P_{K}, \tag{4}
\end{equation*}
$$

where $\varepsilon_{0}$ and $\varepsilon_{\infty}$ are permittivity of free space and the saturated value of the relative constant in the limit that frequency is infinity. The frequency dependent polarization terms $P_{L}, P_{R}$, and $P_{K}$ are linear polarization, Raman scattering, and Kerr effect, respectively. The permeability is assumed to be nondispersive constant and equal to that of free space $\mu_{0}$ in the whole region. As discretizing eq.(4) with respect to time by Z-transform technique[8]-[10], it can be evaluated by using latest and second-latest electromagnetic field components.

## 3 Numerical Results

In this section, we demonstrate the optical wave propagation in the waveguide illustrated in Fig. 1. The FDTD analysis region is surrounded by Mur's absorbing boundary[11] and perfectly matched layer[12]. We also evaluated the radiated field pattern on the circle whose center locates on grid $(116,2500)$ with diameter $130 \mu \mathrm{~m}$ using outermost electromagnetic field components in FDTD analysis region. Nonlinearity is assumed only in core region with $\chi_{0}(3)=$ $0.07[\mathrm{~m} / \mathrm{V}]^{-2}$ and linear dispersion exsists in entire region. The fundamental TE mode of linear waveguide is given at input and propagates to $+z$ direction. The wavelength $\lambda=1.5[\mu \mathrm{~m}]$, $\varepsilon_{\infty, \text { core }}=2.25, \varepsilon_{\infty, s u b s t .}=\varepsilon_{\infty, \text { cover }}=2.0$ are used. The other parameters are same as Ref.[13]'s.

Fig. 2 (a) and (b) show the radiated electric field in time domain and Fourier transformed domain using FFT, respectively. The amplitude of incident electric field $E_{y, i n}=0.4[\mathrm{~V} / \mathrm{m}]$ and the observation time interval is from 17953 to $20000 \Delta t$ ( 2048 time steps). The observing point locates on the circle and on $-x$ axis in fig 1. The time domain wave form shows periodicity, however, it is found that the spectrum includes small amount of second harmonics.

Fig. 3 shows the radiation pattern for $E_{y, \text { in }}=0.4[V / m]$ (solid line) and $0.1[V / \mathrm{m}]$ (dashed line). These patterns do not correspond due to the nonlinear refractive index modulation caused by optical field. The radiation efficiencies of pumping wave $(\lambda=1.5 \mu \mathrm{~m})$ and secondharmonic wave $(\lambda / 2)$ are plotted in Fig. 4. The efficiency in each angle is normalized by total radiated power. The radiation efficiency of second harmonics increases in some angles as $E_{y, \text { in }}$ increases, while that of pumping wave shows relatively small change. However, except several radiation angles, the intensity of second harmonics can be almost negligible with respect to the pumping wave.

Fig. 5 indicates a relation between maximum radiated power and $E_{y, \text { in }}$ by circle. The angle of maximum radiation is also plotted by x . We can find that the radiation power increase linearly as the increase of $E_{y, i n}$, while the angle of maximum radiation shows a rapid change around $E_{y, i n}=0.25[\mathrm{~V} / \mathrm{m}]$. Fig. 6 shows the extinction ratio evaluated by total radiation pattern of $E_{y, \text { in }}=0.1$ and $0.4[\mathrm{~V} / \mathrm{m}]$ in each angle as follows;

$$
10 \log _{10} \frac{P_{1}\left(E_{y, \text { in }}=0.4, \theta\right)}{P_{2}\left(E_{y, \text { in }}=0.1, \theta\right)}
$$

where $P_{1}$ and $P_{2}$ are radiated power to the angle $\theta$. Around 60 and 260 degree, the extinction ratio over 40 dB is observed, while the extinction less than -10 dB is obtained around 100 and 350 degree. This result means that we can distinguish the optical pulse signals with different amplitude by receiving the radiation in respective angles.

## 4 Concluding Remarks

From the numerical results in the previous section, it is able to make use of the nonlinear dielectric waveguide with grating structure as an optical switch for multi-amplitude pulse communication systems. As achieving the switching characteristics using nonlinear dielectric, the radiation of second harmonic wave will be negligibly small with respect to signal (or pumping) wave. Selection of the material, fabrication of the device, and application to actual communication system will be future subjects.

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Figure 1: Illustration of nonlinear dielectric waveguide with rectangular grating structure.

(a) Time domain.

(b) Spectral domain.

Figure 2: Radiated electric field in time domain (a) and spectral domain (b) which is observed from $-x$ directtion for $E_{y, i n}=0.4[\mathrm{~V} / \mathrm{m}]$.


Figure 3: Radiation pattern for $E_{y, \text { in }}=0.4$ and $0.1[\mathrm{~V} / \mathrm{m}]$.


Figure 4: Radiation efficiency of the incident wave and second harmonic wave for $E_{y, \text { in }}=0.4$ and $0.1[\mathrm{~V} / \mathrm{m}]$.


Figure 5: The maximum radiated power and its radiation angle as functions of incident electric field $E_{y, \text { in }}$.


Figure 6: The extinction ratio evaluated by making use of radiated power for $E_{y, \text { in }}=0.4$ to $0.1[\mathrm{~V} / \mathrm{m}]$ as a function of radiation angle.

