Efficient Interpolation of Characteristic Modes

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***Abstract* – Characteristic modes (CMs) provide physical insights for antenna design but it is computationally intensive. In this contribution, an interpolation scheme is proposed for frequency sweep of CMs for the first time. Orthogonality of the eigeicurrents is checked using interpolated impedance matrix. Two numerical examples are provided for verification.**

***Index Terms* — Eigencurrent, eigenvalue, interpolation.**

1. Introduction

Characteristic mode analysis (CMA) was developed in 1970s [1] and found lots of success in the last decade for antenna design, such as the PIFA [2], wire antenna [3], blade antenna [4], and notch array [5]. However, CMA of large platforms (e.g. automobile, aircraft, ship) is limited to low frequencies like 100 MHz as the computational complexity is cubically proportional to the number of unknowns [4][6].

Interpolation methods have been widely used to evaluate the impedance matrix [*Z*] of the method of moments (MoM) in a large frequency band [7]-[9]. As the CMA shares similar procedures with the MoM, it is a natural step to apply well-defined interpolation to the CMA as presented in this paper. Moreover, as the eigencurrents of small radiators vary slowly with frequency [10], interpolation of the eigencurents may be more stable than the impedance matrix.

2. Interpolation Method of Characteristic Modes

(1) Computation of Characteristic Modes

Characteristic modes of a conducting object are computed in the framework of MoM by solving

$$[X]\mathbf{J}_n = \lambda_n [R]\mathbf{J}_n \,, \tag{1}$$

where [*X*] and [*R*] are the imaginary and real parts of the impedance matrix [*Z*], respectively. $\mathbf{J}_n$ is the *n*th order eigencurrent and λ_n is the corresponding eigenvalue.

(2) Proposed Interpolation Method

Interpolation of $\mathbf{J}_n$ and λ_n can speed up the computation within a wide frequency band. Meanwhile the impedance matrix [*Z*] is also interpolated for testing the orthogonality of $\mathbf{J}_n$. The interpolation methods are described as followed.

Interpolation of [*Z*]

Since interpolation of [*Z*] was intensively studied in [7]-[9], standard procedure is followed. Several discrete frequencies are selected and the interval between adjacent frequencies is determined by [7]

$$\Delta f_m = c/(2L) \,, \tag{2}$$

where *L* is the largest dimension of the radiator and *c* is the speed of light. Elements of *[Z]* at intermediate frequencies are approximated by [8]

$$Z_{mn}(f) = \frac{a_{0,mn} + a_{1,mn} f + \cdots a_{p,mn} f^p}{1 + b_{1,mn} f + \cdots b_{d,mn} f^d} \,, \tag{3}$$

where *f* is the frequency, *p* is the order of numerator polynomial, and *d* is the order of denominator polynomial. The coefficients are extracted by solving the matrix equation

$$[A \quad -B][C] = [D] \,, \tag{4}$$

where $[A] = [[1\cdots1]^T_{1\times k} \quad [f_1 \cdots f_k]^T_{1\times k} \quad \cdots \quad [f_1^{\,p} \cdots f_k^{\,p}]^T_{1\times k}]$, $[B]$=$[[H]^1 \quad \cdots \quad [H]^d], [H]^v = [Z(f_1)\cdot f_1^v \cdots Z(f_k)\cdot f_k^{\,v}]^T_{1\times k}$ $[C] = [\mathbf{a}\,\mathbf{b}]^T$, $\mathbf{a}$=$[a_0 \; a_2 \cdots a_p]^T$, $\mathbf{b}$=$[b_1 \; b_2 \cdots b_d]^T$, and $[D] = [Z(f_1) \quad \cdots \quad Z(f_k)]^T$.

To get a better condition number for the matrix in (4), the frequency is normalized by $f = f_{\text{original}}/f_{\max}$. In addition, for electrically large objects, the interpolation accuracy can be improved using following modification [7]

$$Z'_{mn} = Z_{mn} / e^{-jkR_{mn}} \,. \tag{5}$$

Interpolation of $\mathbf{J}_n$ and λ_n

The function in (3) is used for the interpolation of $\mathbf{J}_n$ and λ_n. The polynomial coefficients are computed using (4) with a normalization of the frequency by $f = f_{\text{original}}/f_{\max}$.

Orthogonality Check

Numerical quality of the interpolated $\mathbf{J}_n$ can be evaluated by the orthogonality between different order modes as [1]

$$\text{Orthogonality Index} = \mathbf{J}_m^T \cdot [R]\mathbf{J}_n \,. \tag{6}$$

Note that the interpolated [*Z*] is used in (6). The index should be close to 0 for two different modes and approach one for the same modes. If the orthogonality check fails at certain frequencies, a higher order of polynomial in (3) will be used to improve the index. In addition, if the mode tracking of $\mathbf{J}_n$ fails at certain frequencies [11], the frequency interval is reduced accordingly. Therefore, the interpolation method is performed in an adaptive way.

3. Numerical Results

The [*Z*] has been generated by a MATLAB code [12] that was developed by Makarov [13] and the characteristic modes are computed using built-in functions (e.g. eigs) in MATLAB. To verify the proposed method, a rectangular plate and a missile has been studied as shown in Fig. 1.

(1) Rectangular Plate

The plate has been meshed and results in 1594 unknowns in MoM. The frequency band of interest is from 820MHz to 4820MHz and the lowest ten modes are calculated. The characteristic modes are first computed at five frequencies with an interval of 1000 MHz. 18 frequencies are adaptively inserted for the interpolation until passing the orthogonality check. The interpolated results have been validated at 21 frequencies with a uniform interval of 200 MHz as shown in Fig. 2 and Fig. 3. Good agreements between the exact and interpolated results are observed for this simple example.

(2) Missile

The missile has been meshed and results in 2076 unknowns in MoM. The frequency band of interest is from 50MHz to 300MHz and the lowest seven modes are calculated. First, six frequencies at every 50MHz are chosen for [*Z*] interpolation. The performance has been validated at 16 frequencies with equal step as shown in Fig. 4 and Fig. 5. Fairly well agreements between the interpolated and exact results are obtained.

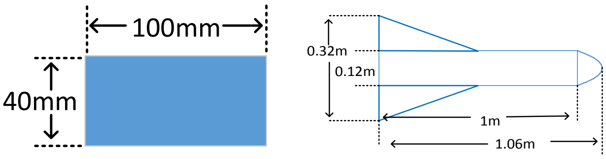

Fig. 1. The geometry of the plate and missile

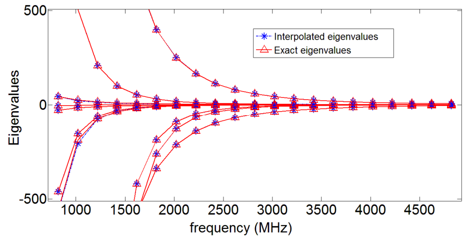

Fig. 2. The results of interpolated and exact eigenvalues

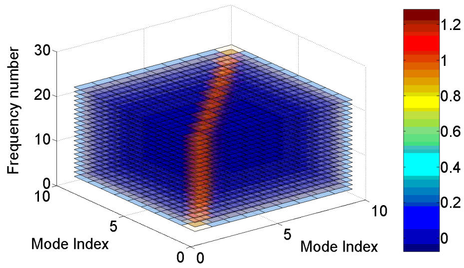

Fig. 3. The orthogonality index of the eigencurrents

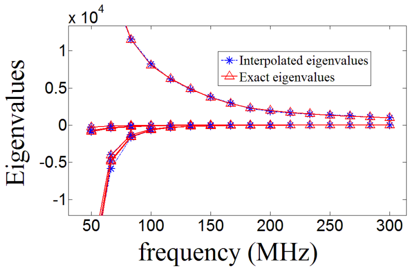

Fig. 4. The results of interpolated and exact eigenvalues

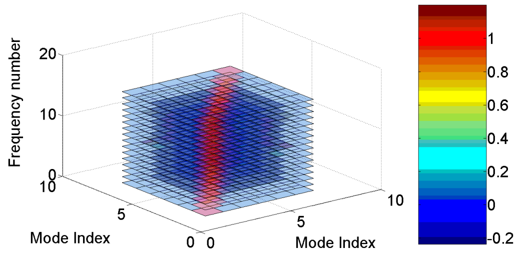

Fig. 5. The orthogonality index of modes at each frequency

4. Conclusion

In this paper, an interpolation method for broadband evaluation of characteristic modes was presented. The impedance matrix is interpolated to check the orthogonality of interpolated eigencurrents. Some sampling frequencies are added if the orthogonality check or the mode tracking fails. Two examples are calculated for validation purpose. Orthogonality index of some interpolated eigencurents is not strictly within zero to one. This deficiency may be caused by the numerical errors of computed [*Z*] in MoM and more investigations are required to locate and solve the problem.

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