Relations between the Characteristic Modes(CMs) and the X Modes(XMs)

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Abstract – **The differences between the characteristic modes(CMs) and X modes(XMs) are pointed out then the relations between these two types of modes are discussed through an example of a thin strip dipole. Finally it is concluded that the resonant information of an antenna, including resonant frequency and resonant current, exists only in the imaginal part of the impedance operator (reactance operator) of the antenna.**

Index Terms **—Characteristic Modes (CMs), X Modes (XMs), relations, impedance operator, antenna theory.**

1. Introduction

The theory of characteristic modes (TCM) was first proposed by Garbacz and Turpin in [1]. Then Harrington and Mautz refined the theory under the help of electric field integral equation and they obtained the most common format we know now. In [2] and [3] they connected the CMs with the electric field integral operator Z (also called the impedance operator since it has the dimension of impedance) and proposed a numerical method in general to calculate the CMs. The application of TCM in modern antenna design is illustrated in [4]. It emphasized that any antenna has a set of orthogonal CMs which only relate to the impedance operator. The real radiation field of the antenna can be represented by the superposition of the associated modal fields. In addition to the conventional CMs, three new types of modes were proposed in [5] and they are also only related to the impedance operator as CMs acts. But until we write this paper, it seems that the effects of the impedance operator on the aforementioned modes have not been systematically studied. In this paper, the relation between CMs and XMs are discussed first. Basing this, we study the real part and the imaginal part of the impedance operator and their different contributions to the antennas. In the following sections, these two parts will be called the resistance and the reactance operator, respectively, for brevity.

2. CMs and XMs

CMs are defined by the following generalized eigenvalue equation:

$$XJ_n^{CM} = \lambda_n^{CM} RJ_n^{CM} \tag{1}$$

where J_n^{CM} represents the n_{th} CM, R and X represent the resistance and reactance operator, namely

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$$R = (Z + Z^*)/2, X = (Z - Z^*)/2j \tag{2}$$

To ensure the uniqueness, CMs are normalized as

$$\left\langle J_m^{CM}, RJ_n^{CM} \right\rangle = \delta_{mn} \tag{3}$$

$$\left\langle J_m^{CM}, XJ_n^{CM} \right\rangle = \delta_{mn}\lambda_n^{CM} \tag{4}$$

XMs are defined by the following eigenvalue equation:

$$XJ_n^{XM} = \lambda_n^{XM} J_n^{XM} \tag{5}$$

where J_n^{XM} represents the n_{th} XM.

To ensure the uniqueness, XMs are normalized as

$$\left\langle J_m^{XM}, J_n^{XM} \right\rangle = \delta_{mn} \tag{6}$$

$$\left\langle J_m^{XM}, XJ_n^{XM} \right\rangle = \delta_{mn}\lambda_n^{XM} \tag{7}$$

$J_n^{CM}, J_n^{XM}, \lambda_n^{CM}$ and λ_n^{XM} are all real since both R and X are real symmetric operators.

We consider the frequency and current distribution where an antenna stores null energy as the resonant frequency and the resonant current of the antenna. Notice that an antenna's stored energy can be evaluated by

$$Antenna\ Stored\ Energy = \left\langle J, XJ \right\rangle / 2\omega \tag{8}$$

Combining (4)(7), we can know that for both CMs and XMs, the frequency and modal current where the eigenvalue $\lambda = 0$ can be seen as the antenna's resonant frequency and resonant current aforementioned, respectively.

Different from CMs, only the reactance operator is used to figure out XMs. This fact makes it possible for us to distinguish the different impacts of resistance and reactance operator on an antenna.

To figure out the CMs and XMs, we use RWG functions [6] as the basis and test functions in order to convert the aforementioned operator equations into matrix equations:

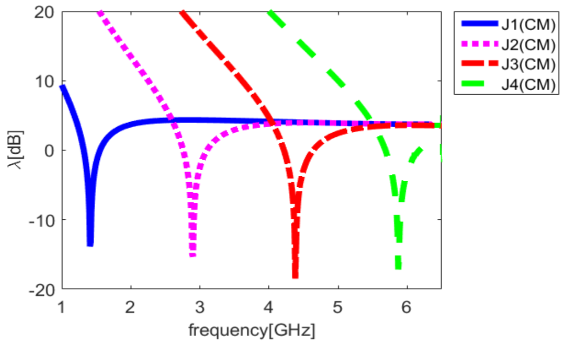
$$\left[X\right]\left[I_n^{CM}\right] = \lambda_n^{CM}\left[R\right]\left[I_n^{CM}\right] \tag{9}$$

$$\left[X\right]\left[I_n^{XM}\right] = \lambda_n^{XM}\left[I_n^{XM}\right] \tag{10}$$

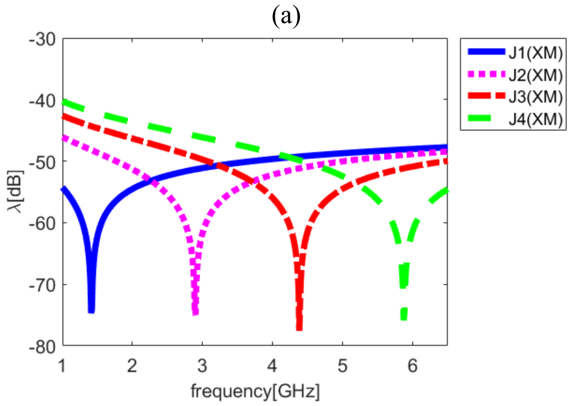
3. Relations between CMs and XMs of a thin strip dipole

In this section we illustrate the relations between CMs and XMs by comparing these two types of modes of a thin strip dipole. Having a length of $L = 100mm$ and a width of $W = 1mm$, this dipole is meshed with about 200 triangles.

Fig.1 shows the variation with frequency of the eigenvalues of the first four CMs and XMs. It can be found that CMs and XMs have the same modal resonant frequencies which locate at 1.4GHz, 2.9 GHz, 4.4 GHz and 5.9 GHz, respectively. In other words, both CMs and XMs contain the resonant frequency information of the dipole. It should be noted that the eigenvalue in logarithm scale are using $10\lg(|\lambda|)$ since eigenvalue has a meaning of power.



(a)



(b)

Fig. 1.The first four (a)CMs and (b)XMs

In spite of their very different magnitudes, the normalized modal current distributions of CMs and XMs are found to be nearly the same at each mode's resonant frequency, as shown in Fig.2.

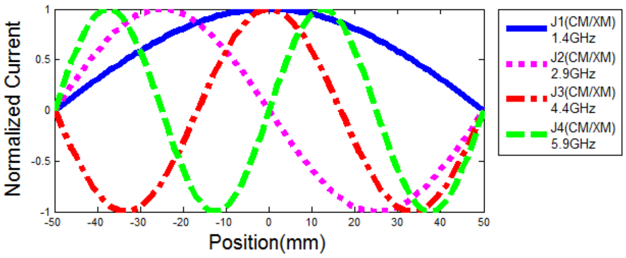


Fig. 2.The modal current distributions of CMs/XMs

To further study how CMs resemble XMs in a quantitative way, it seems to be beneficial to use the correlation coefficient (CC) which is defined as

$$corr(\boldsymbol{I}_n^{CM},\boldsymbol{I}_n^{XM})=\frac{\left|\sum_i (I_{n,i}^{CM}-\bar{\boldsymbol{I}}_n^{CM})(I_{n,i}^{XM}-\bar{\boldsymbol{I}}_n^{XM})\right|}{\sqrt{\sum_i (I_{n,i}^{CM}-\bar{\boldsymbol{I}}_n^{CM})^2\sum_j (I_{n,j}^{XM}-\bar{\boldsymbol{I}}_n^{XM})^2}} \qquad (11)$$

$$0\le corr(\boldsymbol{I}_n^{CM},\boldsymbol{I}_n^{XM})\le 1 \qquad (12)$$

From (11), it can be seen that the CC is normalized implicitly and it does not depend on the magnitude of the modal current of CM or XM. Then the CCs of CMs and XMs are computed according to (11) and they are found all equal to 1 as exhibited in Table I. This fact means that the modal currents of CM and XM are really the same at every resonant frequency. It also tells us both CMs and XMs contain the resonant current information of the dipole.

TABLE I
Correlation Coefficients of CMs and XMs

CM,XM	$\boldsymbol{I}_1^{CM},\boldsymbol{I}_1^{XM}$	$\boldsymbol{I}_2^{CM},\boldsymbol{I}_2^{XM}$	$\boldsymbol{I}_3^{CM},\boldsymbol{I}_3^{XM}$	$\boldsymbol{I}_4^{CM},\boldsymbol{I}_4^{XM}$
FRE	1.4GHz	2.9GHz	4.4GHz	5.9GHz
CORR	1	1	1	1

Now we can discuss the different effects of R and X operators. Looking back Fig.1, we see that although without using R operator at all, XMs still carry the dipole's resonant information as CMs do. The reasonable explanation is that only the X operator contains the dipole resonant information. But this doesn't deny the significance of R operator for an antenna. In fact, its functions are required further study.

It should be noted although we only consider the first four CMs and XMs in the above analysis for brevity, the relations between higher order CMs and XMs and the discussions above still remain to be true.

4. Conclusion

In order to distinguish the different contributions of R and X operator to antennas, the XMs have been proposed. By comparing XMs with the conventional CMs, it has been found that both these two types of modes carry the same resonant information, including resonant frequency and resonant current of antennas. We have drawn a conclusion that the resonant information of an antenna only exists in the reactance operator but remain the resistance operator to be further studied in future.

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